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Teachers' orientations toward using student mathematical thinking as a resource during whole-class discussion

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Abstract

Using student mathematical thinking during instruction is valued by the mathematics education community, yet practices surrounding such use remain difficult for teachers to enact well, particularly in the moment during whole-class instruction. Teachers' orientations—their beliefs, values, and preferences—influence their actions, so one important aspect of understanding teachers' use of student thinking as a resource is understanding their related orientations. To that end, the purpose of this study is to characterize teachers' orientations toward using student mathematical thinking as a resource during whole-class instruction. We analyzed a collection of 173 thinking-as-a-resource orientations inferred from scenario-based interviews conducted with 13 teachers. The potential of each orientation to support the development of the practice of productively using student mathematical thinking was classified by considering each orientation's relationship to three frameworks related to recognizing and leveraging high-potential instances of student mathematical thinking. After discussing orientations with different levels of potential, we consider the cases of two teachers to illustrate how a particular collection of thinking-as-a-resource orientations could support or hinder a teacher's development of the practice of building on student thinking. The work contributes to the field's understanding of why particular orientations might have more or less potential to support teachers' development of particular teaching practices. It could also be used as a model for analyzing different collections of orientations and could support mathematics teacher educators by allowing them to better tailor their work to meet teachers' specific needs.

Keywords Teacher orientations · Teacher beliefs · Student mathematical thinking · Using student thinking · Building on student thinking

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Using student mathematical thinking during instruction has been valued and widely discussed in the mathematics education community for over 25 years (e.g., National Council of Teachers of Mathematics 1989, 2000, 2014), yet practices surrounding such use remain difficult for teachers to enact well, particularly in the moment during whole-class instruction (Peterson and Leatham 2009; Scherrer and Stein 2013). To begin to address the challenge of helping teachers develop their in-the-moment use of student mathematical thinking, the field must develop its understanding of both how teachers use student thinking during their instruction and why they use it in those ways. Although the field has begun to develop an understanding of the how (e.g., Lineback 2015), less is known about the why. Understanding teachers' reasons for using student mathematical thinking in particular ways could assist researchers and teacher educators in being more responsive to teachers in efforts to support them in enhancing their teaching practice. To that end, the purpose of this study was to characterize teachers' orientations toward using student mathematical thinking as a resource during whole-class instruction.

Conceptualization of orientations

Informed by the orientation and resource components of Schoenfeld's (2011) theory of goal-oriented decision making, we focus on teachers' orientations—their "dispositions, beliefs, values, tastes and preferences" (p. 29)—toward using student thinking as a resource to support instruction. We view orientations as "dispositions to actions" (Rokeach 1968, p. 113), meaning that orientations inform how teachers perceive, interpret and react to the classroom environment (Leatham 2006). Thus, one important aspect of understanding teachers' actions is understanding the orientations that influence those actions.

We recognize, however, that teachers may not be explicitly aware of their orientations (Leatham 2006), meaning that one cannot simply ask a teacher to describe their orientations and expect to receive a complete account. Rather, orientations must be inferred using other means, such as through observing a teacher's practice or by putting teachers in a context where they reflect on their practice. In drawing such inferences, we take the perspective that decisions about using student thinking must always be viewed as sensible to the teacher enacting (or proposing) the response (Leatham 2006). Other researchers posit similar perspectives. Simon and Tzur (1999), for example, discussed developing an account of a teacher's practice that "portrays the reasonableness of all the teacher's observed actions" (p. 255). Similarly, Herbst and Chazan (2012) used the term *practical rationality* to explain the sensibleness of teachers' actions based on factors such as their dispositions, "obligations to the mathematics teaching profession" (p. 611), and accepted norms of classroom activity. Thus, we see the process of inferring orientations as a process of making sense of teachers' actions and their reasons for those actions.

Conceptualization of student thinking as a resource

Given our focus on exploring teachers' orientations toward using student mathematical thinking as a resource during whole-class instruction (subsequently referred to as thinking-as-a-resource orientations), we describe how we conceptualize student thinking as a resource during instruction. We see student thinking as a critical resource for enacting ambitious teaching practices (Lampert et al. 2013). Our work focuses on a critical subset of these practices that encompasses productively using student mathematical thinking as a resource during whole-class instruction. This subset of practices includes noticing students' mathematics (Jacobs et al. 2010) and responding in productive ways (Stockero et al. 2017).

In order to better understand this subset of ambitious teaching practices, we conceptualized teachable moments—high-potential instances of student thinking that, if made the object of consideration by the class, could help students better understand important mathematical ideas. We refer to these instances as MOSTs—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking (Leatham et al. 2015); we refer to the practice of taking advantage of MOSTs as *building* (Van Zoest et al. 2016). In the following paragraphs, we describe these theoretical constructs and the principles that, together, form the underpinnings of the study at hand.

As elaborated elsewhere (Van Zoest et al. 2016), our conception of productive use of student thinking as a resource during instruction is based on core principles of quality mathematics instruction that we distilled from current research and calls for reform (e.g., NCTM 2014): (a) students' mathematics is at the forefront (Mathematics Principle); (b) students are positioned as legitimate mathematical thinkers (Legitimacy Principle); (c) students are engaged in sense making (Sense Making Principle); and (d) students are working collaboratively (Collaboration Principle). Throughout this study, we viewed teachers' thinking-as-a-resource orientations through the lens of these *Core Principles*.

As we have described elsewhere in greater detail (Leatham et al. 2015), MOSTs occur at the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunity. For each characteristic, two criteria determine whether an instance of student thinking embodies that characteristic. For student mathematical thinking, the criteria are: "(a) one can observe student action that provides sufficient evidence to make reasonable inferences about *student mathematics* and (b) one can articulate a mathematical idea that is closely related to the student mathematics of the instance—what we call a *mathematical point*" (Leatham et al. 2015, p. 92). The criteria for significant mathematics are: "(a) the mathematical point is *appropriate* for the mathematical development level of the students and (b) the mathematical point is *central* to mathematical goals for their learning" (p. 96). Finally, "an instance embodies a pedagogical opportunity when it meets two key criteria: (a) the student thinking of the instance creates an *opening* to build on that thinking toward the mathematical point of the instance and (b) the *timing* is right to take advantage of the opening at the moment the thinking surfaces during the lesson" (p. 99). The six *MOST Criteria* are considered linearly and an instance of student mathematical thinking is classified according to the last criterion it satisfies (student mathematics, mathematical point, appropriate, central, opening, and timing). Those instances that appear mathematical, but for which the student mathematics cannot be inferred, are designated *cannot infer* (CNI). When an instance satisfies all six criteria, it embodies the three requisite characteristics and is a MOST.

Applying the Core Principles of quality mathematics instruction to our vision of taking advantage of MOSTs led to a definition of the practice of building on student mathematical thinking. We define building as making an instance of student mathematical thinking "the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al. 2017, p. 36). An enactment of the practice of building aligns with all four Core Principles: (a) student mathematics is at the forefront (Mathematics Principle) given the focus on better understanding an important instance of student mathematical thinking; (b) students are positioned as legitimate mathematical thinkers (Legitimacy Principle) as it is their mathematical thinking that is both the object of consideration and the substance of the class discussion; (c) students are engaged in sense making (Sense Making Principle) as they unpack each other's ideas; and (d) students are working collaboratively (Collaboration Principle) in this whole-class discussion.

