PROMOTING PRODUCTIVE MATHEMATICAL DISCOURSE: TASKS IN COLLABORATIVE DIGITAL ENVIRONMENTS

Arthur B. Powell
Rutgers University-Newark
powellab@andromeda.rutgers.edu

Muteb M. Alqahtani
Rutgers University-New Brunswick
muteb.alqahtani@gse.rutgers.edu

Tasks can be vehicles for productive mathematical discussions. How to support such discourse in collaborative digital environments is the focus of our theorization and empirical examination of task design that emerges from a larger research project. We present our task design principles that developed through an iterative research design for a project that involves secondary teachers in online courses to learn discursively dynamic geometry by collaborating on construction and problem-solving tasks in a cyber learning environment. In this study, we discuss a task and the collaborative work of a team of teachers to illustrate relationships between the task design and productive mathematical discourse. Implications suggest further investigations into interactions between characteristics of task design and learners mathematical activity.

Keywords: Technology; Geometry; Teacher Education-Inservice; Classroom Discourse

Mathematical tasks are central as they convey what mathematics is and what it means to do mathematics. Sierpinska's (2004) considers that "the design, analysis, and empirical testing of mathematical tasks, whether for purposes of research or teaching, is one of the most important responsibilities of mathematics education" (p. 10).Mathematical tasks shape significantly what learners learn and structure their classroom discourse (Hiebert & Wearne, 1993).Such discussions when productive involve essential mathematical actions and ideas such as representations, procedures, relations, patterns, invariants, conjectures, counterexamples, and justifications and proofs about objects and relations among them. Nowadays, these mathematical objects and relations can be conveniently and powerfully represented in digital environments, and many contain functionality for collaboration. However, for such collaborative, digital environments, the design of tasks that promote productive mathematical discussions is an enduring challenge and requires continued theorization and empirical examination (Margolinas, 2013). In this brief research report, guided by the question—What features of tasks support productive mathematical discourse in collaborative, digital environments?—we articulate theoretical and practical principles for designing such tasks for small teams of individuals working in online, collaborative environments.

Our work employs a specific online environment that supports synchronous collaboration and discussions and provides tools for creating graphical and semiotic objects for doing mathematics. The environment, Virtual Math Teams (VMTwG), has a multiuser version of a dynamic geometry environment, GeoGebra. Here, we first indicate our task design to promote potentially productive mathematical discourse among small groups of learners working in VMTwG. Afterward, we present an example of a task along with the mathematical insights a small team of teachers developed discursively as they engaged with it. We conclude with implications and suggestions areas for further research.

Task-Design Principles

Our principles of task design embody particular intentionalities for a virtual synchronous, collaborative environment that has representation infrastructures (GeoGebra) and communication infrastructures (social network and chat features). The intentions are for mathematical tasks to be vehicles "to stimulate creativity, to encourage collaboration and to study learners' untutored, emergent ideas" (Powell et al., 2009, p. 167) and to be sequenced so as to influence the coemergence of learners instrumentation and building of mathematical ideas. To these ends, sensitive to the infrastructural features of VMTwG, we developed and tested the following seven design principles for digital tasks that are intended to promote productive mathematical discourse by encouraging collaboration in such environments:

- Provide a pre-constructed figure, instructions for constructing a figure, or invitation to construct a figure with particular properties.
- Invite participants to interact with a figure by looking at and dragging objects to notice how the objects behave, relations among objects, and relations among relations.
- Invite participants to reflect on the mathematical meaning of what they notice.
- Invite participants to wonder or raise questions about their noticings and meanings.
- Pose suggestions as hints or new challenges that prompt participants to notice particular objects, attributes, or relationships.
- Provide formal mathematical language that corresponds to awarenesses that they are likely to have explored and discussed or otherwise realized (Hewitt, 1999, 2001).
- Respond with feedback based on participants' work that pose new situations as challenges that extend what participants noticed, wondered, or constructed; invite participants to revisit a task, to generalize noted relationships and to construct justifications and proofs of conjectures; suggest that participants consider the attributes of a situation (theorem, figure, actions such as drag) in order to generate a "what if?" question and explore it.

The hints aim to maintain learners' engagement with a task and the challenges to encourage them to extend what they know. The hints support participants' discourse by eliciting from them statements that reveal what they observe and what mathematical sense or meanings they make of their observations. The challenges provide new, related situations to investigate. Hidden initially, the hints and challenges can be revealed by learners clicking a check box.

These design principals guided how we developed tasks in our research project, a collaboration among investigators at Rutgers University and Drexel University. VMTwG records users' chat postings and GeoGebra actions. The project participants are middle and high school teachers in New Jersey who have little to no experience with dynamic geometry environments and no experience collaborating in a virtual environment to discuss and resolve mathematics problems. The teachers took part in a semester-long professional development course. They met for 28 two-hour synchronous sessions in VMTwG and worked collaboratively on 55 tasks. In the next section, we provide examples of two tasks to illustrate the actualization of our task design.

Task Example and Analysis

Based on our design principles, we developed dynamic-geometry tasks that encourage interlocutors of a team to discuss and collaboratively manipulate and construct dynamic-geometry objects, notice dependencies and other relations among the objects, discuss meanings and wonderings associated to their noticings.

