TEACHERS' RESPONSES TO INSTANCES OF STUDENT MATHEMATICAL THINKING WITH VARIED POTENTIAL TO SUPPORT STUDENT LEARNING

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We investigated teachers' responses to a common set of varied-potential instances of student mathematical thinking to better understand how a teacher can shape meaningful mathematical discourse. Teacher responses were coded using a scheme that both disentangles and coordinates the teacher move, who it is directed to, and the degree to which student thinking is honored. Teachers tended to direct responses to the same student, use a limited number of moves, and explicitly incorporate students' thinking. We consider the productivity of teacher responses in relation to frameworks related to the productive use of student mathematical thinking.

Keywords: Classroom Discourse, Instructional Activities and Practices

Recommendations for ambitious mathematics teaching have identified the importance of instruction that honors and incorporates student thinking (e.g., National Council of Teachers of Mathematics [NCTM], 2014). Such instruction involves the teacher facilitating meaningful mathematical discourse by eliciting and making public student thinking, as well as appropriately *responding* to that thinking. Research has shown that the way in which teachers respond to student mathematical thinking (SMT) affects student learning in the classroom. For example, research has found that teacher responses that press students to further engage in thinking about the mathematics in their contributions provide students with increased learning opportunities (Kazemi & Stipek, 2001). More recently, Ing et al. (2015) found that responses that encourage students to engage with each other around mathematics correlate with increased student participation and higher student achievement.

Since SMT varies in the degree to which it provides leverage for accomplishing mathematical goals (Leatham, Peterson, Stockero, & Van Zoest, 2015), it follows that not all thinking should be responded to in the same way. Research examining teachers' responses to different types of SMT has produced mixed findings. Franke et al. (2009) found that the types of questions teachers asked did not vary depending on the clarity, correctness or completeness of a student's initial explanation, but other studies have found that teachers' responses do vary based on different types of SMT. For example, Bishop, Hardison, and Przybyla-Kuchek (2016) found that short or routine student contributions were related to teacher actions that were not responsive to SMT, whereas those that included strategies or reasoning were related to responses that engaged students in conversations about the SMT. Similarly, Drageset (2015) reported that brief answers to non-complex questions were typically responded to with a recall or procedural question, whereas unexplained answers were typically followed by responses that focused on an elaboration or rationale. Although prior research provides a foundation for understanding teachers' responses to instances of SMT, more work is required to fully understand variations in such responses, including whether particular responses might be more or less productive in supporting student learning in particular situations. Fortunately, scholars have developed a number of constructs for characterizing teacher responses that support this line of research.

Researchers have characterized teacher responses in various ways based on the focus of their studies. Brodie (2011) developed a coding scheme for teacher responses that captured two key aspects of responses—responsiveness to student ideas and student engagement. Schleppenbach, Flavares, Sims, and Perry (2007) analyzed teachers' responses to student errors using a scheme that captured two additional aspects of responses—the form of the response (statements or questions) and who questions were directed towards (the same student or other student(s)). Peterson et al. (2017) aimed to develop a comprehensive coding scheme that foregrounded responsiveness while also capturing other important ideas included in existing teacher response constructs. These researchers developed the Teacher Response Coding Scheme (TRC) to both disentangle and coordinate a number of components important in a teacher response, including the actors invited to respond, the type of action, and responsiveness to the SMT. Such a scheme provides a way to study teacher responses to student contributions that simultaneously addresses components of responses valued by other scholars working in this area.

Our study extends earlier research on teacher responses by using the TRC (Peterson et al., 2017) to examine teachers' responses to a common set of SMT with varied potential to support student mathematical learning. In particular, this study focuses on answering the question: *How do teachers' responses vary depending on the potential of an instance of SMT to support student mathematical learning?* We use these findings to discuss the extent to which various teacher responses are productive given the mathematical potential of an instance of SMT.