We further conceptualize four *building subpractices* associated with productively enacting the practice of building (Van Zoest et al. 2016). The first subpractice is to *make precise* the student mathematics of the MOST so that what students are meant to consider is clear. The second subpractice, the *grapple toss*, turns the student mathematical thinking over to the class with parameters that put them in a sense-making situation. The third subpractice involves *orchestrating* a whole-class discussion in which students collaboratively make sense of the object of consideration, the MOST. The fourth subpractice is to facilitate the extraction and articulation of the important mathematical idea from the discussion; that is, to *make explicit* that idea.

As can be seen in our descriptions of the Core Principles and the associated conceptualizations of MOSTs and the practice of building on MOSTs, we see student mathematical thinking as a valuable resource during whole-class instruction—a resource that is both mathematically rich and pedagogically potent. We also see mathematics teacher education as an enterprise that endeavors both to develop teachers' knowledge of effective teaching practices and to provide experiences that confront orientations that would potentially hinder effective teaching practice and, conversely, that strengthen orientations that would potentially support effective practice (Leatham 2006; NCTM 2014).

Literature review

We found no research reporting specifically on teachers' thinking-as-a-resource orientations. Across the literature, however, within the context of studies on mathematics teachers' beliefs, we did find evidence of such orientations. The thinking-as-a-resource orientations inferred from the literature broadly fall under three themes: (a) student mathematical capability, (b) student thinking informing instruction, and (c) the utility of student errors in instruction. Within our discussion of each theme, we contrast orientations that potentially support effective teaching practices with those that potentially hinder such practices, and thus would also potentially support or hinder the development of such practices.[1]

Orientations toward student mathematical capability

Teacher orientations toward student mathematical capability fall on a continuum from those with potential to support effective teaching by recognizing students as legitimate mathematical thinkers to those that would potentially hinder effective teaching by not recognizing them as such. Some teachers see students as capable of engaging with mathematical ideas (Beswick 2007; Cross 2009; Schleppenbach et al. 2007), talking about mathematics, and teaching one another (Lloyd 2005), while others do not see students as capable of such engagement with mathematical ideas (Bray 2011). Although some teachers view

[1] Because of the logical connection we are making between potential to support or hinder teaching practices and potential to support or hinder the development of such practices, from here on we use "potential to support the practice" and "potential to support the development of the practice" interchangeably.

students as capable of solving mathematical problems without prior instruction (Vacc and Bright 1999), others believe students' thinking processes must be highly scaffolded in order to "send kids off on appropriate paths, not just let them wander through the minefield" (Beswick 2007, p. 111). Teachers with orientations similar to the latter often doubt whether students are capable of solving mathematical problems on their own or should be allowed to use their own strategies (Vacc and Bright 1999), with some believing that students may not be able to face new problem situations without prior explanations to equip them with the specific skills for those situations (Aguirre and Speer 1999).

Orientations toward student thinking informing instruction

The literature also reveals teacher orientations related to how student thinking should inform instructional decisions. Like orientations toward students' mathematical capabilities, these orientations also fall on a continuum, varying from teachers who believe in tapping into student thinking as it surfaces in the moment (thus potentially supporting effective teaching practice) to those for whom emerging student thinking in the classroom has limited impact on their planned lessons (thus potentially hindering effective teaching practice). Beswick (2007) described a teacher who believes that it is important to take up student thinking in the moment: "If… kids come up with a 'what if' idea or 'I wonder what happens if we do this', then I'd absolutely grab it all the time" (p. 105). In addition, this teacher believed that student ideas have potential that may go unnoticed unless there is a deliberate effort to clarify and interpret student responses:

> If you just listen to their words, because they haven't got the language and they haven't got the background… it's easy to dismiss stuff as ludicrous, but if you got a culture where you can sit and try to tease it out and explain it, often they come up with amazing sorts of things (p. 106).

The description of this teacher seems to suggest an orientation that positions student thinking as a valuable resource to be used during instruction to support the development of mathematical ideas. More recently, this kind of positioning of student thinking was also identified by Lee and Cross Francis (2018), as reflected in teachers' comments about their objectives for using student thinking such as, "starting with where they understand to build on what students understand" (p. 122).

A similar orientation that would likely support leveraging the potential in student thinking during instruction is a belief in encouraging students to engage in sense-making activities in order to facilitate their learning (Cross 2009). Consistent with this orientation is a belief that students learn through discussing and explaining ideas to each other and, therefore, that working collaboratively supports their construction of knowledge (Aguirre and Speer 1999). A similar belief is reported in Bray (2011)—that students can make productive contributions to one another's ideas, so whole-class discussions should be deliberately structured to allow students to comment on their peers' ideas. These orientations seem to position student mathematical thinking at the forefront of instructional decision making.

The literature also reveals teacher views that reflect a diminished role of student mathematical thinking in making instructional decisions. Examples of such views are highlighted in Stockero (2013), where teachers were found to focus on students' more general needs rather than their mathematical thinking and on how student thinking aligns with the teacher's expectations. Similarly, Lloyd (2005) described a teacher who seemed to value student thinking in general and was excited that students were talking about mathematics.

The teacher also believed, however, that mathematics is something to be explained and that students do not need to grapple with and respond to one another's unanticipated ideas. These latter beliefs would potentially limit the ways this teacher leveraged student thinking during instruction. In this case, the role of student thinking in instructional decisions is restricted by orientations that position the teacher as the provider of knowledge and students as recipients of that knowledge. Such a restrictive view of how student thinking may be applied in instruction is also described by Cross (2009), who described one teacher's belief as "students learn mathematics best through demonstration and practice" (p. 334). Another orientation consistent with a view of teacher as knowledge provider is highlighted by Aguirre and Speer (1999) in their description of a teacher who believes students can only learn when the teacher explains things thoroughly. Teachers with beliefs that reflect a diminished role of student mathematical thinking often use student thinking to inform instructional decisions only in a limited way, and tend to stick to their lesson plans regardless of the student thinking that emerges, making little effort to clarify and interpret that thinking (Vacc and Bright 1999).

Orientations toward the utility of student errors in instruction

A third substantial subset of orientations related to how student thinking informs instructional decisions consists of orientations related to the use of student errors. Teacher orientations toward student errors vary from views of student errors as useful for furthering learning and thus something that should be elicited, to views of student errors as having potential to impede learning and thus something that should be avoided. Researchers, however, have argued that errors can be useful in fostering learning (e.g., Borasi 1994; Kazemi and Stipek 2001) because bumping into errors engages students in the process of equilibration that is fundamental to the development of knowledge (Piaget 1964).

Errors are viewed by some teachers as useful in providing them with information about where students may need support in developing an understanding of mathematical ideas (Schleppenbach et al. 2007). Beyond viewing errors as formative assessment, other teachers view them as learning opportunities both for the student who made the error and for the class (Bray 2011). Some teachers who believe in the usefulness of errors are oriented toward making errors public by being intentional about eliciting problematic student thinking (Bray 2011; Schleppenbach et al. 2007) and providing opportunities for students to consider the validity of those ideas (Schleppenbach et al. 2007).

