PERSPECTIVES ON MATHEMATICS CLASSROOM DISCOURSE

Shari L. Stockero, Michigan Technological University

Jessica Bishop, The University of Georgia

AnnaMarie Conner, The University of Georgia

DR K-12 PI Meeting

August 2014 Washington D.C.

This work is supported by the National Science Foundation.

Introductions

- How do you examine/use/interact with classroom discourse in your work?

Overview of the Session

- Three 20-minute presentations with follow-up questions
- Small group analysis of video transcript
- Whole group discussion

Perspectives on Discourse

Analysis of Discourse

Discourse Analysis

- Analysis of discourse—consider discourse as evidence for some other construct or means by which a social/ cognitive phenomena is expressed.
- Discourse analysis—discourse itself is the object of analysis (often emerging out of studies from linguistics)
- Is discourse foregrounded or backgrounded?
- Where do you locate yourself?

Which is larger, x or $x + x$?

- (Video)

CAREER: Noticing and Capitalizing on Important Mathematical Moments in Instruction

- DRL-1052958

Collaborative Research: Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

- DRL-1220141, DRL-1220357, DRL-1220148
- Co-Principal Investigators
  - Keith R. Leatham, Brigham Young University
  - Blake E. Peterson, Brigham Young University
  - Laura R. Van Zoest, Western Michigan University

The Motivation for the Work

- Student teachers and graduates of our teacher education programs were successful in eliciting student thinking
- That student thinking was not being used to further students' mathematical understanding
- Saw "teachable moments" not get acted on
- Need to better understand these moments
- Need to understand how to prepare teachers to take advantage of them

From the literature…

- “critical moments in the classroom when students created a moment of choice or opportunity” (Walshaw & Anthony, 2008, p. 527)
- “novel student idea[s] that prompt teachers to reflect on and rethink their instruction” (Schifter, 1996, p. 130)
- “potentially powerful learning opportunities” (Davis, 1997, p. 360)
- “significant mathematical instances” (Davies and Walker, 2005, p. 275)
- “[student’s] comment provides the fodder for a content-related conversation” (Schoenfeld, 2008, p. 57)
- “crucial mathematic hinge moment[s]” (Thames and Ball, 2013, p. 31)

From the literature…

- “critical moments in the classroom when students created a moment of choice or opportunity” (Walshaw & Anthony, 2008, p. 527)
- “novel student idea[s] that prompt teachers to reflect on and rethink their instruction” (Schifter, 1996, p. 130)
- “potentially powerful learning opportunities” (Davis, 1997, p. 360)
- “significant mathematical instances” (Davies and Walker, 2005, p. 275)
- “[student’s] comment provides the fodder for a content-related conversation” (Schoenfeld, 2008, p. 57)
- “crucial mathematic hinge moment[s]” (Thames and Ball, 2013, p. 31)

From the literature…

- "critical moments in the classroom when students created a moment of choice or opportunity" (Walshaw & Anthony, 2008, p. 527)
- "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (Schifter, 1996, p. 130)
- "potentially powerful learning opportunities" (Davis, 1997, p. 360)
- "significant mathematical instances" (Davies and Walker, 2005, p. 275)
- "[student's] comment provides the fodder for a content-related conversation" (Schoenfeld, 2008, p. 57)
- "crucial mathematic hinge moment[s]" (Thames and Ball, 2013, p. 31)

From the literature…

- “critical moments in the classroom when students created a moment of choice or opportunity” (Walshaw & Anthony, 2008, p. 527)
- “novel student idea[s] that prompt teachers to reflect on and rethink their instruction” (Schifter, 1996, p. 130)
- “potentially powerful learning opportunities” (Davis, 1997, p. 360)
- “significant mathematical instances” (Davies and Walker, 2005, p. 275)
- “[student’s] comment provides the fodder for a content-related conversation” (Schoenfeld, 2008, p. 57)
- “crucial mathematic hinge moment[s]” (Thames and Ball, 2013, p. 31)

Three Characteristics

- Students
- Mathematics
- Pedagogy

Three Characteristics

- Students —> Student Mathematical Thinking
- Mathematics —> Mathematically Significant
- Pedagogy —> Pedagogical Opportunity

Mathematically significant
pedagogical **O**penings
to build on **S**tudent
Thinking

Mathematical
Opportunities
in **S**tudent
Thinking

MOSTs are "In-the-Moment Opportunities"

MOSTs are “In-the-Moment Opportunities”

But opportunities for what?

MOSTs are opportunities for…

…the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

MOSTs are opportunities for…

…the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

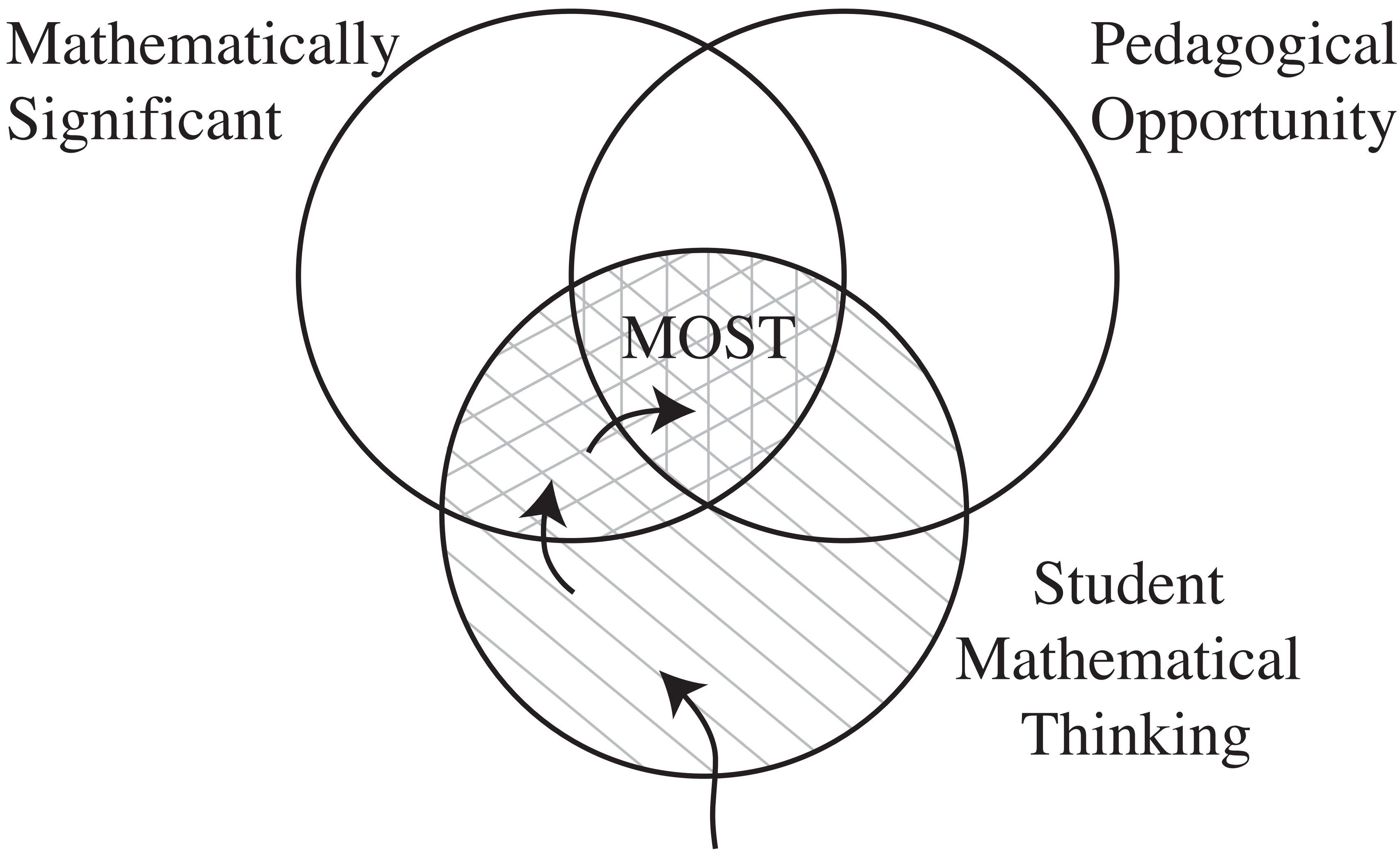
***Building** on student thinking*

Instance of student thinking

Teacher: We have x. Is it larger than x plus x? Which one is larger? [Most students have hands up.] Who haven't I called on. Leianna?

Leianna: x plus x.

Is it a MOST?