Theoretical Framework

Our work draws on two distinct, but related theoretical constructs. To make sense of the potential of an instance of SMT to support student mathematical learning, we use the MOST Analytical Framework (Leatham et al., 2015). To interpret the productivity of teachers' responses to these instances of varying potential, we use a set of principles drawn from the literature that underlie productive use of SMT. Descriptions of these constructs follow. Leatham et al. (2015) characterized particular high potential instances of SMT as MOSTs—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking. The MOST Analytic Framework defines three characteristics of these instances—student mathematical thinking, significant mathematics, and pedagogical opportunity—each having two criteria that are used to determine whether an instance of SMT embodies that characteristic. For student mathematical thinking the criteria are that the *student mathematics* is inferable and that one can articulate a closely related *mathematical point* that the contribution could be used to better understand. For the significant mathematics characteristic, the criteria are that the mathematical point is *appropriate* for the students in the class—not too easy or too hard—and *central* to mathematical goals for their learning. To satisfy the pedagogical opportunity characteristic, the instance must create an *opening* to build on the SMT and the pedagogical *timing* must be right to take advantage of the opening when it occurs. The six criteria are considered linearly and an instance of SMT is classified according to the last criteria it satisfies (Student Mathematics, Mathematical Point, Appropriate, Central, Opening). Instances that meet all six criteria are classified as MOSTs. Those instances that appear mathematical, but for which the student mathematics cannot be inferred, are designated *cannot infer* (CNI).

To determine whether a teacher response to an instance of SMT is likely to be productive, we focus on the extent to which it coordinates core ideas about effective teaching and learning of mathematics drawn from the literature (e.g., NCTM, 2014)—what we refer to as principles underlying productive use of SMT (see Figure 1). We see these four principles as simultaneously coordinated in the teaching practice of *building* on SMT—making student thinking "the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36). Further, we see MOSTs as instances of SMT that are prime opportunities for a teacher to engage in building.

1. The mathematics of the instance is at the forefront. (Mathematics Principle)
2. Students are positioned as legitimate mathematical thinkers. (Legitimacy Principle)
3. Students are engaged in sense making. (Sense-Making Principle)
4. Students are working collaboratively. (Collaboration Principle)

Figure 1. Principles underlying productive use of student mathematical thinking.

Methodology

The Scenario Interview

The Scenario Interview (Stockero et al., 2015) is a tool to investigate how teachers respond to SMT during instruction and their reasoning underlying those responses. Teachers are presented with eight instances of SMT—four each from geometry and algebra contexts. The instances represent a range of SMT that satisfy different sets of MOST criteria, including those for which the SMT cannot be inferred and those that are mathematically significant but have poor timing. Four instances—two from each context—are MOSTs. Figure 2 provides four sample instances, their contexts and the last criteria they met on the MOST Analytic Framework.

Scenario	Context	Instance	Criteria
G1	Students were sharing their solutions to the following task (a corresponding picture was on the board). *Given two concentric circles, radii 5cm and 3cm, what is the area of the band between the circles?*	Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."	MOST
G2		Before the teacher had a chance to respond to Chris, Pat says, "I also got 4π cm^2, but I did it a different way."	SM
A1	Students had been discussing the following task and had come up with the equation y = 10x + 25. Task: *Jenny received $25 for her birthday that she deposited into a savings account. She has a babysitting job that pays $10 per week, which she deposits into her account each week. Write an equation that she can use to predict how much she will have saved after any number of weeks.*	Terry says, "If you deposit $20 per week instead of $10 per week, the number in front of the x in the equation would change, but the number that is added would stay the same."	Central
A2		Casey said, "You could also change the story so the number in front of the x is negative."	MOST

Figure 2. Sample Scenario Interview instances, contexts and the last MOST criteria met.

The Scenario Interview situates the interviewee as the teacher in the context presented. They are asked to describe what they might do next were the instance to occur in their classroom and to explain why they would respond in that way. The interviewee may ask for contextual information they feel is needed before giving their initial response and is later provided common contextual information, after which they can revise their response if desired. If they did so revise, the revised response was used for this analysis. Using a common set of instances of SMT and providing common contextual information allowed for direct comparisons among teacher responses to a collection of instances that satisfy different subsets of the MOST criteria. This comparison allowed us to determine whether teachers seem to differentiate their responses based on the type of SMT to which they are responding.