On the opposite end of the spectrum of orientations related to student errors are orientations that limit opportunities for students to consider such errors, and thus limit their opportunities to better understand the important mathematics that often underlies an error. For example, Bray (2011) described a teacher who values tightly controlled discussion that limits opportunities for other students in the class to contribute to the discussion of an error. Other limiting orientations are those that involve the teacher responding to errors with evaluative statements or quickly correcting the error by providing a brief explanation (Schleppenbach et al. 2007). As one teacher stated, "Not only tell them that they are wrong but tell them why they are wrong and to help them find their mistakes and correct them" (Cross 2009, p. 334). Yet other orientations view errors as something to be avoided, with some teachers preferring to focus on correct student reasoning rather than incorrect reasoning (Lee and Cross Francis 2018). Other teachers believe that confronting errors can hurt students' feelings (Bray 2011) or embarrass students (Lee and Cross Francis 2018), while still others see errors as likely to impede students' learning of new content and thus hold the orientation that "incorrect solutions should be intercepted" (Aguirre and Speer 1999, p. 339).

Summary

We drew from the literature on mathematics teacher beliefs to paint a picture of what is currently known about teachers' thinking-as-a-resource orientations. Although we have some understanding of teachers' orientations toward students' mathematical capabilities and the usefulness of their thinking (primarily of their incorrect thinking) during instruction, the picture at this point is incomplete. This paper begins to provide a more complete picture by explicitly seeking to infer teachers' thinking-as-a-resource orientations and by analyzing these orientations according to their potential to support or hinder teachers' productive use of student thinking.

Methodology

In previous work (e.g., Peterson and Leatham 2009; Stockero and Van Zoest 2013), we have used classroom observations and video recordings of instruction to explore teachers' responses to student ideas. These methodologies, however, proved problematic in our efforts to infer teachers' thinking-as-a-resource orientations because of the difficulty in making comparisons among the practices of teachers who are teaching different content, in different contexts, with different student responses, and thus, have different opportunities to use student ideas. To provide a mechanism for better understanding how different teachers respond to student mathematical thinking in similar situations, we developed a scenario-based interview (Scenario Interview) as a tool for prompting teachers' reflection on their use of student thinking and inferring the orientations (Schoenfeld 2011) that underlie this use.[2] In the sections that follow, we describe the Scenario Interview and then our approach to inferring and characterizing teachers' thinking-as-a-resource orientations.

The Scenario Interview

The Scenario Interview was created to provide evidence of how a teacher thinks about using student thinking during instruction and to infer the teacher's orientations in the context of using student thinking. In the current analysis, we used Scenario Interview data to focus specifically on *teachers' thinking-as-a-resource orientations*.

During the Scenario Interview the teacher is presented with statements from eight individual students—four each from an algebra and a geometry context—that represent a range of instances of student thinking that satisfy different sets of MOST Criteria, including statements in which the student mathematics cannot be inferred, those that are mathematically significant but have poor pedagogical timing, and those that are MOSTs. (See "Appendix 1" for all eight scenarios instances, their contexts, and MOST classifications.) Each student statement and a description of what happened in the classroom immediately preceding

[2] We acknowledge that no single instrument would provide sufficient data to infer a teacher's entire set of orientations. We also acknowledge that observation data and reflection data both have affordances and constraints when inferring orientations.

the statement was developed based on instances that the research team had observed in classrooms. During the interview, the interviewee is situated as the teacher and is asked to describe what they might do next were the student statement to occur during whole-class discussion in their mathematics classroom. Although normally a classroom teacher would know the context of the situation in which the student thinking occurred (e.g., the task that students are working on, what prompted the student statement), the Scenario Interview initially does not reveal to the interviewee any contextual information. After being presented with an instance of student thinking, the teacher is given an opportunity to ask questions about the context. These questions offer insight into possible orientations that underlie their decisions about how to respond to student thinking. The interview then provides five more opportunities for the teacher to reveal their orientations by asking them to: (1) describe what they would do immediately after the student's statement was made, (2) explain why they would respond in that way, (3) articulate any assumptions they were making that informed their decision, (4) explain their reason for wanting to know the contextual information they asked about, and (5) describe whether their response to the student thinking would have been different had they known particular contextual information (provided to all interviewees if not heretofore requested).

Video recorded interviews were conducted with a convenience sample of 43 secondary school mathematics teachers from several sites across the USA. Initially ten teachers' interviews were chosen for analysis to provide a spectrum of types of use of student thinking—from very teacher-centered uses like explaining the student's idea, to very student-centered uses like engaging students in a discussion of ideas that have been shared. This selection of these ten teachers was based on the researchers' knowledge of the teachers' classroom practice and initial impressions from having conducted the interviews. Following the initial analysis of ten teachers, we found that we had not identified very many orientations at the "upper end" of the orientation range. Thus, three additional teachers were added to the data set. These teachers were chosen because, based on the researchers' observations of their teaching and their responses during the interview itself, they were thought to have the potential to flesh out the upper end of the spectrum of orientations. The added collection of inferred orientations did not significantly alter the overall range of orientations, but did contribute to our data by adding more examples of orientations at the upper end of the spectrum.

Analysis

Identifying orientations

The 13 interview videos were transcribed and the conversations broken down into teacher responses (an entire answer to an interview question, 1041 in total), which made up the initial unit of analysis. Using the video analysis tool Studiocode© (SportsTec 1997–2015), at least three members of the research group individually analyzed each teacher response to determine whether it included evidence of any potential thinking-as-a-resource orientations and, if so, wrote a statement capturing each potential orientation. The researchers then compared and discussed their statements, and came to an agreement on what potential orientation(s) each teacher response revealed. This analysis resulted in 451 instances that provided evidence of potential thinking-as-a-resource orientations. The complete collection of potential orientations identified from a particular teacher's responses was then sorted to identify themes that emerged. For example, three interview responses from one teacher included evidence that the teacher believed that it is important for students to have an opportunity to think about mathematics. This evidence included the statements, "I'm thinking I still have a lot of kids in the classroom who have not had a chance to resolve the dilemma between these two [solutions] and actually work through another problem…. I want every student in the classroom to have a chance to look through it and go, 'How am I going to process this?'" and "But I still want the rest of the kids to have some think time before he gets to justify his reasoning." The three responses together provided corroborating evidence that this teacher values student mathematical thinking and actively cultivates such thinking. Since having student thinking to work with is a necessary prerequisite to using student thinking as a resource, we thus inferred that the teacher had the following thinking-as-a-resource orientation: *it is important for students to have an opportunity to think about mathematical work*. This analysis of potential orientations resulted in 173 inferred thinking-as-a-resource orientations.

Alignment of orientations and frameworks

For the next phase of analysis, we anonymized the inferred orientations and took them as the unit of analysis. We then identified each orientation's potential or lack thereof for supporting the practice of building on student thinking and thus for supporting teachers' development of this practice.