During a collaborative session, a team of three teachers worked on identifying the dependencies involved in the constructions of different triangles (see Figure 1). The team dragged the triangles vigorously to explore dependencies among objects in the task. The vertices of first triangle, ABC, were constructed as independent objects, so the team did not belabor discussing it. The second figure is an isosceles triangle DEF. The lengths of DE and DF are equal. Point F is constrained to a hidden circle with radius DE. Points D and E are independent objects. In the following excerpt, concerning the second figure the Team discusses their noticings and the meaning they derive from them:

386 ceder: so in the second one, f is dependent on g
387 ceder: I mean d

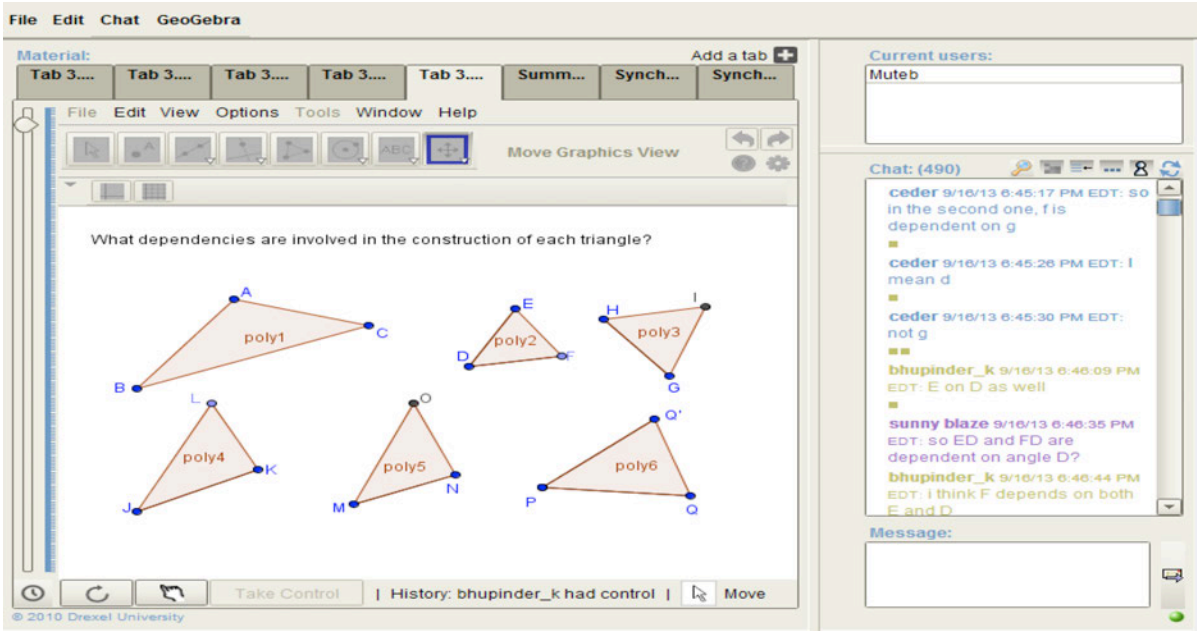

Figure 1: Dynamically examining dependences among different triangles

388 ceder: not g
389 bhupinder_k: E on D as well
390 sunny blaze: so ED and FD are dependent on angle D?
391 bhupinder_k: i think F depends on both E and D
392 ceder: f doesnt look dependent on anything now...am I imissing something?
393 ceder: ok, what am I missing? F can move independently, but when E is moved, F moves, so that makes which one dependent?
394 bhupinder_k: when you move F, ED stays fixed
395 ceder: right, so F is free to move anywhere
396 ceder: but not when E is moved
397 ceder so F is sometimes dependent?

The team discusses dependencies among points, segments and angles. In lines 386 to 388, ceder states that F is dependent on D then dismisses her assertion in line 392. In line 390, sunny blaze asks whether the two line segments are dependent on angle D. Though, week before, the team had already seen and constructed dependent objects, they struggled with this new, more complex situation. The idea of dependency is key and permits interlocutors to identify and build relationships in dynamic constructions. In a latter task, the team uses the concept of dependency to identify relations among objects. The task presents two circles constructed using the same radius, AB, where each endpoint is a center of a circle. Their points of intersections, C and D, were connected to create a perpendicular bisector to radius AB that intersects it at point E. In the session, one teacher states that points C, D, and E are dependent on A and B. Another teacher states that the two circles share the same radius and that dragging the center of one circle affects the size of the other, which makes the circles dependent on the centers. The teachers appropriate the idea of dependency and use it to understand components of the task's constructions.

In these two tasks, the teachers attended to the co-active feedback of the environment to their actions, which enabled them to appropriate the concept of dependency. For the team of teachers, engaging with tasks where dependencies are key relations among geometrical objects was an important step. These tasks triggered a discussion about how to use dependency to create valid geometric constructions.

Discussion and Conclusion

Our focus was to describe how we address task design challenges to promote productive mathematical discourse in an online synchronous environment. In the virtual environment, a teacher or facilitator is present largely as an artifact of the environment's digital tools and most specifically in the structure and content of tasks. An important feature of our task design are principles 2 to 4 since when collaborating interlocutors respond to those prompts they generate propositional statements that can become the focus of their discussions. Their discussions are mathematically productive as their noticings, statements of meaning, and wonderings involve interpretations, procedures, patterns, invariants, conjectures, counterexamples, and justifications about objects, relations among objects, and dynamics linking relations.

Our task-design principles aim to engage interlocutors in productive mathematical activity by inviting them to explore figures, notice properties, reflect on relations and meanings, and wonder about related mathematical ideas. The design provides support through hints and feedback to help learners with certain parts of the tasks. The tasks also include challenges that ask the participants to investigate certain ideas and extend their knowledge. Further investigation is needed to understand how the task-design elements, the affordances of collaborative digital environments, and learners' mathematical discourse interact to shape the development of learners' mathematical activity and understanding.

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