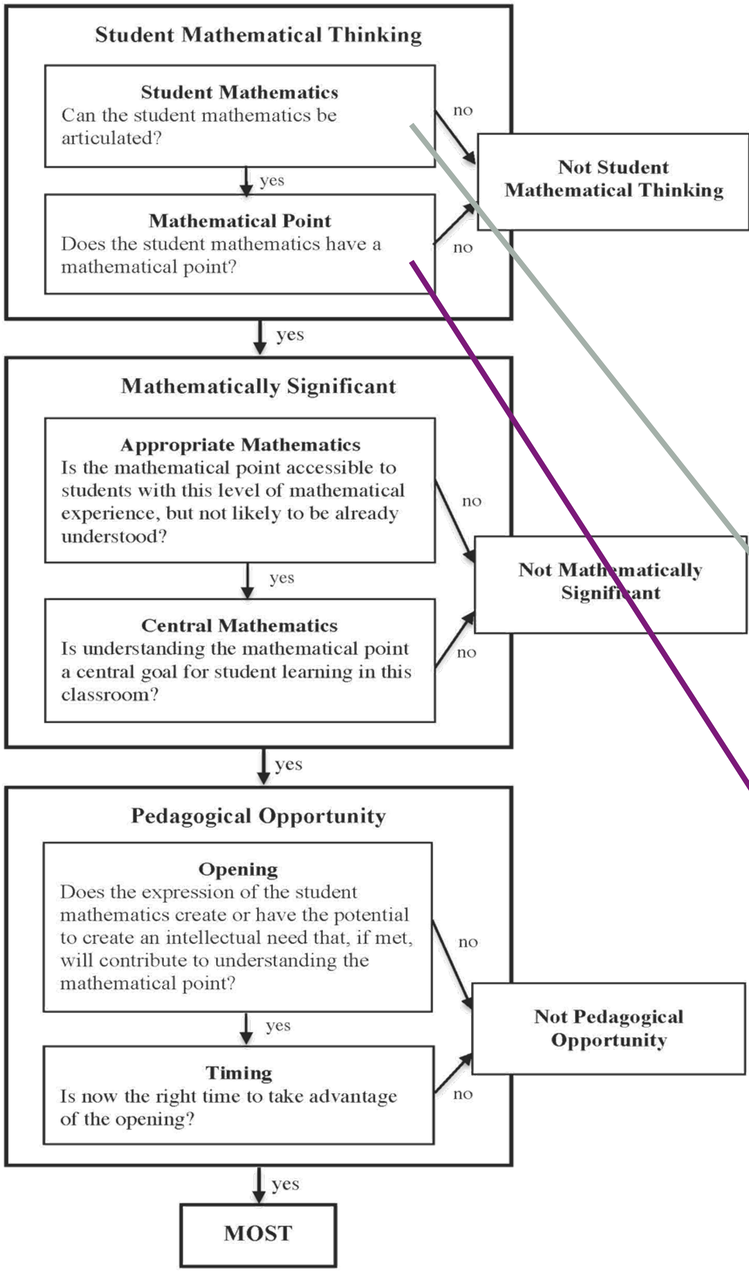

Mathematically
Significant
Pedagogical
Opportunity
MOST
Student
Mathematical
Thinking

Student Mathematical Thinking: An evidence-based inference about student mathematical thinking that can be used to develop a mathematical idea.

Criteria:

Student Mathematics: an inference (that can reasonably be made based on a student's actions) about what a student is thinking mathematically, regardless of the correctness of that thinking.

Mathematical Point: a concise statement of a mathematical idea that mathematics learners could better understand as a result of making the student mathematics of the instance an object for consideration.



Instance: Teacher: We have x. Is it larger than x plus x? Which one is larger? [Most students have hands up.] Who haven't I called on. Leianna?

Leianna: x plus x.

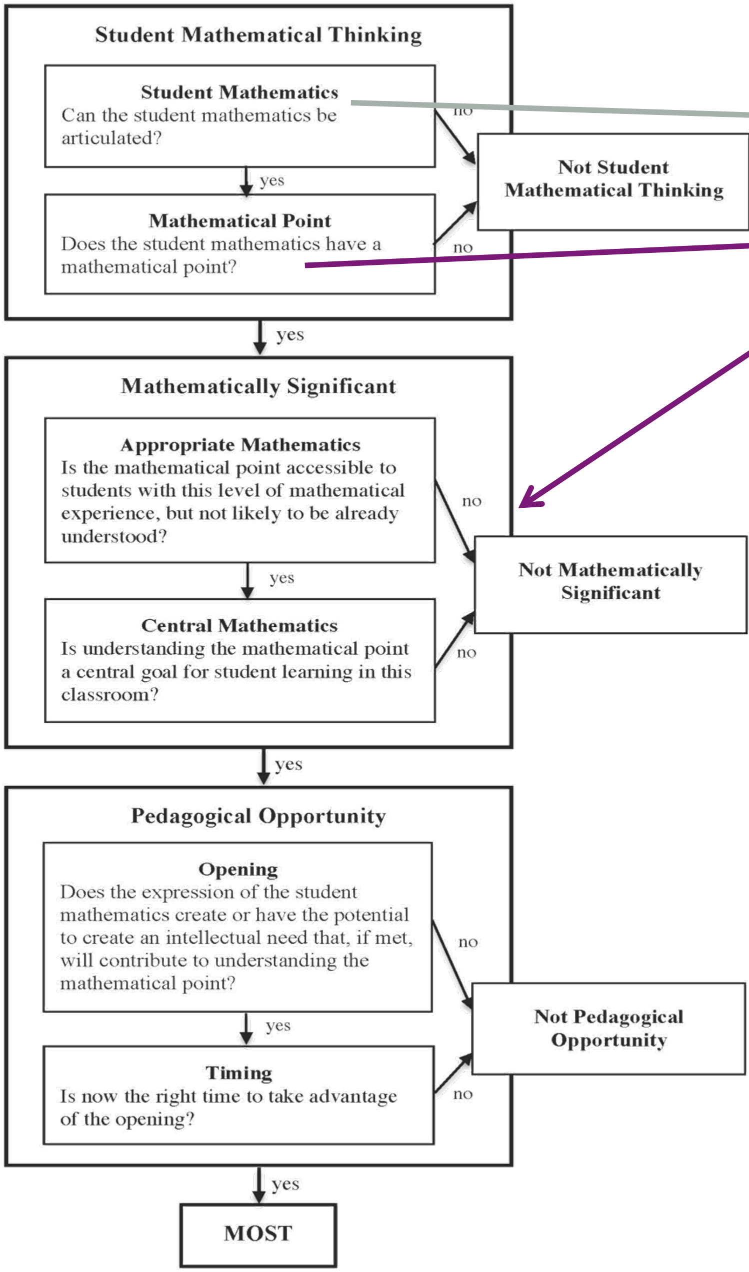
SM: x + x is larger than x.

MP: The ordering of two variable expressions often depends on the value of the variable.

<u>Mathematically Significant</u>: When the mathematical point of an instance warrants the use of limited instructional time; used in the context of teachers engaging a particular group of students in the learning of mathematics.

Criteria:

> **Appropriate Mathematics**: When the mathematical point is accessible to students given their prior mathematical experiences but not typically mastered by most of them.
>
> **Central Mathematics**: When the mathematical point is a central goal for this group of students—central either to the lesson or to the discipline of mathematics.



SM: : x + x is larger than x.

MP: The ordering of two variable expressions often depends on the value of the variable.

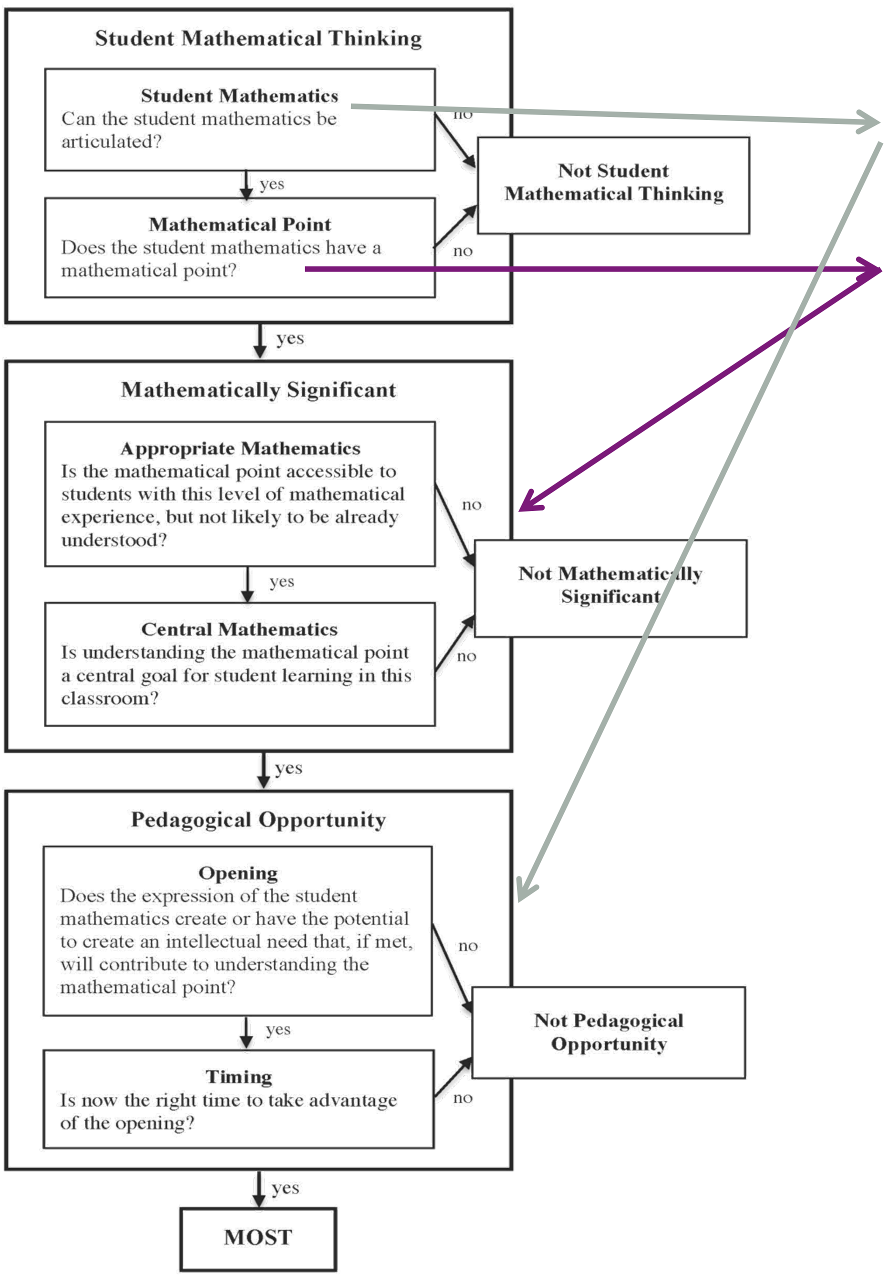
Is it Mathematically Significant?