We carried out this analysis by identifying how each previously identified orientation related to the following *building-related constructs* from our theoretical framework: (a) the principles underlying productive use of MOSTs (*Core Principles*), (b) the six MOST criteria (*MOST Criteria*), and (c) the subpractices of the practice of building on MOSTs (*Building Subpractices*). Table 1 summarizes the building-related constructs and their related codes and includes the key question asked and potential evidence looked for to determine whether an orientation aligned with a particular construct-related code; that is, whether a rational and reasonable teacher response motivated by the orientation would enact or satisfy a particular principle, criterion, or subpractice. Consistent with our perspective that teachers' responses are always sensible to the teacher enacting the response, this analysis involved inferring how a teacher would reasonably respond to an instance of student thinking given that they held a specific orientation. In particular, we used the following logic to determine alignment: Assume a MOST is on the table and that a particular orientation was to motivate a teacher's action in response to that thinking. What teacher action would be rational and reasonable and how would that response be related to the Core Principles, MOST Criteria and Building Subpractices? Additional details about these analyses are given in the paragraphs that follow; a summary of the alignments to the building-related constructs that were inferred for a subset of the orientations can be found in "Appendix 2".

For each orientation, we began by inferring how it related to the Core Principles. (See Table 1 and "Appendix 2".) For example, consider the inferred orientation, *it is valuable for students to see and hear other students' mathematical explanations.* Were this orientation to be the driving force behind this teacher's response to an instance of student thinking that is a MOST (in this case, a student's mathematical explanation), it would be rational and reasonable that they would allow other students to consider the explanation. Because this response would allow the MOST to be considered by members of the class and to be available as an object of discussion, this orientation is aligned with the Mathematics Principle. It also aligns with the Legitimacy Principle because the response, allowing students to see and hear other's explanations, would position students as capable of thinking about

Table 1 Guiding questions and potential evidence for determining the alignment of orientations with the building-related constructs

Building-related constructs	Construct-related codes	Key questions: If this orientation were motivating a teacher's action in response to a MOST….	Potential evidence of alignment between orientation and construct
Core Principles	Mathematics	…would the student mathematics of the instance be likely to be the focus of the class discussion?	Students' ideas are made public.
			Students are oriented to each other's thinking.
	Legitimacy	…would students likely be positioned as capable of thinking about mathematical ideas?	Students' ideas are valued.
			Students are seen as capable of engaging with each other's ideas.
	Sense Making	…would opportunities likely be provided for students to make sense of the mathematical ideas under consideration?	Students are given opportunities to think about mathematical ideas.
			Students are given opportunities to explain their reasoning.
			Students can ask their peers questions related to one another's ideas.
	Collaboration	…would opportunities likely be provided for students to collaboratively consider mathematical ideas?	Students are working together.
			Students are discussing each other's ideas.
MOST Criteria	Student Mathematics (SM)	…would the teacher be likely to accurately infer the SM?	Students' ideas are clarified when necessary.
			The reasoning behind students' responses is sought.
	Mathematical Point (MP)	…would the teacher likely be able to articulate the MP?	Understanding student thinking is important.
			The meaning and logic underlying mathematical procedures is important.
			Connections across representations and mathematical ideas are important.
	Appropriate	…would the teacher likely consider whether the MP is accessible to students?	Students' understanding of ideas is monitored and guides the teacher's instructional decisions.
	Central	…would the teacher likely consider how the MP aligns with goals for student learning?	Lesson goals guide the teacher's decisions about how to respond to student thinking.

Table 1 (continued)

Building-related constructs	Construct-related codes	Key questions: If this orientation were motivating a teacher's action in response to a MOST….	Potential evidence of alignment between orientation and construct
			Student ideas are considered in relation to how the underlying mathematical ideas are situated in the curriculum.
	Opening	…would the teacher likely consider whether the SM creates intellectual need for students in the class?	Students' incorrect thinking is valued.
			Students' understanding of ideas is monitored.
			Students' ideas are considered in the context of other students' likely reactions to the ideas.
	Timing	…would the teacher likely consider whether the pedagogical timing is right to pursue the instance?	The flow and progression of student contributions are considered in directing discussion.
			Student thinking is addressed in light of the stage of learning (introductory or solidifying) in the classroom.
Building Subpractices	Make Precise	…would the student mathematics of the instance under consideration likely be clarified?	Students are asked to clarify ideas they have shared.
			The whole class is involved in understanding an idea that has been shared by a student.
	Grapple Toss	…would the student mathematics of the instance likely be turned over to the class for consideration in a way that necessitates sense making of the idea?	Students are invited to discuss one another's ideas.
			Students are oriented to each other's thinking.
	Orchestrate	…would class discussion likely be directed towards making sense of the student mathematics of the instance?	Students see, listen to, comment on and critique one another's ideas.
	Make Explicit	…would the mathematical idea underlying the student mathematics of the instance likely be made explicit?	The big mathematical ideas in student contributions are brought out in the discussion.

those shared ideas. Additionally, the response would provide a context for students to work together to make sense of the explanations; thus, the orientation aligns with the Collaboration Principle. As a second example, consider the orientation, *the teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students.* Were this orientation to be what motivated this teacher's response to a MOST, it would be reasonable that the teacher would explain their own understanding of the given mathematics, placing their mathematics rather than the student's at the forefront; thus, the response would misalign with the Mathematics Principle. The response would also be misaligned with the Legitimacy and Sense Making Principles because the response—explaining mathematical ideas to students—would not position the students as legitimate thinkers who are capable of directly engaging in making sense of the mathematics of the MOST. Rather, the students would likely be positioned as passive recipients of the teacher's thinking about the student contribution.

Next we inferred how the orientation related to the MOST Criteria. (See Table 1 and "Appendix 2".) We provide several examples to illustrate our analysis of the alignment between orientations and the MOST Criteria. First consider the orientation, *it is important for the teacher to make sense of student thinking, both to avoid making wrong assumptions about it and also to know how best to respond to it*. Were this orientation to be the driving force behind this teacher's response to a MOST, the rational and reasonable teacher action would be to ask clarifying questions if they were unable to fully make sense of the student's contribution. This response would support the teacher in considering the Student Mathematics (SM) Criterion, since they would be focused on figuring out what the student is trying to say, which is exactly what is necessary to infer the student mathematics of an instance. Now consider another orientation, *it is more important for students to understand the underlying logic and big ideas in mathematics than to memorize procedures*. Were this orientation to motivate the teacher's response to a MOST, it would be reasonable that the teacher would take the time to consider what key mathematical ideas are most closely related to the student thinking. This consideration would support the teacher in articulating the mathematical point of the instance of student thinking, thus the orientation aligns with the Mathematical Point (MP) Criterion. Finally, consider the orientation, *the goals of a lesson are important in guiding a teacher's decisions related to using student thinking.* Were this orientation to underlie a teacher's response, it would be rational and reasonable for the teacher to focus on determining whether an instance of student thinking contains mathematics that is central to students' learning—that is, whether incorporating the student thinking into the lesson would help the teacher meet their learning goals. Thus, this orientation would potentially support the teacher in considering the Central Criterion, which focuses on whether an instance is aligned with learning goals for students in the class. By contrast, analysis revealed some orientations that could potentially hinder a teacher's ability to analyze some MOST Criteria. For example, were the orientation *looking at multiple solutions is useful, but it may not always be feasible to use class time to do this* to be the driving force behind a teacher's response, it would be reasonable that the teacher might consider clock time, rather pedagogical timing, as the basis for determining whether students should consider an important student contribution. Such a focus would hinder the teacher in considering the Timing Criterion.