Data Collection and Analysis

Data consisted of video-recorded Scenario Interviews conducted with 25 grade 6-12 mathematics teachers from across the United States. We segmented each interview into the 8 instances of SMT and the 25 teachers' responses to each individual instance. A *teacher response* was defined as *the collection of actions that a teacher describes they would take immediately following an instance of SMT*. There were a total of 198 teacher responses because one teacher was not able to envision one of the instances occurring in their classroom, and another teacher's interview was cut short before the last scenario was completed.

To begin to understand teachers' responses, we focused on teachers' initial responses to the instances in the Scenario Interview. Thus, preparing data for coding required making inferences about how the teacher would respond *in the moment* to each instance by considering both the teacher's description of their initial response and their rationale. Three coders individually analyzed each instance for a participating teacher, distilled the teacher's response to its essence, and met to discuss their inferred responses until a final teacher response was agreed upon. Any disagreements were brought to the larger research team for further discussion.

The teacher responses were coded using the TRC (Peterson et al., 2017), a scheme that disentangles the teacher move (Move) from other aspects of the teacher response, including who is publicly given the opportunity to consider the SMT (Actor) and the degree to which the SMT is honored (Recognition-Student Action and Recognition-Student Ideas). Figure 3 provides the TRC coding categories and codes discussed in this paper. Note that the TRC allows for multiple moves to be present in a teacher response. To facilitate our analysis, in cases of multiple actions in a given teacher response we identified the *predominate code* for each category—*the code that the students are most likely to experience as the instructional intent of the response*. The analysis used the predominate codes for each of the 198 teacher responses.

Category	Coding Category Description	Codes
Actor	Those who are publicly given the opportunity to consider the instance of SMT	teacher, same student(s), whole class
Recognition-Action	The degree to which the teacher response (either verbal or non-verbal) uses the student action	explicit, implicit, not
Recognition-Ideas	The extent to which the student who contributed the instance of SMT is likely to recognize their idea(s) in the teacher response	core, peripheral, other, not applicable
Move	What the actor is doing or being asked to do with respect to the instance of SMT	adjourn, clarify, collect, connect, develop, dismiss, justify

Figure 3. Subset of the *Teacher Response Coding Scheme* (*TRC*) discussed in this paper.

We use two teachers' responses to Scenario G1 (see Figure 1) to illustrate our application of the TRC. T1's response, "I would just ask [Chris] to explain by using pictures and words how he came up with the 4 pi," asks the *same student* a question that is a *develop* move in that the student is asked to explain how he arrived at the answer. T1 uses the student's words so Action is coded *explicit*, and the question stays *core* to the student's Ideas because it focuses on how Chris arrived at his answer. In T2's response, "Who else has another answer? Did everybody get that? Give me some more answers," Actor is coded as *whole class* because all students are invited to participate. Move is *collect* as T2 requests that other students share their answers. The student's words are not used, but referred to (by "that"), so Action is *implicit.* Asking other students to share their answers to the same task is *peripheral* to the contributing student's Ideas.

Results and Discussion

To understand how teachers' responses vary depending on the potential of an instance of SMT to support student mathematical learning, we begin by comparing results related to the Actor, Move, and Recognition categories for MOSTs and non-MOSTs. Then, we discuss how particular responses might be more or less productive in particular situations by considering how they adhere to the principles underlying productive use of SMT. Note that there were 99 MOSTs and 99 non-MOSTs in the data set; since the frequencies and percentages are essentially equivalent, we report only the frequencies.

Comparison of Teacher Responses

Actor. Responses coded *same student* were the most prevalent in the data and occurred at about the same frequency for MOSTs (65 of 99) and non-MOSTs (63 of 99). This even distribution among MOSTs and non-MOSTs did not occur for instances with a *whole class* or *teacher* actor. More MOSTs (26) than non-MOSTs (6) were coded whole class, while the opposite was true for instances coded teacher (4 MOSTs; 30 non-MOSTs). This suggests that teachers may distinguish, at least to some extent, instances that have potential to be discussed by the class from those that the teacher might just quickly deal with and move on.