Pedagogical Opportunity *(to build on student thinking)*: An observable student action that creates an *intellectual need* (Harel, 2013) that can be acted on in that moment to contribute to students' understanding of a mathematical point.

Criteria:

Opening: an instance in which the expression of a student's mathematical thinking creates, or has the potential to create, an intellectual need for students to make sense of the *student mathematics*, thus providing an opportunity to understand the *mathematical point*.

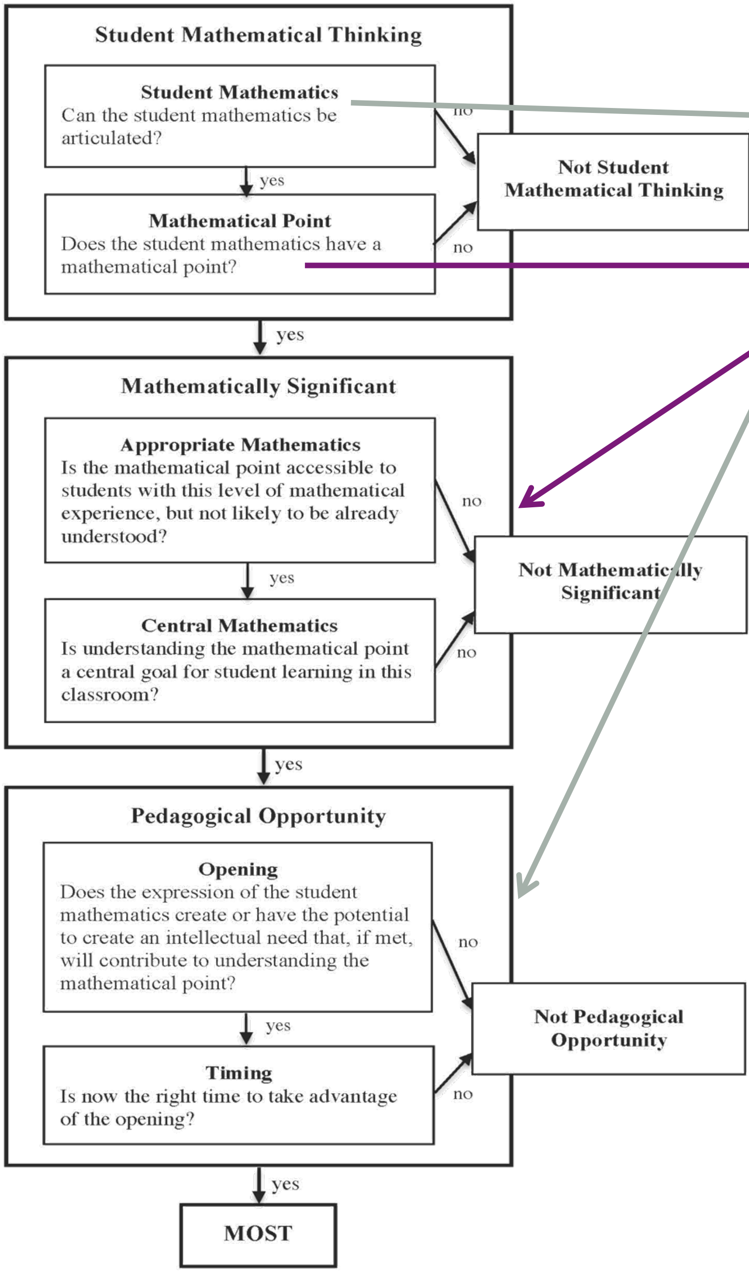
Timing: an opportune time when taking advantage of the opening *at that moment* is likely to further students' understanding of the mathematical point of the instance.



SM: : x + x is larger than x.

MP: The ordering of two variable expressions often depends on the value of the variable.

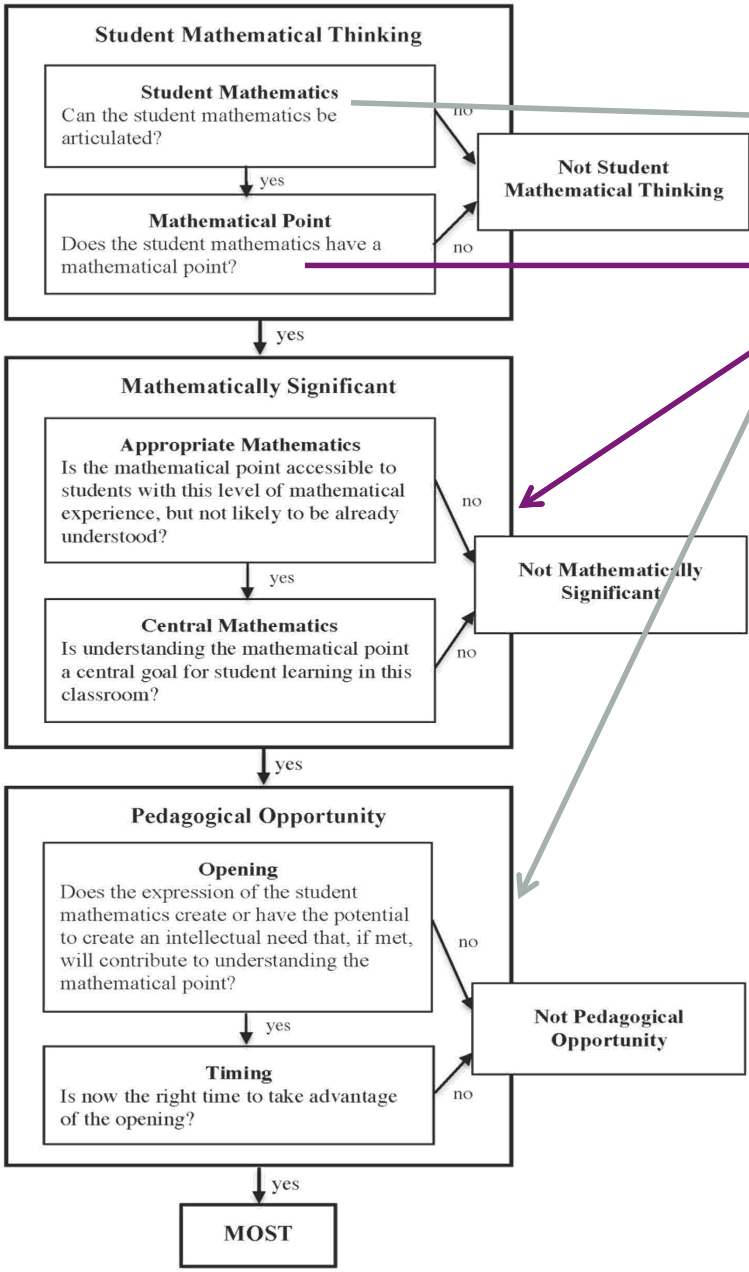
Is it a Pedagogical Opportunity?



SM: : x + x is larger than x.

MP: The ordering of two variable expressions often depends on the value of the variable.

Is it a MOST?



SM: : x + x is larger than x.

MP: The ordering of two variable expressions often depends on the value of the variable.

Is it a MOST?

YES!

Using the Framework—Prospective Teachers

- Analysis of unedited classroom video
  - early field experience
  - focus on *noticing* (Sherin & van Es, 2002) important student ideas
- Results
  - Begin to attend to students and their mathematics
  - Articulation of the mathematical point seems to be particularly productive
  - At end of experience, prospective teachers notice about 1/3 of opportunities identified by researchers

Using the Framework

- Work with practicing teachers—focus on helping them engage in better mathematical discussion with student teachers
- Future work
  - Teacher development experiments
  - Professional development materials

Questions & Comments?

Teachers’ Support for Collective Argumentation

CAREER: Learning to Support Productive Collective Argumentation in Secondary Mathematics Classes

PI: AnnaMarie Conner

Research Team: Richard Francisco, Carlos Nicolas Gomez, Hyejin Park, Ashley Suominen

This work is supported by the National Science Foundation, grant number DRL-1149436.