Our final analysis with respect to identifying each orientation's potential or lack thereof for supporting an enactment of the practice of building on student thinking involved inferring how the orientations might be related to the Building Subpractices (see Table 1 and "Appendix 2"). We again use examples to illustrate how orientations might align or not align with particular subpractices. First consider the orientation, *students can identify* *mistakes and question the shared work of fellow students without the teacher intervening to ask questions.* Were this orientation to guide a teacher's response to a MOST, the teacher would reasonably turn over student ideas—even those that are not correct or complete—for the class to consider. Thus, the orientation aligns with the Grapple Toss Subpractice. The response would also provide an opportunity for the class to discuss student ideas that are shared, providing support for the orientation's alignment with the Orchestrate Subpractice. In contrast, consider the orientation, *students should share their ideas one at a time and the teacher should resolve one student's idea before another idea is shared*. Were this orientation to motivate a teacher's response to a MOST, it would be reasonable that the teacher would not turn over ideas for the class to consider since such an action could open the floor to additional student ideas. This, in turn, could prevent them from orchestrating a discussion about students' ideas. Thus, the orientation is misaligned with the same two subpractices—Grapple Toss and Orchestrate.

Potential of orientations

For each orientation, we tallied the number of identified alignments between it and the building-related constructs, keeping track of whether those alignments were either supporting or hindering.[3] The frequency of tallies for each orientation allowed us to rank them according to their potential to support or hinder the development of the practice of building on student thinking, thus creating a continuum of sorts for these orientations.

This orientation analysis was done without knowledge of which orientations belonged to which teacher; the teacher identifications were only reconnected to the orientations later in order to facilitate an analysis of patterns for individual teachers in order to select illustrative cases.

Results

We begin by reporting and illustrating the range of potential of the complete set of inferred thinking-as-a-resource orientations. We then present the cases of two teachers to illustrate what it would look like for a teacher's collection of orientations to position them at very different levels of readiness to develop the practice of building on student mathematical thinking.

Orientation levels of potential

The orientations fell on a continuum from those that had high potential to support teachers in developing the practice of building on student thinking to those that had the potential to hinder the development of this practice, with tallies ranging from eight to negative four. Fifty-five orientations had only one tally, meaning that they aligned with only a single Core

[3] Although the MOST Criteria and Building Subpractices are undergirded by the Core Principles, we did not feel that the alignment analysis over counted alignments because the Core Principles alignment captured core ideas or beliefs underlying the productive use of student thinking, while the MOST Criteria and Building Subpractice alignment captured discrete skills or practices in which teachers would engage when enacting building; in other words, the MOST Criteria and Building Subpractices represent things teachers might *do*.

Principle, MOST Criterion, or Building Subpractice. To give a sense of orientations at different points on the continuum, we describe and illustrate typical thinking-as-a-resource orientations (a) with high potential, (b) with low potential, and (c) likely to hinder a teacher in building.

High-potential orientations

Thinking-as-a-resource orientations at the upper end of the continuum (see "Appendix 2") aligned with several building-related constructs. These orientations commonly aligned with the Legitimacy, Sense Making, and Mathematics Principles; they also supported at least one (and often several) of the MOST Criteria or Building Subpractices.

These orientations position student thinking as a particularly valuable resource during whole-class instruction, with value placed on students directly interacting with one another by hearing other students' explanations, and questioning, comparing, critiquing, and discussing their peers' ideas. Furthermore, these orientations assume that no two people view mathematics in exactly the same way, so all ideas should be carefully interpreted and clarified when necessary in order to avoid making incorrect assumptions about what students are saying. Orientations with high-potential position student thinking, even thinking that is not completely correct, as a resource from which everyone in the classroom—including the teacher—could potentially learn. Such orientations view student thinking as providing valuable information related to making instructional decisions about the flow, progression, and duration of mathematical conversation. These orientations position students as capable of evaluating ideas, asking questions, stopping the discussion, and introducing new ideas; they do not position teachers as the sole mathematical authority in the classroom.

With respect to the MOST Criteria, these orientations most often support the Appropriate, Central and Timing Criteria. They place value on monitoring, selecting, and sequencing student solutions in service of the lesson goal, thus supporting teachers' decision making about when and whether students' ideas are incorporated into the class discussion. Many of the high-potential orientations also support the Grapple Toss and Orchestrate Subpractices because these orientations support the sharing of student thinking with the whole class and making space for discussion of such thinking.

Low-potential orientations

Although low-potential thinking-as-a-resource orientations were related to effective mathematics instruction, they seemed to provide little leverage on their own for supporting teachers in developing the practice of building. These orientations aligned with only a single building-related construct, rather than the multiple alignments that were the hallmark of high-potential orientations. Low-potential orientations might reflect an interest in student thinking or in pressing students to provide justifications for their mathematical claims. Others reflect good instructional practices like having all students "on board" or engaged in discussion, giving students time to think through a problem before whole-class discussion, or valuing the monitoring of students' understanding. In addition, some low-potential orientations suggest that there is value in considering the goals of the lesson while making instructional decisions related to the use of student thinking that emerges or when prioritizing certain mathematical ideas because of student needs. Although the instructional practices supported by low-potential orientations are all positive, when they occur in isolation, as they did for this group of orientations, they provide minimal leverage for developing the teaching practice of building.

The most common alignment for the low-potential orientations was with the Legitimacy Principle, with nearly half (43%) of such orientations having this alignment. Thus, many of these orientations position students as capable of contributing ideas and acknowledge that legitimate thinking takes time. Several other of these orientations aligned with a single MOST Criterion, most often the MP, Appropriate, or Central Criterion. For example, some suggest that it is valuable to prioritize the mathematical ideas that students should know, which might aid a teacher in considering the MP Criterion. Other orientations suggest the need to assess student understanding before moving on, an orientation that might support a teacher in considering the Appropriate Criterion. Yet other orientations suggest that it is important to consider lesson goals when determining how to respond to student thinking, which could support a teacher in considering the Central Criterion. Few low-potential orientations aligned with the Building Subpractices.

Hindering orientations

We inferred a relatively small collection of thinking-as-a-resource orientations that misaligned with our building-related constructs (see "Appendix 2"). Some orientations suggested that student learning needed to be highly scaffolded, and that the teacher's primary role when responding to student mathematical thinking is to explain and demonstrate related mathematical ideas. Furthermore, these orientations tended to view correction of errors as needing to be done quickly and before other student ideas surfaced. There were also several hindering orientations related to student capabilities, where students' ideas were seen as guesses or luck rather than as the result of sense-making activity.

This collection of orientations was typically misaligned with the Legitimacy and Sense Making Principles. That is, these orientations tend to delegitimize students as mathematical thinkers, placing the teacher as the one with legitimate mathematics. Similarly, these orientations tend not to put anyone in a sense-making situation or, at best, to put the teacher in the position of sense making in front of the students.

With respect to the MOST Criteria, the view that students might be guessing or have just gotten lucky would seem to hinder teachers from considering the SM Criterion. The greatest hindrance to the Building Subpractices would seem to be with respect to the Grapple Toss Subpractice, as many of these hindering orientations would likely stand in the way of the teacher looking for opportunities to ask other class members to consider a given student's thinking. These hindering orientations position the teacher as the one responsible to engage in such grappling. In addition, several of these orientations expressed a reluctance to engage in whole-class discussion, suggesting that students understand and retain better when they are taught one-on-one. Such an orientation would seem to be a serious impediment to developing the Orchestrate Subpractice.