Moves. Two dominant moves in the data, *develop* and *justify*, occurred more frequently in response to MOSTs than non-MOSTs (Table 1). Develop moves accounted for 37 MOST responses and only 25 non-MOST responses, while justify moves accounted for 18 MOST and 11 non-MOST responses. Together these two moves accounted for over half of responses to MOSTs. Two other dominant moves, *adjourn* and *clarify*, occurred more frequently in response to non-MOSTs. These moves accounted for 21 and 23 non-MOST responses, respectively, and for only 3 and 5 MOST responses, respectively. Thus, adjourn and clarify moves accounted for nearly half of non-MOST responses. As with Actor, the differences in Move for MOSTs versus non-MOSTs suggest the teacher actions differ depending on the type of SMT.

Table 1: Actor and Move Totals for MOSTs and non-MOSTs

	Actor			Move			
	Same Student	Whole Class	Teacher	Develop	Justify	Adjourn	Clarify
MOST	65	26	4	37	18	3	5
non-MOST	63	6	30	25	11	21	23

Actor/Move interactions. We also considered the distribution of Moves by Actor. Three moves—*develop, clarify* and *justify*—accounted for 111 of the 128 responses with a *same student* actor. We found that the distribution of these moves between MOSTs and non-MOSTs paralleled that for the data set overall; develop and justify moves occurred more frequently in response to MOSTs and clarify moves occurred more frequently in response to non-MOSTs. The predominant moves directed to the *whole class*—*collect* and *connect*—each occurred twice as often for MOSTs as non-MOSTs (although the numbers are small). Still different moves were the most common when the *teacher* was the actor, with *adjourn* being the most common move, followed by *dismiss*. Adjourn and dismiss moves necessarily had a teacher actor since when a teacher uses these moves, they do not provide an opportunity for students to publicly consider the instance. The majority of both of these moves were in response to non-MOSTs.

Recognition of Student Actions and Ideas. The Recognition codes operationalize the extent to which the student who provided the instance would recognize their thinking in the teacher's response. More responses to MOSTs than to non-MOSTs were classified as *explicit* or *implicit* for Student Action (86 MOSTs; 65 non-MOSTs), while more responses to non-MOSTs were classified as *not* aligning with student actions (13 MOSTs; 34 non-MOSTs). In terms of Student Ideas, 139 of the 198 total responses remained *core* to the SMT, and 19 of 198 were *peripheral*. As with the student actions, more responses to MOSTs than non-MOSTs were classified as core or peripheral (89 MOST; 69 non-MOST), while more responses to non-MOSTs were classified as *other* and *not applicable* (10 MOSTs; 30 non-MOSTs). These findings suggest that teachers' responses generally valued students' contributions by incorporating their actions and ideas.

Discussion of Productivity of Responses

We consider the productivity of teacher responses by examining the extent to which a response adheres to the four principles for productive use of student mathematical thinking (see Figure 1). We discuss several instances, including both MOSTs and non-MOSTs, to illustrate how responses with different coding can be more or less productive given the type of SMT to which the response is given.

The majority of *adjourn* moves (21 of the 24) occurred in response to two particular non-MOSTs instances. The productivity of this move is not the same for each instance, however. The first instance, classified as Opening, involved a student, Sam, giving an answer before other students had time to think about the task. Here, the common response, "Let's give everyone a chance to work it out and see what everyone else gets" (T10) is productive because of the poor timing of Sam blurting out his answer. Adjourning Sam's response provides all of the students in the class sufficient time to engage in sense making. In scenario G2 (where Pat claims to have arrived at the same answer in a different way; classified as Student Mathematics), a similar adjourning response to "address Chris's [the previous student's] comment first" (T3) might be less productive. Because we do not know Pat's "different way," making a move to *develop* his idea (as 12 teachers did) could lead to an opportunity to compare and contrast two different solution methods. Thus, develop responses, such as, "Talk to us Pat. What did you do?" (T9) seem more productive than adjourning this instance. T9's develop move would position Pat as a legitimate mathematical thinker and provide an opportunity for *all* students to make sense of the relationship between Pat's and Chris's contributions.