Overview of Project

- 1 cohort of secondary mathematics prospective teachers
- 3 semesters of coursework, student teaching, 2 years of teaching
- Research focus: Examine teachers' opportunities to engage in and learn about argumentation and how they engage their students in mathematical arguments
- Currently finished year 2 of 5-year study

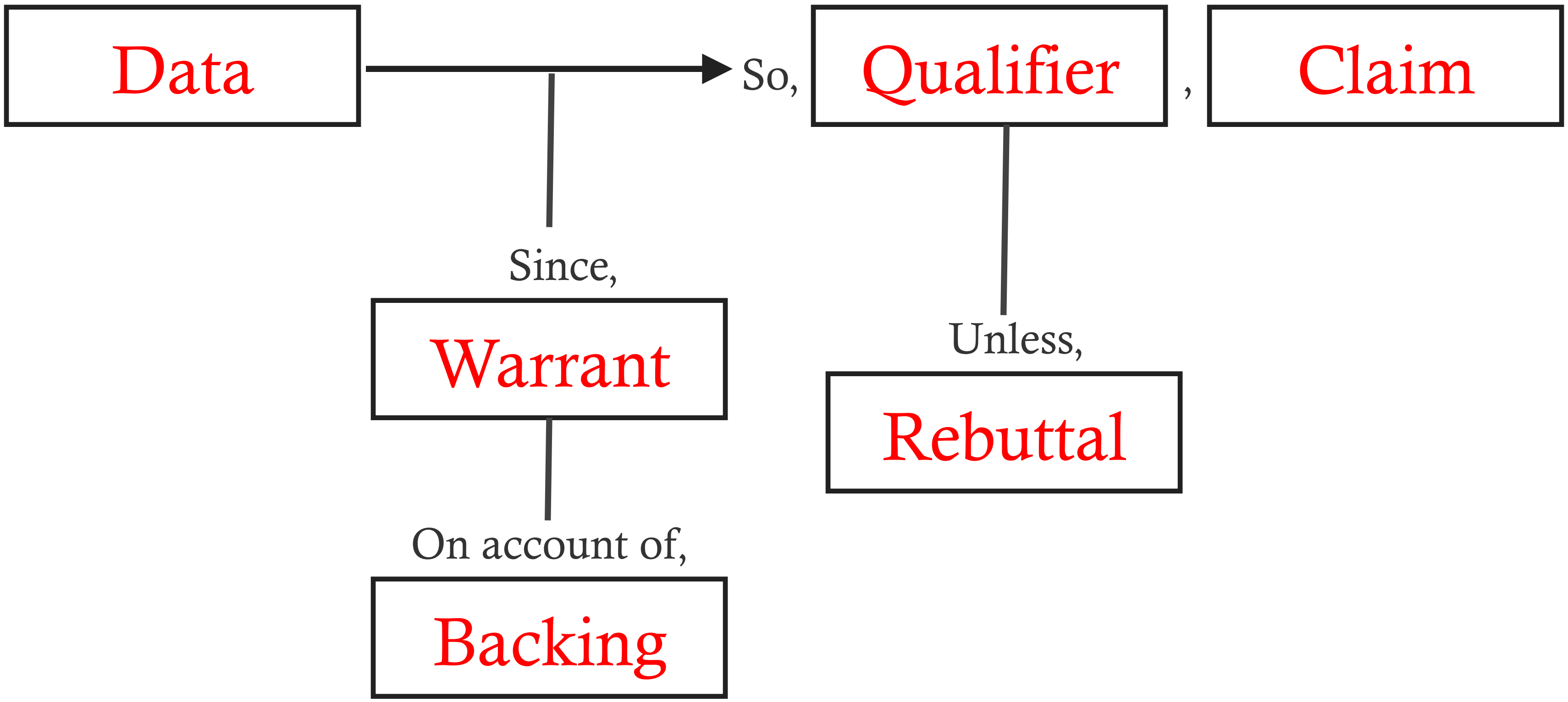
Background

- Facilitating mathematical discussions is difficult for teachers (Hufferd-Ackles, Fuson, & Sherin, 2004)
- Mathematical practices: “Construct viable arguments and critique the reasoning of others.” (CCSS-M)

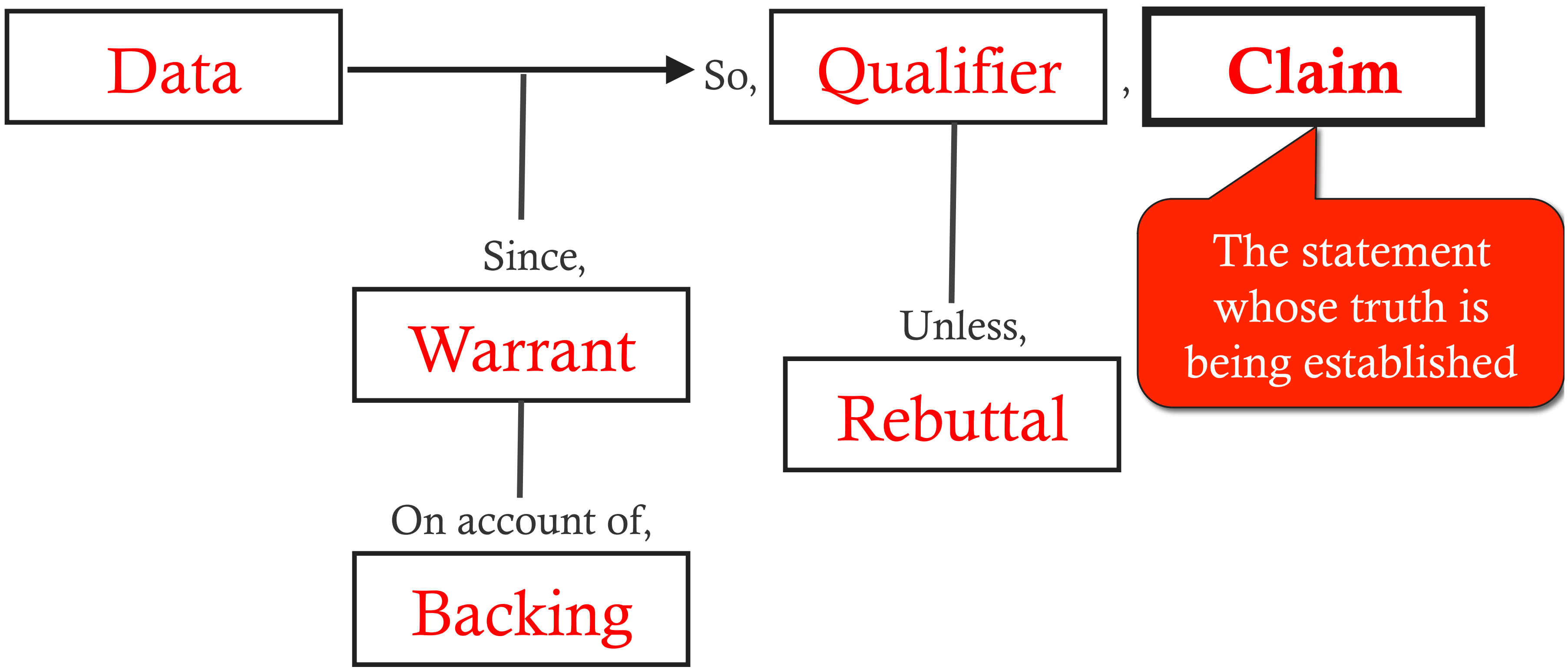
Background

- Collective argumentation
  - We define collective argumentation broadly as any discourse leading to a conclusion that is accepted (or not challenged) by the group
- The role of the teacher (Yackel, 2002)