Two illustrative cases

Mr. Taft: primed to build

Mr. Taft is an example of a teacher who has several orientations on the upper end of the potential continuum that suggest he is positioned well to develop the practice of building on student mathematical thinking during whole-class discussion. Mr. Taft held the orientation that *digging into student's thinking or having a student clarify their thinking can surface more reasoning and is beneficial to the class, so the main purpose/power of using student thinking is to promote learning for all students and to get them engaged*. For example, he stated that if he is "just using student thinking for my own benefit then I've kind of lost the power and opportunity in that student thinking…. It's to promote learning for all of the other students in the classroom." Thus, this orientation can support Mr. Taft in digging into or clarifying a student's ideas to benefit both that student and all the students in the class. Furthermore, beyond the more immediate pedagogical value of such probing, he wants to maintain the "norm and expectations" of "making sure everyone's with us, justifying our reasoning, all of those kinds of things," thus ensuring that he is supporting the thinking of all students in the class. This orientation aligns closely with the Legitimacy and Collaboration Principles because clarifying student ideas sends the message that these ideas are valuable, and using these ideas to promote the learning of all students in the class suggests collaborative engagement with the ideas. It also supports the first three Building Subpractices because clarifying student ideas supports the Make Precise Subpractice, and getting students engaged with the ideas to promote all students' learning supports the Grapple Toss and Orchestrate Subpractices.

Mr. Taft's orientation that *students can identify mistakes and question the shared work of fellow students without the teacher intervening to ask questions* suggests a willingness to allow students to have authority in the classroom. In many cases, he "wouldn't need to say much if I just put it back to the class" or allowed questions to "arise naturally" as students consider the ideas of others in the class. These orientations are particularly well-aligned with building, especially with the Legitimacy and Sense Making Principles, since students are believed to be able to independently think about and critique one another's ideas. They also support the Grapple Toss Subpractice since the teacher would allow space for students to consider shared work.

Mr. Taft's orientations about his own role in the classroom also lay a foundation for the practice of building. For example, Mr. Taft's orientation that *monitoring student work is important to direct class discussion and consider the flow and progression of student contributions* and his related orientation that *a teacher's actions related to student thinking, including their reasoning about selecting, sequencing, pursuing or dismissing student thinking, depends on the teacher's goals and the task at hand* allow him to "be more deliberate in the path that we take and kind of the things that we examine and discuss." The former of these orientations was one of the most highly-rated, aligning with three Core Principles (Mathematics, Legitimacy and Collaboration), as well as supporting three MOST Criteria (Appropriate, Central and Timing) and one Building Subpractice (Orchestrate). These orientations would clearly support the practice of building as they support the likelihood that the student-generated ideas that are incorporated into class discussion are those that will enhance student learning.

In addition to the high-potential orientations discussed previously, Mr. Taft also holds several orientations that fell at the mid-range of the potential continuum. Such orientations do not provide much leverage individually, but when taken in conjunction with one another or with the higher-potential orientations, they could support the development of the practice of building. Mr. Taft's orientation that *to use student thinking effectively, the teacher needs as much of it available as possible, thus it is important for student thinking to be accessible and shared with others* provides him with ample opportunity to use student thinking to benefit the class. To use such thinking effectively, he "needs as much of it on the table as possible" so students have access to the thinking. He further notes that "it's hard to dig in and make [ideas] shared and meaningful for the whole class if there's only the one student or a small number of students that have access to the conversation." This orientation aligns with the Legitimacy and Collaboration Principles because making student ideas available and accessible indicates that the ideas are important and sharing with others suggests collaborative engagement. Mr. Taft also recognizes the value of a good task in bringing out "some really good logical thinking or reasoning," noting that "if you have a good task or a good problem for the kids to start with, that often happens." His orientation that *a good task or problem can bring out higher level reasoning from students* aligns with the both the Legitimacy and Sense Making Principles because it indicates a belief that students are capable of engaging with and making sense of high-level tasks. Additionally, Mr. Taft stated that "it's really important that all of the students, as much as possible, are all thinking and reasoning about the tasks and problems." His related orientation that *it is important for all students to have an opportunity to think about problems or tasks* also aligns with the Legitimacy and Sense Making Principles. Although none of these orientations on their own provide significant leverage to develop the practice of building, collectively they can provide such support.

Taken together, Mr. Taft's orientations at the middle and upper end of the potential continuum provide a solid foundation on which to develop the practice of building. Importantly, we have evidence that his orientations collectively align with all four Core Principles underlying building. Additionally, he appears to already allow a lot of student ideas to emerge in his class. His orientation toward clarification suggests that Mr. Taft would work to make student ideas clear and precise, not only to himself, but also to other students in the class (SM Criterion and Make Precise Subpractice). There is also some indication that he is willing to toss ideas to the class for consideration (Grapple Toss Subpractice) and allow students to discuss those ideas (Orchestrate). Thus, we see a collection of orientations that align well with the early subpractices. This means that professional development could potentially initiate work with Mr. Taft around the Building Subpractices in which he already has begun to engage, moving on to develop the Make Explicit Subpractice of building.

Ms. Dean: serious roadblocks to building

Ms. Dean is an example of a teacher whose current thinking-as-a-resource orientations would likely cause significant roadblocks to developing the practice of building on student mathematical thinking. Perhaps most telling (pun intended) of these orientations is that *the teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students*. We had more evidence for this orientation than for any other of Ms. Dean's orientations. In general, "explanation" is her default response to presented student thinking. Such an approach, if repeated time and time again, would misalign with the Legitimacy Principle, constantly sending the message that students are not capable of engaging directly with their peers' mathematical thinking. This approach also places the teacher, rather than the students, in the role of making sense of the mathematics at hand, thus misaligning with the Sense Making Principle. Given that our inferred orientations for Ms. Dean suggest her tendency to respond to student thinking by moving immediately toward explanation, seldom asking for clarification, it seems likely that Ms. Dean would pursue her own mathematics more often than the mathematics that is intrinsic in the shared student mathematical thinking—a response that misaligns with the Mathematics Principle.

Another prominent and likely building-hindering orientation is related to the correctness of students' ideas: *it is the teacher's responsibility to correct student mistakes and misconceptions as quickly as possible*. If students' ideas are incorrect, she believes that it is her role to correct it, to say, "It's not quite correct. Here's why." Furthermore, if possible, Ms. Dean would seek to limit shared student mathematical thinking to correct thinking: "I try really hard to make sure that when I do pick on a kid that their probability of success in getting the answer right is higher than the probability of not success." This orientation toward correctness and correcting misaligns with the Sense Making Principle by again placing the teacher in the role of making sense of mathematics for students rather than placing the students in sense-making situations. The orientation further assumes that students are not capable, as individuals and as a class, of presenting worthwhile mathematical ideas and arguments in the presence of incorrect or incomplete thinking (and thus misaligns with the Legitimacy Principle).