Develop moves with a *same student* actor were common for MOSTs. However, because MOSTs are opportunities for *building* on SMT—making student thinking the object of consideration by the class—asking the same student to develop or justify their idea may not always be necessary and may actually limit other students' opportunities to jointly participate in making sense of mathematical ideas. For example, consider scenario A2 (Figure 2), for which nearly half of the develop move responses with same student actor for MOSTs occurred. The most common teacher response in this instance was to ask Casey, the student who made the suggestion, to explain how they would change the story (e.g., "Well what do you mean? What sort of an equation, or what sort of a real-life situation can you think of where that would be a negative?" (T6)). Contrast this response with a similar response directed instead to the whole class: "Interesting comment; who can come up with a story, a situation that would match what Casey is saying?" (T7). In this case, directing the response to the whole class might be more productive, as it would engage all of the students in trying to come up with a situation where the coefficient is negative, increasing the likelihood of advancing the entire class's understanding of the mathematics of linear equations. This type of response aligns with all four core principles underlying productive use of a MOST, as it positions the students as capable of collaboratively making sense of the SMT.

Sometimes it is productive, however, to direct a response back to the same student. Consider an instance in which the student, Jesse, said, "It would have to be divided by x," an imprecise statement that needs clarification because "it" is unclear. The most common move in response to this instance was *clarify*, typically by asking Jesse, "what do you want to divide by x?" (T8). This response might be quite productive in helping members of the class figure out what Jesse was saying, and in doing so, would position Jesse's statement as legitimate mathematical thinking. Although the collaboration principle for productive use of SMT (Figure 1) privileges turning SMT over to the whole class whenever appropriate, cases like this one illustrate instances where directing the teacher response to the same student may be a productive first step. Productivity also depends on the recognition of student actions and ideas. A large percentage of the responses in our data were both *explicit* and *core*, meaning that the teachers in this study honored the SMT by incorporating the student's verbal or non-verbal actions and staying focused on the student's core ideas. For example, T8's response to Jesse discussed above is *explicit* and *core* as it incorporates both the student's words (divide by x) and his ideas (what he wanted to divide). A response such as this positions the student as a legitimate mathematical thinker by keeping the students' mathematics at the forefront, important aspects of productively using SMT.

Conclusion and Implications

By studying teachers' responses to a common set of instances of SMT with varying potential for incorporation into instruction, this study contributes to the teacher response literature by illuminating (a) how teachers' responses vary depending on the potential the SMT that is shared has to support student mathematical learning and (b) why some teacher responses are more productive than others in particular situations.

The results revealed that the teachers were generally able to distinguish when different moves might be more productive, as different moves were often employed in response to MOSTs and non-MOSTs. For example, most *whole class* actor responses occurred with MOSTs and most *teacher* actor responses occurred with non-MOSTs. However, our finding that the majority of teacher responses to both MOSTs and non-MOSTs were directed to the *same student* raises some concerns, as MOSTs are prime opportunities for teachers to engage in the teaching practice of *building* on SMT. Thus, directing responses to such instances to a single student results in a missed opportunity. In terms of responsiveness, we found that teacher responses to both MOSTs and non-MOSTs often stayed *core* to the ideas in the SMT and *explicitly* incorporated the students' actions, signaling that these teachers valued students' contributions and often positioned students as legitimate mathematical thinkers who can make valid contributions to the development of the mathematics in the classroom.

Our findings have the potential to help teacher educators develop a more nuanced understanding of what teachers are doing well and where they may need support, thus providing more focus to their teacher development efforts. For example, suppose the majority of a teacher's responses honor SMT, but engage only the contributing student. Professional development work could focus specifically on expanding the ways that they honor SMT by studying the potential in directing a response to the whole class, and when it would and would not be appropriate to do so. Such focused efforts would allow teacher educators to leverage teachers' strengths and thus develop teachers' practice more effectively.

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