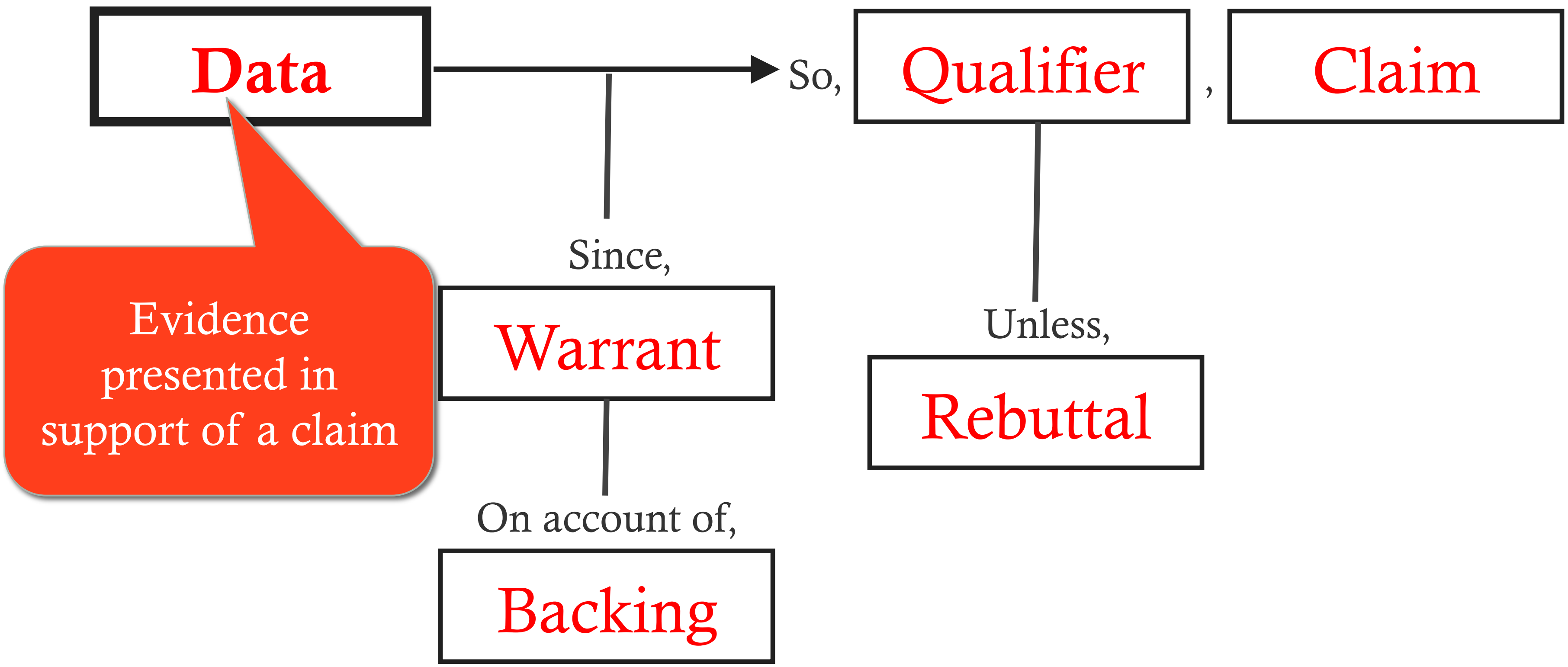
Toulmin's Initial Model



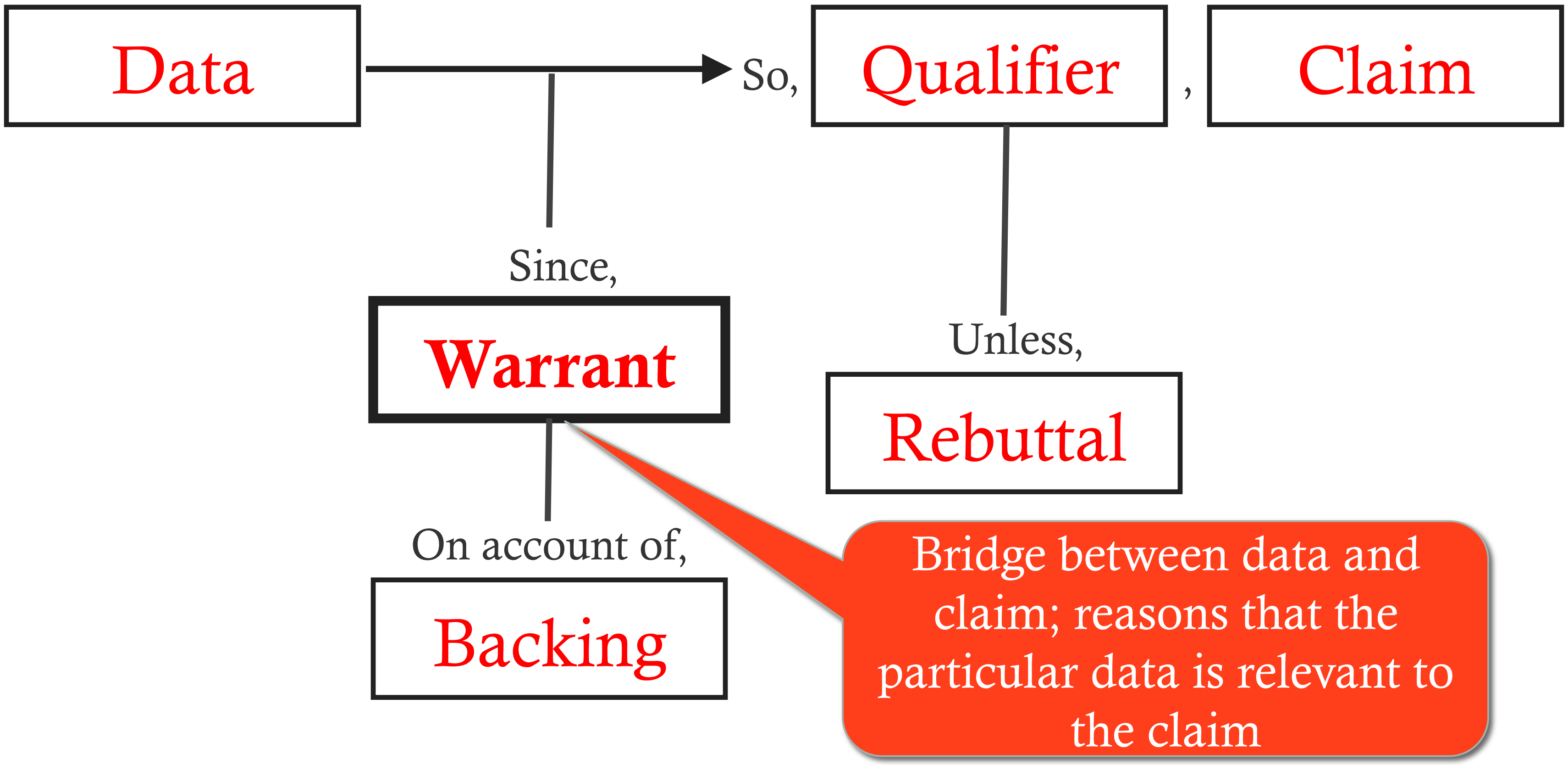
Toulmin's Initial Model



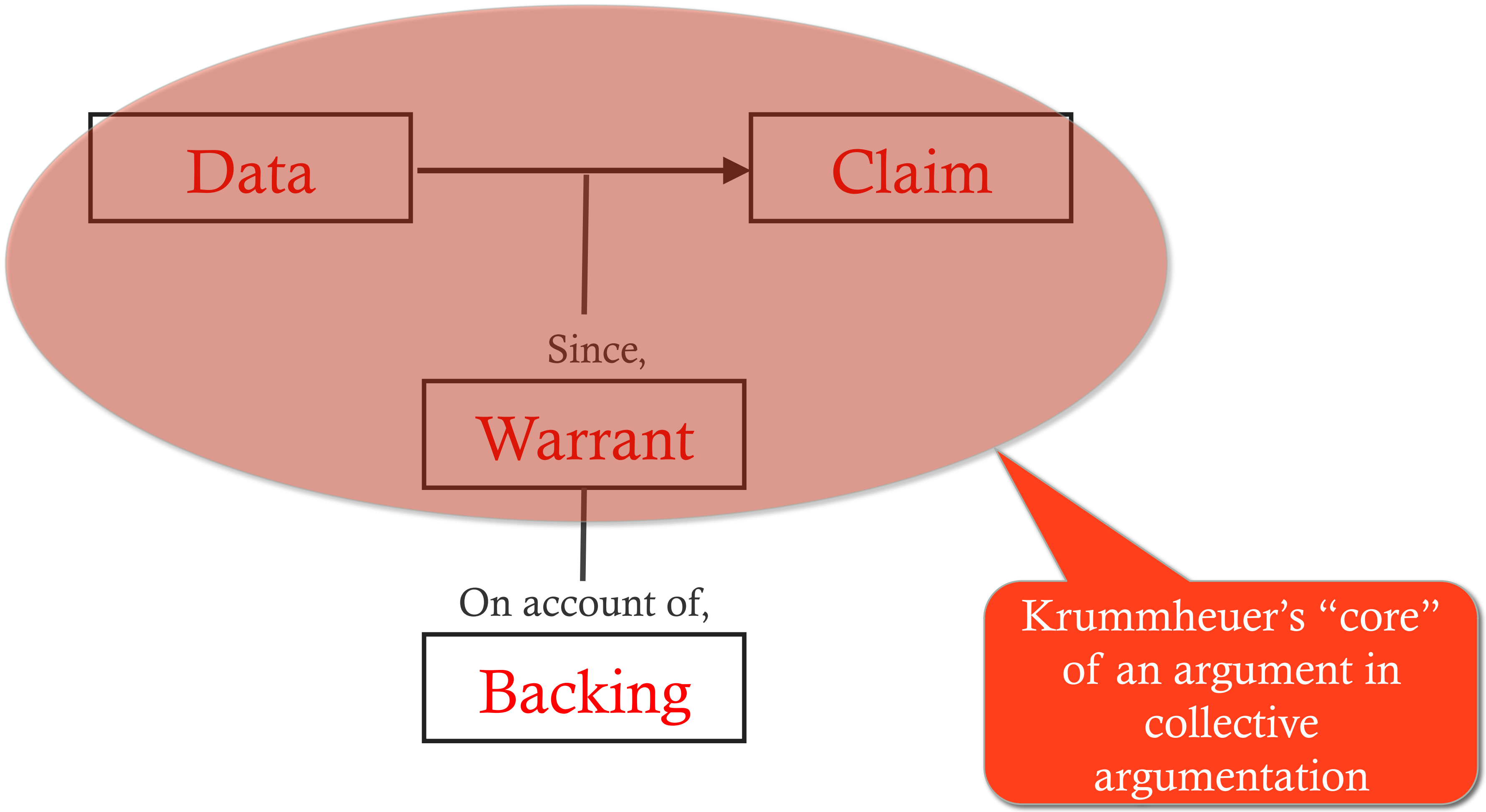
Toulmin's Initial Model



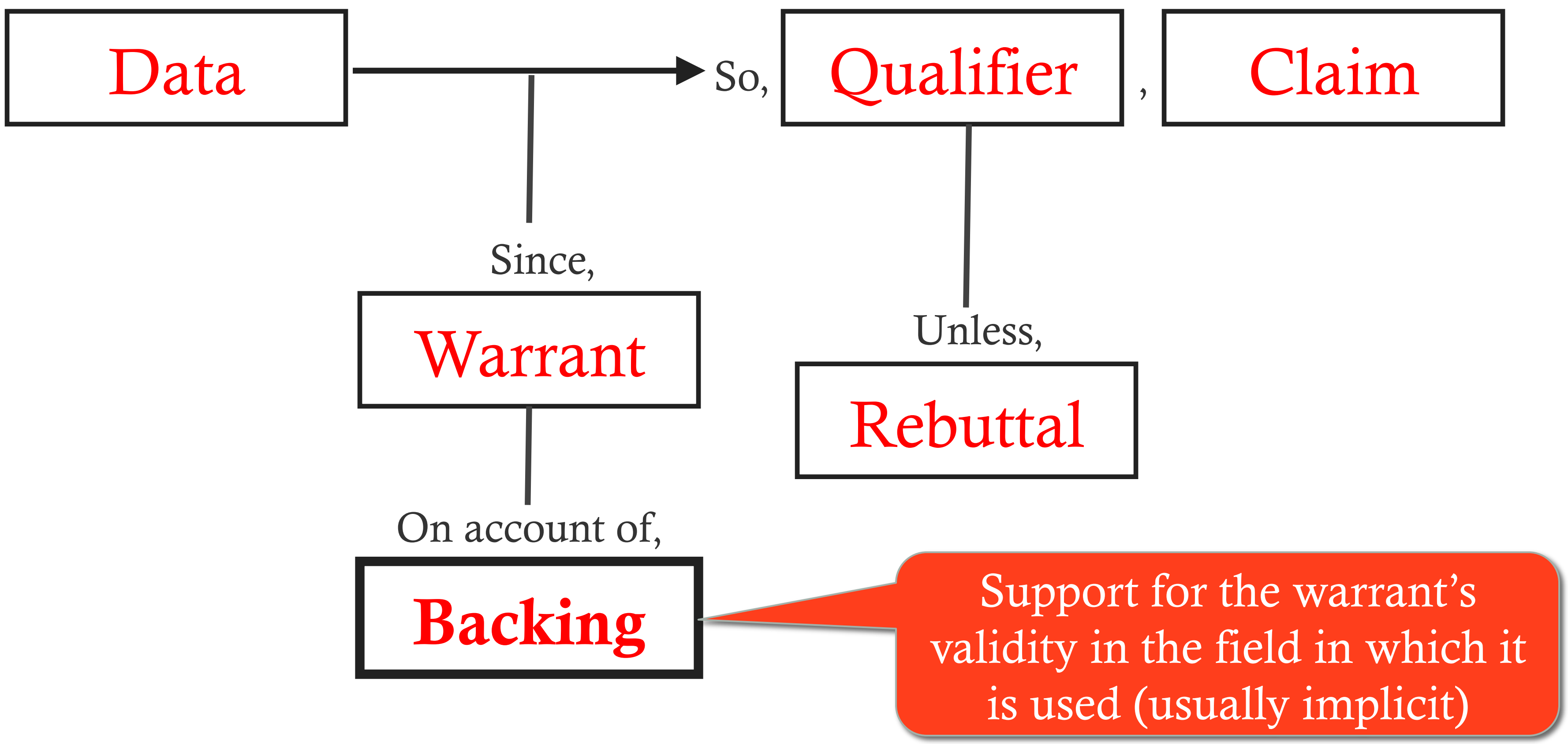
Toulmin’s Initial Model



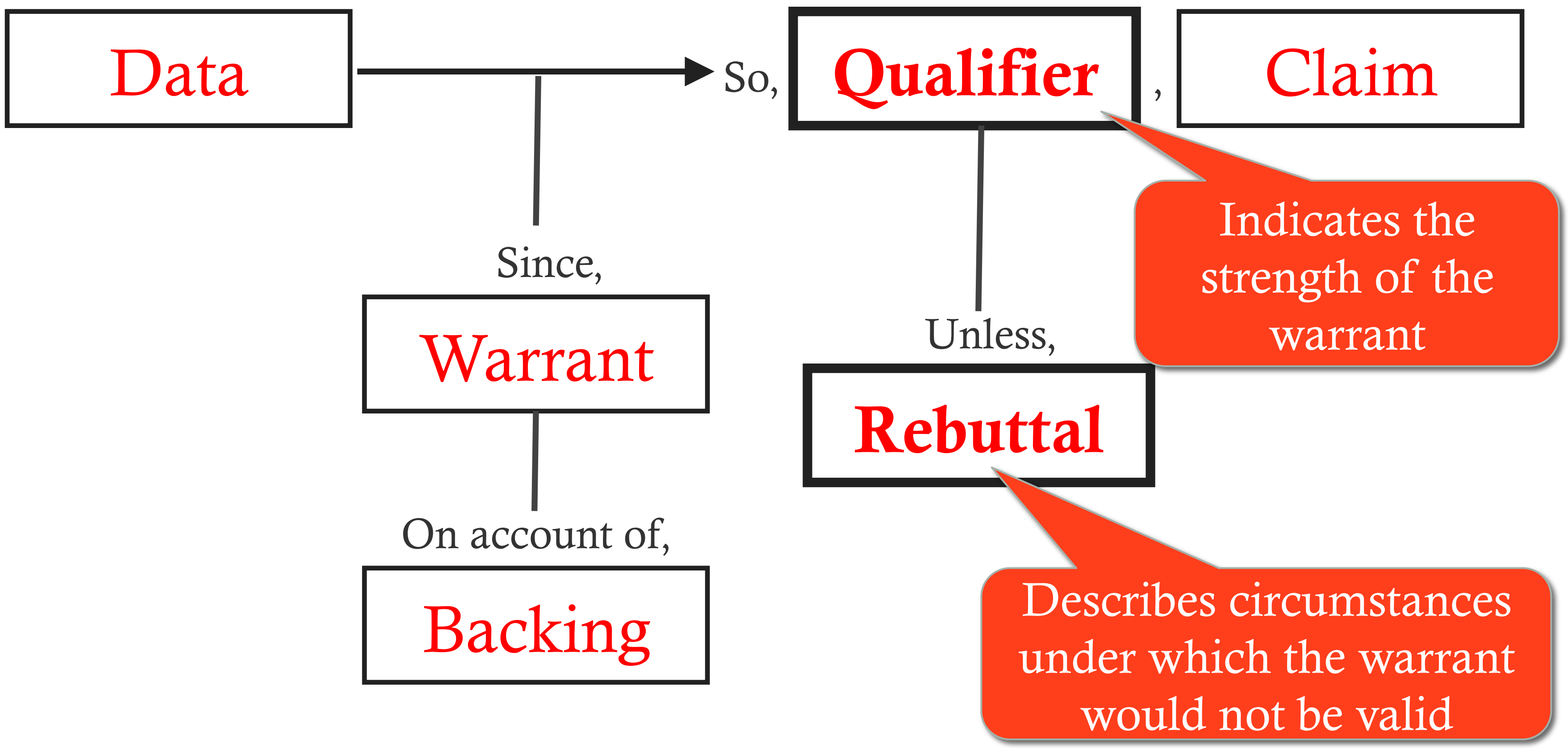
Toulmin's Initial Model