Beyond a substantial number of hindering orientations, Ms. Dean does have several orientations that would provide at least low-potential support for developing the practice of building. That said, a number of these low-potential orientations, when taken together, would seem to send mixed messages to students with regards to the Legitimacy and Sense Making Principles. For example, although Ms. Dean does believe that students should have an opportunity to work individually on problems before someone shares their answer to those problems (thus making space for individual sense making), when it comes to sharing those ideas she would have students share one at a time, only moving on to another student's idea "when I'm done explaining." As another example, Ms. Dean does want students to contribute their ideas, and she does use them in the sense that she goes on to explain the mathematics behind what they are saying. But given the fact that she much prefers students to share correct thinking, students are likely to come away thinking that the teacher's mathematics is the only legitimate mathematics; their divergent thinking would be undervalued. In fact, Ms. Dean hopes that her quickly inserted explanations would cause other students with differing ideas to realize, "Oh, you know, that is not the right way then" and thus not share their thinking.

We thus characterize Ms. Dean's thinking-as-a-resource orientations as providing little leverage and rather, on the whole, significant barriers to developing the practice of building on student mathematical thinking. Were we to work with Ms. Dean in a professional development setting we would likely want to begin by discussing the possibility of and potential value in students reacting to other students' emerging (and thus often incorrect or incomplete) mathematical ideas. If Ms. Dean could allow space for students to respond to one another in place of immediate teacher explanation she may create space in her own learning for developing the practice of building. Such work would likely require paying close attention to student mathematical thinking and considering the pedagogical potential therein. The MOST Criteria and the Building Subpractices have potential to provide a meaningful structure for beginning such work.

Discussion

In the literature on mathematics teachers' beliefs, the potential of particular teacher orientations or beliefs to develop specific teaching practices is often implied, without any real support for making such an implication. Coding each thinking-as-a-resource orientation in terms of how it might support the development of the practice of building clarifies why some orientations related to using student mathematical thinking as a resource have more potential than others to support the development of this practice. For example, in the literature we saw evidence of teacher beliefs related to students' mathematical capability, including beliefs that students are capable of engaging with mathematical ideas (Beswick 2007; Cross 2009; Schleppenbach et al. 2007), talking about mathematics, and teaching one another (Lloyd 2005). These beliefs are implied to have potential to support teachers' use of student thinking without explicit arguments for why this is the case. We saw similar orientations in our study—for example, that *it is important to find out what students are thinking/understanding by having the class comment on or ask questions of the student whose thinking has been shared*. Rather than relying on intuition to infer that this orientation has high potential to support the development of the teaching practice of building, our coding scheme helps make sense of why this is so. In this case, the orientation aligns with all four Core Principles, supports inferring the SM (since the student would be required to clarify their idea), and also provides leverage for the first three Building Subpractices—Make Precise, Grapple Toss and Orchestrate. In fact, this orientation was scored as having the most potential to support building of any orientation in our data set.

On the other end of the continuum, the literature revealed beliefs related to students' mathematical capability that could be inferred to have less potential to support building, such as that students' thinking processes must be highly scaffolded in order to "send kids off on appropriate paths, not just let them wander through the minefield" (Beswick 2007, p. 111). We saw a similar orientation in our data, that *the teacher should respond to student thinking by explaining, showing, using examples and demonstrating mathematical ideas to students.* As discussed previously, our analysis of this orientation found that it misaligned with three Core Principles (Mathematics, Legitimacy and Sense Making). The orientation would also hinder the Grapple Toss Subpractice, since a rational and reasonable teacher response motivated by this orientation would involve the teacher explaining rather than the teacher inviting students to consider the thinking. Again, the coding provides insight into why this intuitively low-potential orientation would likely hinder the development of the practice of building on student thinking.

Similarly, the literature revealed teachers' orientations toward student errors that varied from a view of errors as learning opportunities—something that should be elicited—to errors as likely to impede learning, and thus something that should be avoided or intercepted. We identified orientations related to student errors that varied in similar ways. Our coding again helps illuminate why some orientations related to student errors have more potential to support the practice of building than others. For example, the orientation that *students can identify mistakes and question the shared work of fellow students without the teacher intervening to ask questions* was classified as high potential. This classification resulted from its alignment with the Legitimacy, Sense Making and Collaboration Principles, since the reasonable teacher response to a MOST would position students as capable of engaging with each other's ideas, allow the students to be involved in sense making when questioning the work of other students, and provide an opportunity for students to be involved in collaboratively making sense of ideas with their peers. Additionally, this orientation supports the Grapple Toss and Orchestrate Subpractices because were this orientation to motivate a teacher's response to a MOST, they would allow students to consider each other's ideas and make space for discussion. Knowing that this orientation supports building in several ways helps to explain why it is a high-potential orientation. On the other end of the continuum, the literature revealed orientations that position the teacher as responsible for promptly correcting errors because allowing students to discuss errors might cause the errors to become further ingrained—orientations that could be implied to hinder the use of student thinking. Cross (2009), for example, discussed a teacher who reported that they, “not only tell them that they are wrong but tell them why they are wrong and to help them find their mistakes and correct them” (p. 334), while Aguirre and Speer (1999) discussed a teacher who believes that incorrect solutions are likely to impede students’ learning so “should be intercepted” (p. 339). We identified a similar orientation related to errors—that *it is the teacher’s responsibility to correct student mistakes and misconceptions as quickly as possible*. Again, our coding scheme illuminates why this orientation would likely hinder building. The orientation misaligns with the Legitimacy and Sense Making Principles since the reasonable teacher response to a MOST would not allow students to respond to each other’s mistakes and instead would position the teacher as the one who is responsible for making sense of the error. Additionally, this orientation would not support the Grapple Toss Subpractice since the teacher would be unlikely to turn over student thinking, particularly errors, to the class. For these reasons, the orientation was classified as hindering the practice of building.

Conclusion

Although some thinking-as-a-resource orientations could be inferred from the literature on mathematics teachers’ beliefs, our analysis revealed a broader range of mathematics teachers’ thinking-as-a-resource orientations and characterized them according to their potential to support or hinder the development of the practice of building on student mathematical thinking. We identified high-potential orientations that position student thinking as a particularly valuable resource during whole-class instruction by placing value on students directly interacting with one another by hearing other students’ explanations, and questioning, comparing, critiquing, and discussing their peers’ ideas. We also identified hindering orientations that position student thinking as needing to be evaluated and corrected, and place value on highly scaffolded learning experiences in which the teacher’s role is to explain and demonstrate mathematical ideas. The orientations at the extremes of the continuum are likely to have substantial impact on a teacher’s development of the practice of building and thus would be of interest to the field. (A complete list of these orientations is included in “Appendix 2”.) Analyzing teacher orientations in the way that we have can contribute to the field’s understanding of why particular orientations might either support or hinder teachers’ development of particular teaching practices, and could be used as a model for analyzing different collections of orientations. Future research could build on these results by studying the orientations that motivate teachers’ actions during this developmental work.

Knowing the range of thinking-as-a-resource orientations has practical implications for mathematics teacher educators’ work. For example, knowing which orientations particular teachers hold could allow professional development providers to situate teachers on a learning-to-build trajectory, allowing the providers to better tailor their work to meet teachers’ specific needs. These efforts might involve identifying teachers who are ready to develop the practice of building and those who would require prerequisite work. One might expect, for instance, that a teacher who holds a hindering orientation such as that *student thinking should come from highly scaffolded instruction that minimizes student struggle* would need opportunities to consider other ways that student thinking might be supported, as well as potential advantages of these alternative methods, before they would be positioned to think about the practice of building. Knowledge of particular hindering orientations can help teacher educators develop ways to address them. Teachers who hold orientations on the upper end of the potential continuum, on the other hand, might be immediately ready to begin to develop the Building Subpractices. Again, future research could investigate the extent to which such varying orientations do indeed influence mathematics teachers' development of particular teaching practices.

Understanding teachers' orientations is particularly important to understanding teachers' actions since such orientations influence how teachers perceive, interpret and react to the classroom environment (Leatham 2006; Schoenfeld 2011). Thus, better understanding thinking-as-resource orientations themselves, as well as their potential to support the development of the practice of building on student mathematical thinking, could assist researchers and teacher educators in developing teachers' abilities to engage in ambitious teaching practices that effectively support student learning of mathematics.

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Appendix 1

See Table 2.

Table 2 Scenario Interview instances, with MOST classifications

Scenario	Context	Instance	HLI classification
G1	Students were sharing their solutions to the following task (a corresponding picture was on the board): *Given two concentric circles, radii 5 cm and 3 cm, what is the area of the band between the circles?*	Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."	MOST
G2		Before the teacher had a chance to respond to Chris, Pat says, "I also got 4π cm^2, but I did it a different way."	SM
G3		Pat explained how he got the same answer as Chris (4π cm^2) a different way: "π times r^2 for the big circle is π times 5^2, which is 10π and π times 3^2 is 6π for the little circle. I minused (sic) them and got 4π as my answer."	MOST
G4	The teacher has just posed a problem parallel to the one above with circles of radii 4 cm and 1 cm.	Sam says, "The answer is 15π cm^2."	Opening
A1	Students had been discussing the following task and had come up with the equation $y=10x+25$: *Jenny received $25 for her birthday that she deposited into a savings account. She has a babysitting job that pays $10 per week, which she deposits into her account each week. Write an equation that she can use to predict how much she will have saved after any number of weeks.*	Terry says, "If you deposit $20 per week instead of $10 per week, the number in front of the x in the equation would change, but the number that is added would stay the same."	Central
A2		Casey said, "You could also change the story so the number in front of the x is negative."	MOST
A3	The teacher asked, "How do we find the equation given any table?" and put this generic table of values [to the right] on the board for the students to use in their explanation. X Y 0 15 2 19 3 21 5 25	Jamie said, "I found the number in front of the x by subtracting the y-values in the table, $21-19$, so that number is 2."	MOST
A4	Following Jamie's response in the previous instance, the teacher asked, "Do others agree with Jamie?"	Jessie says, "It would have to be divided by x."	CNI

Appendix 2

See Table 3.

Table 3 Alignment of high and hindering thinking-as-a-resource orientations to the building-related constructs

High thinking-as-a-resource orientations	Principles				MOST Criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
It is important to find out what students are thinking/understanding by having the class comment on or ask questions of the student whose thinking has been shared.	+	+	+	+	+						+	+	+	
It is important to monitor student work in order to direct class discussion and consider the flow and progression of student contributions.	+	+		+			+	+		+			+	
Hearing the student thinking behind a response may help the class make sense of the mathematical idea underlying the response.	+	+	+	+								+		
It is preferable to direct a question related to a student's response to the whole class.	+	+	+	+								+		
Monitoring student work and sequencing are an important part of teaching a lesson.	+	+					+	+		+			+	

Table 3 (continued)

High thinking-as-a-resource orientations	Principles				MOST Criteria						Subpractices			
	Math-ematics	Legit-imacy	Sense Making	Collabo-ration	Student Mathemat-ics	Math-ematical Point	Appro-priate	Central	Opening	Timing	Make Precise	Grapple Toss	Orches-trate	Make Explicit
Student thinking is valuable and it is preferable that a student's ideas are tossed out for other students in the class to see and discuss, rather than the teacher being the one to evaluate those ideas.	+	+	+	+								+		
It is important for students to support, critique, listen to, and see other students' work and to explain their own thinking to others.	+	+	+	+									+	
Students learn by comparing their mathematical work, which in some cases could include an alternative solution or method provided by the teacher.	+		+	+		+						+		
Students can identify mistakes and question the shared work of fel-low students without the teacher intervening to ask questions.		+	+	+								+	+	
Digging into a student's thinking or having a student clarify their thinking can surface more reasoning and is beneficial to the class, so the main purpose/power of using student thinking is to promote learning for all students and to get them engaged.	+	+		+							+	+	+	

Table 3 (continued)

High thinking-as-a-resource orientations	Principles				MOST Criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
Student thinking should be made public so that students can discuss, correct, and improve their understanding of the student thinking.	+	+		+							+	+		
It is valuable for students to see and hear other students' mathematical explanations.	+	+		+								+	+	
A teacher's actions related to student thinking, including their reasoning about selecting, sequencing, pursuing or dismissing student thinking, depends on the teacher's goals and the task at hand.	+						+	+		+			+	
Hindering thinking-as-a-resource orientations	**Principles**				**MOST criteria**						**Subpractices**			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
The teacher should respond to student thinking by explaining, showing, using examples, and demonstrating mathematical ideas to students.	–	–	–									–		

Table 3 (continued)

Hindering thinking-as-a-resource orientations	Principles				MOST criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
Students who claim they used a different method than another student may not have understood the original method that had been shared, so their method may be the same.		–	–		–						–			
Student thinking should come from highly scaffolded instruction that minimizes student struggle.		–	–											
If a student's answer is correct but the mathematics of their statement was incomplete, the student got lucky.		–	–											

Table 3 (continued)

Hindering thinking-as-a-resource orientations	Principles				MOST criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
For problems where a diagram may support the process of finding a solution, it is always important for students to first have an accurate diagram before working on the problem.		–	–											
Students are unlikely to make sense of a situation involving an idea that has not been addressed in class.		–	–		–						–			
Students who provide numbers without reasoning are likely trying to guess and check and would rather do this and wait for the teacher's validation than reason through the problem.		–			–	–								
Students stop making sense of a problem once they have the right answer.		–	–											

Table 3 (continued)

Hindering thinking-as-a-resource orientations	Principles				MOST criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
If students believe that a particular student is smart, they assume the student is right and repeat that student's thinking rather than share their own.		–		–										
Students should share their ideas one at a time and the teacher should resolve each idea before another idea is shared.			–									–	–	
It is the teacher's responsibility to correct student mistakes and misconceptions as quickly as possible.	+	–	–									–		
If student thinking reveals a lack of understanding, the teacher should help that student understand by asking them guiding questions or going back to previously taught ideas.			–										–	

Table 3 (continued)

Hindering thinking-as-a-resource orientations	Principles				MOST criteria						Subpractices			
	Mathematics	Legitimacy	Sense Making	Collaboration	Student Mathematics	Mathematical Point	Appropriate	Central	Opening	Timing	Make Precise	Grapple Toss	Orchestrate	Make Explicit
It is better to address a student's thinking one on one rather than in a whole-class setting because students will understand and retain better.				–								–		
It is difficult for a teacher to keep track of students' thinking all at once, particularly in large classes.							–						–	

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