Toulmin's Initial Model

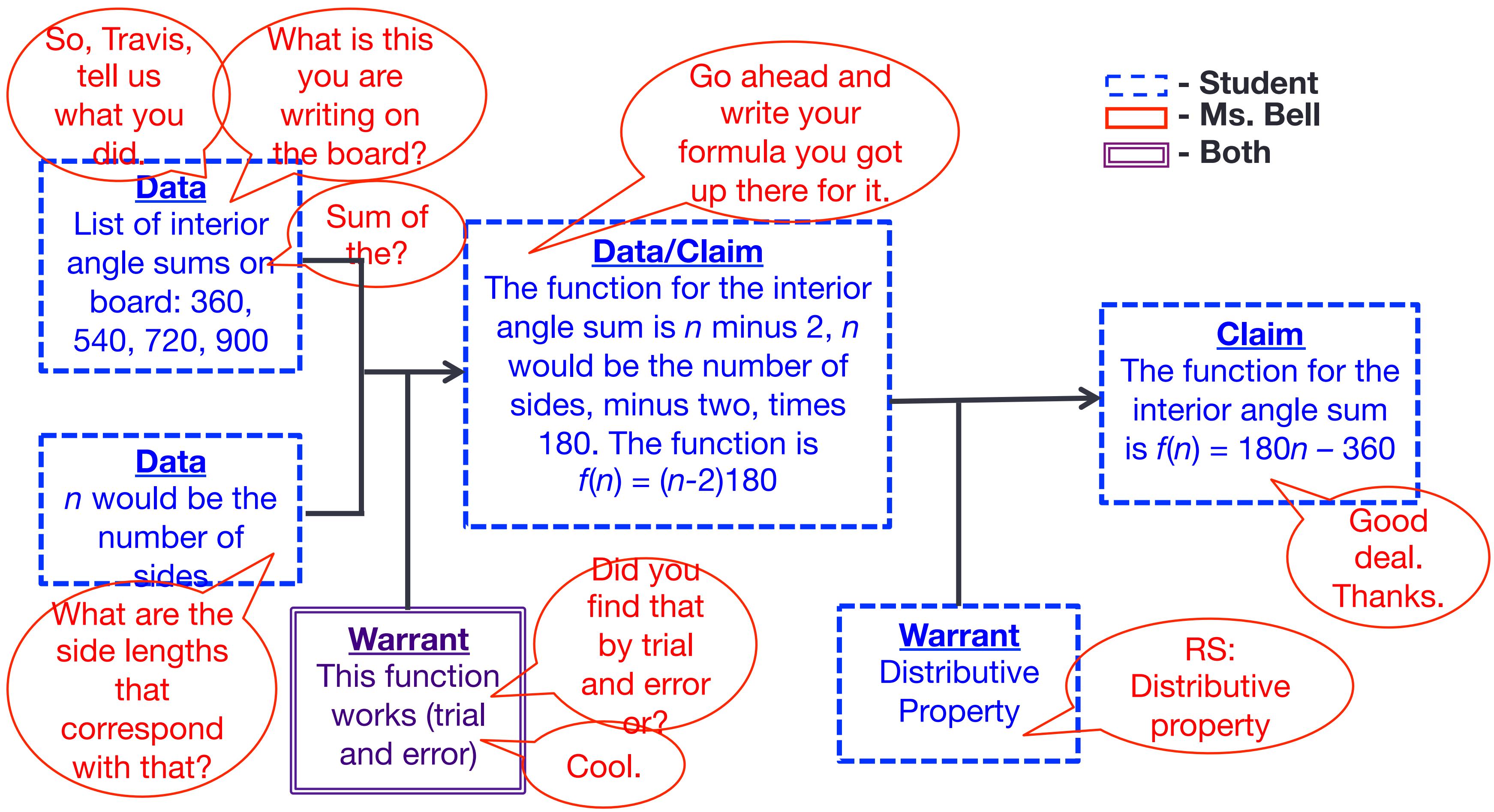


Toulmin's Initial Model



Modifications to Toulmin's Diagram

(Krummheuer, 1994; Conner, 2008)



Constructing a Framework for Teacher Support (previous study)

Research Question: In what ways do teachers support collective argumentation in high school mathematics classes?

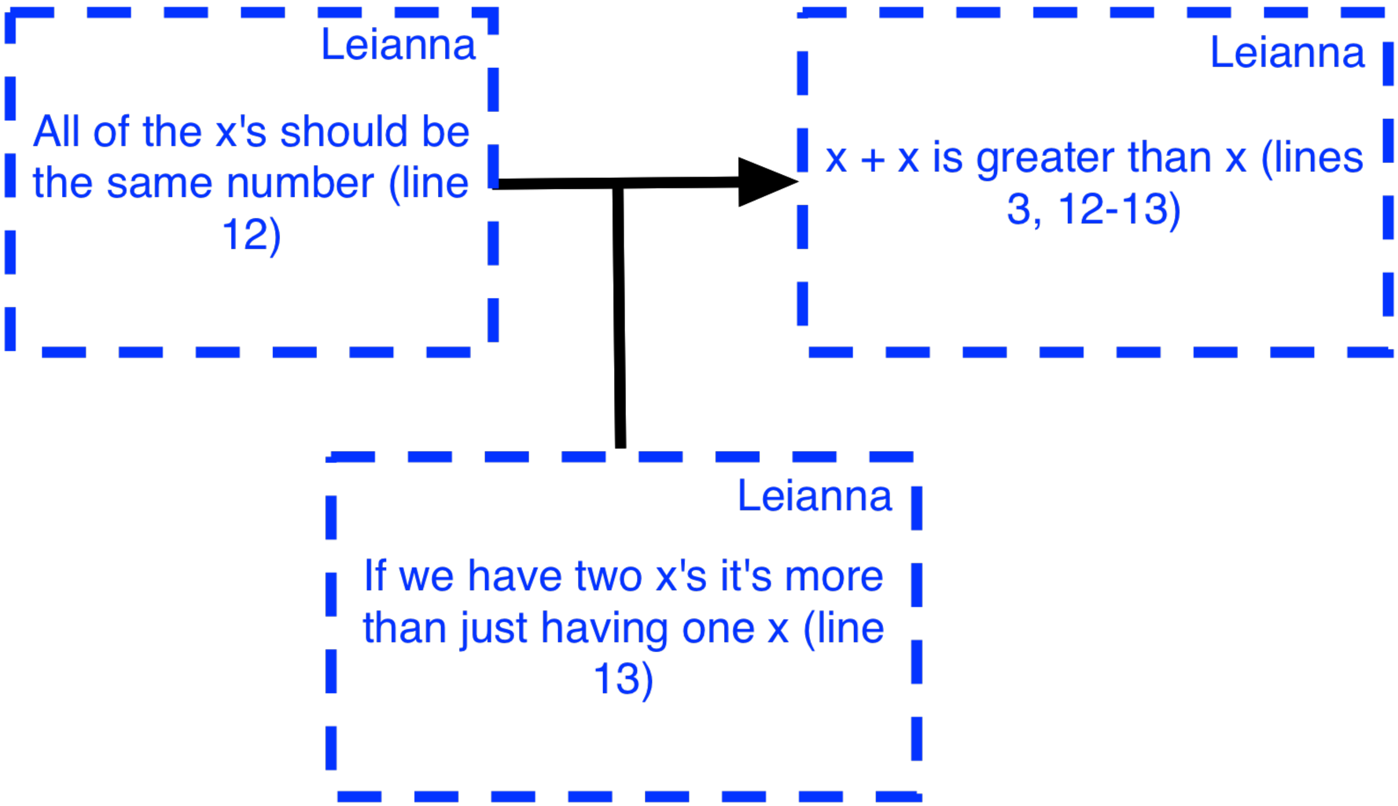
- **Data sources**
  - Observation notes, video recordings, mathematical tasks from 7 days of ninth grade accelerated mathematics class and 7 days of tenth grade geometry class
- **Data Analysis**
  - Identified 277 episodes of argumentation
  - Diagrammed using modified Toulmin scheme
  - Analyzed categories of teacher support

Support for Collective Argumentation Framework

Direct Contributions		Questions		Other Supportive Actions	
Claims	Statements whose validity is being established	Requesting a Factual Answer	Asks a question that can be answered quickly by providing a mathematical fact	Focusing	Actions that serve to focus the students' attention and/or the argument
Data	Statements provided as support for the claims	Requesting a Method	Asks students to demonstrate or describe how they did something	Promoting	Actions that serve to promote mathematical exploration
Warrants	Statements that connect data with claims	Requesting an Idea	Asks students to compare, coordinate, or generate mathematical ideas	Evaluating	Actions that center on the correctness of the mathematics
Rebuttals	Statements describing circumstances under which the warrants would not be valid	Requesting Elaboration	Asks for an elaboration of some idea, statement, diagram	Informing	Actions that provide information for the argument
Qualifiers	Statements describing the certainty with which a claim is made	Requesting Evaluation	Asks students to evaluate a mathematical idea	Repeating	Actions that repeat what has been or is being stated
Backings	Usually unstated, dealing with the field of the argument				

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*(3), 401-429.

Diagram of Leianna's Argument



Leianna's Argument with Support

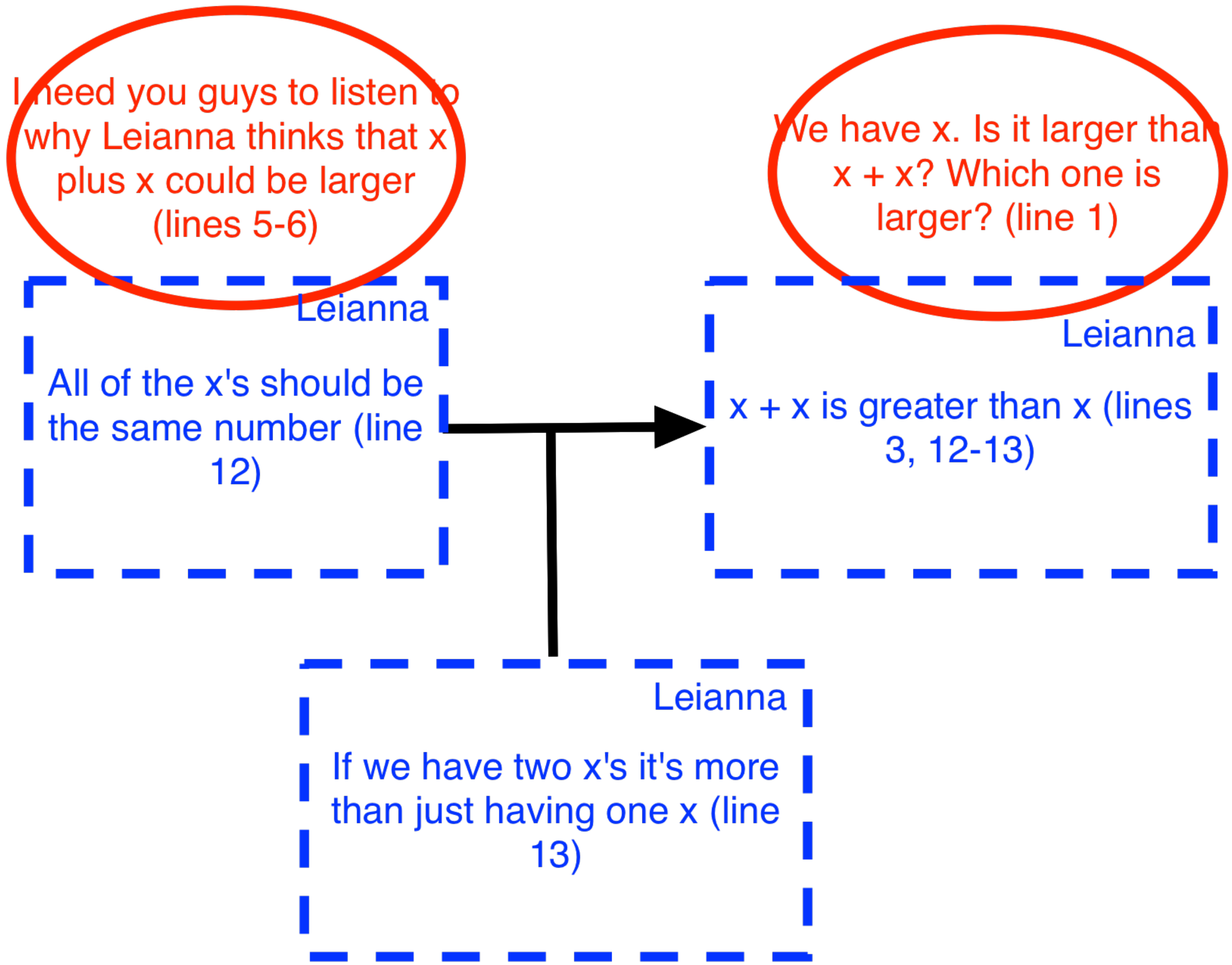
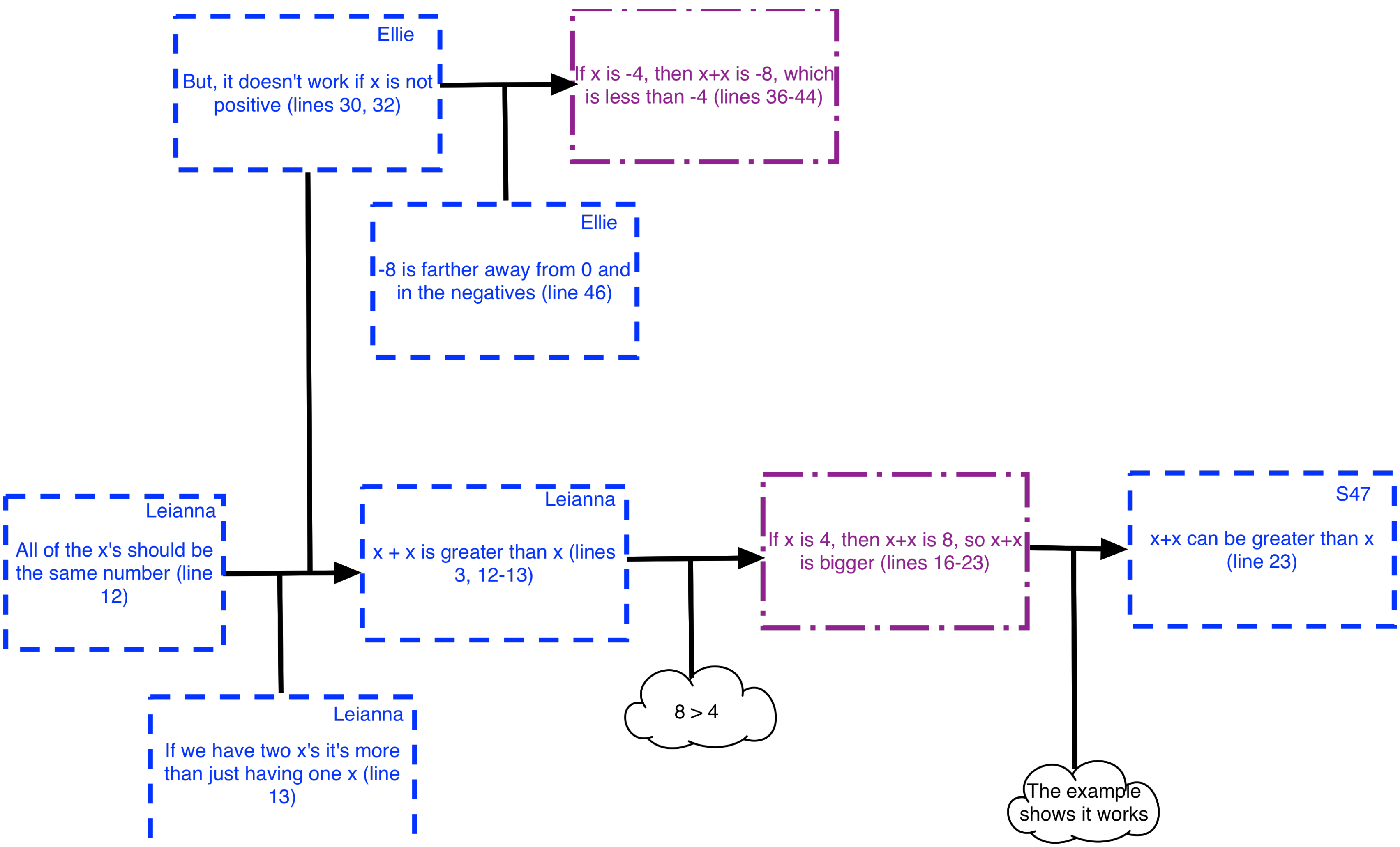


Diagram of Lines 1-45



Questions & Comments?

Perspectives on Mathematics Classroom Discourse

Characterizing critical aspects of productive mathematics classroom discourse

PI: Jessica Pierson Bishop — University of Georgia

Project Personnel: Hamilton Hardison, Eric Siy, Adam Molnar, Clay Kitchings

DR K-12 PI Meeting

August 2014 Washington D.C.

This work is supported by the National Science Foundation, grant number DRL-1265677

Why Discourse?

- Our stance is that mathematics classroom discourse is a critical element of learning mathematics. In fact, in NCTM's *Principles to Action*, 2 of the 8 Teaching Practices have an explicit discursive focus—facilitate meaningful mathematical discourse and pose purposeful questions.
- A large part of learning mathematics is learning the language of mathematics, not only to construct the mathematical content, but also to enact and build positive identities and dispositions toward mathematics. There are also communally-accepted ways of talking and writing mathematics that school helps students to learn.

Study Design

- Year 2 of a 5-year study
- 18 mathematics teachers—6 teachers at *each* of grades 5, 6, and 7. Teachers have been identified as exemplary math teachers with a reputation of consistently using problem solving and discussion in instruction. Last year we collected data for 9 teachers.
- Filmed 15 lessons per teacher across 3 instructional units—algebraic reasoning, integers, and fractions (mainly fraction division and equal sharing tasks). Data corpus consists of 131 filmed lessons.
- Research questions:
  - What discursive constructs and patterns can be identified, described, and operationalized within Grades 5–7 mathematics classrooms?
  - How stable are discursive constructs across curricular topics and grade levels and within individual teachers?

Emerging Discursive Constructs

- Prevalence of students' mathematical ideas and teachers' responsiveness to student ideas
- Use of "uncertain language" as in hedges/modifiers or deictic expressions (pragmatics, see Rowland's work)
  - (How) are these expressions related to making conjectures, modifying constraints, and moving toward using more precise language and technical mathematics terms?
- What kinds of metaphors are used by teachers and students?
  - order of operations is like a funnel
  - the equal sign is like a mirror

Emerging Discursive Constructs (cont.)

- Cohesion or connectedness of discourse? (related to intertextuality and ways in which big mathematical ideas are/are not used as organizing principles)
- Identity and positioning of students through discourse. Every utterance has a relational function. What kinds of positioning moves do teachers/students engage in? How are students positioned with respect to classroom and mathematical communities?
- Students' production of and engagement with different *mathematical genres* recognized by the broader (mathematical) community. Genres are "conventionalized forms of text" with distinct linguistic and stylistic patterns that are determined by contextual features (including things like activity structures) and the underlying purpose.

Mathematical Genres

- Why genre?
- Part of learning mathematics is learning the conventional and specialized styles of mathematics discourse that are valued by the broader mathematical community. One of the functions of school mathematics should be to help familiarize students with characteristic meanings and forms of mathematical expression (such as definitions, argumentation, strategy reporting, making conjectures, posing problems, etc.).
- Familiarity with and mastery of different forms of mathematical genres is an element of mathematical proficiency.

- Strategy Sharing, Ellie, 6th grade, Stairwell Problem

As a contractor you specialize in outdoor brick stairwells. How many bricks will you need to build a 10-brick-high stairwell? Solve the problem 2 ways.

Strategy Sharing

- (Video)

Spoken Mathematical Genres Identified in our Data Corpus

- Strategy sharing
- Generalized procedure
- Mathematical observation
- Definition
- Conjecture
- Justification
- Extended argumentation
- What else?

Unanswered Questions

- Are some genres more desirable than others? Do we want students to have experiences producing each of the different kinds of genres?
- What role do tasks play in the kinds of genres students produce or engage with?
- Can pairs/groups of genres be sequenced in more and less effective ways?
- What is missing from our list of genres?

- Task 1-MOST: Locate an interesting instance of student mathematical thinking in the transcript. Analyze the instance using the MOST Analysis Framework. Is the instance a MOST? If not, what criterion does it fail? If time permits, can you find more instances that are MOSTs?
- Task 2-Support for Arguments: Identify and characterize the teacher's supportive moves in lines 1-45 in the transcript. Alternatively, you may want to engage in diagramming Tompson's argument in lines 50-59 and look at the teacher's support in this argument.
- Task 3-Genre: What genre(s) do you think students are producing in the x versus x + x clip and why? What are characteristic ways of speaking (i.e., indicators or markers) in this genre? As time permits, examine the genres in the other transcripts.

Discussion

- What did each lens allow you to see in the classroom discourse?
- What were the affordances (and limitations) of each perspective?

What do I Mean by Discourse?

- Discourse is the spoken and written words, semiotic systems, representations, and gestures of teachers and students as they use language to communicate, interact, and act. My focus is on the work accomplished with and through **spoken and written discourse** during interaction and communication in mathematics classrooms.
- Within mathematics, there are specific ways of talking and using language. Halliday uses the term **register** to describe the patterns of language use appropriate to particular circumstances and situations. I am interested in the unique characteristics of the mathematics registers of middle grades classrooms as well as broader discourse constructs that cut across disciplines.