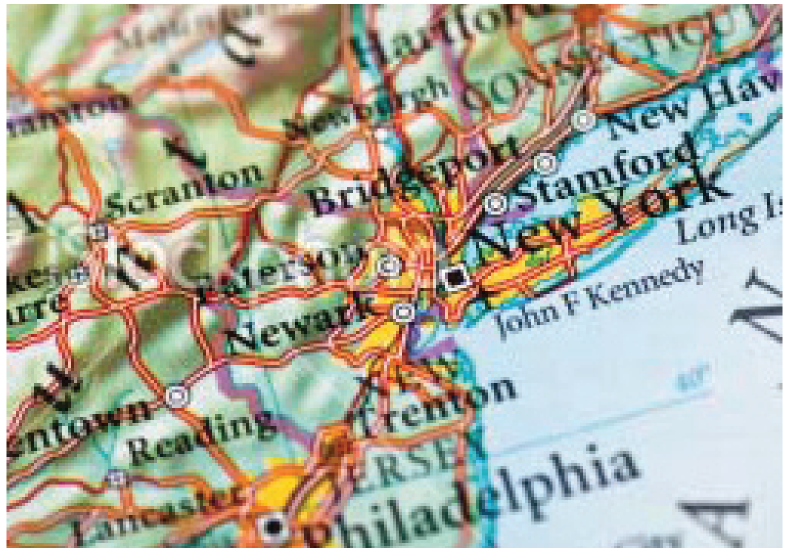

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Learning to Map & Mapping to Learn Our Students' Worlds

Maps at four levels of scale—global, national, regional, and local—provide a context for mathematical investigations that help teachers learn about their students.

The National Council of Teachers of Mathematics (NCTM), through its Connections Standard, highlights the importance of "the opportunity for students to experience mathematics in a context" (2000, p. 66). Seeing how mathematics can be used to describe real-world phenomena can motivate students to learn more mathematics. Connecting mathematics to the real world is complicated, however, by the diversity of students' cultural and linguistic backgrounds, experiences, and interests. One student's real world likely differs from another's, and the students' real and lived worlds likely differ from those of the teacher's. Mathematics teacher education and professional development programs must, therefore, guide teachers toward learning about their students. Knowledge about students helps teachers incorporate students' out-of-school knowledge, experiences, and interests into mathematics teaching and learning.

In the professional development unit presented here, teachers explore maps at four levels of scale—global, national, regional, and local—and these explorations make explicit a variety of ways that maps can be used as a context for mathematical investigations. More significantly, however, the unit provides teachers with opportunities and strategies for learning about their students, students' families, and school communities.

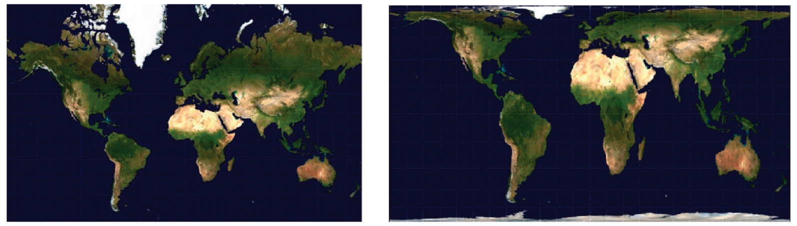
TEACHER LEARNING: FOUR LEVELS OF SCALE

The Global Level

At the global level of scale, we consider the mathematical question of how to represent the Earth's spherical surface on a two-dimensional sheet of paper. We might decide to prioritize the preservation of angle relationships between places, crucial for maritime navigation, as in the commonly used Mercator projection (see **fig. 1a**). Yet in doing so, we distort other features of the Earth's surface, such as relative area. Places farther away from the equator appear relatively bigger and more vertically elongated on the Mercator projection than they do on the globe itself.

A less common mapping of the Earth, the Gall-Peters projection (see **fig. 1b**), is scaled so that land areas accurately represent global land size. The Gall-Peters projection is helpful for an accurate sense of relative land sizes, but it distorts angle relationships between locations.

Some parts of the world look similar on the two projections; other parts of the world appear very different in relative size (Gutstein 2006). Readers can compare the Mercator and Gall-Peters projections in **figures 1a** and **1b** and note their similarities and differences. In addition, although a sphere's surface has no center, two-dimensional representations, such as the Mercator or Gall-Peters projections, are centered and oriented in specific



(a) (b)

Source: http://en.wikipedia.org/wiki/File:Mercator-projection.jpg

Source: http://commons.wikimedia.org/wiki/File:Gall-peters.png

Fig. 1 The Mercator projection (**a**) preserves angle relation while the Gall-Peters projection (**b**) preserves global land size.

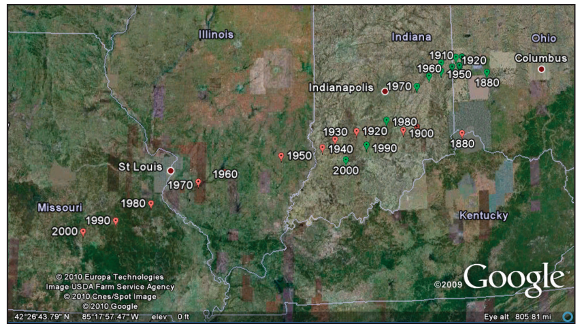


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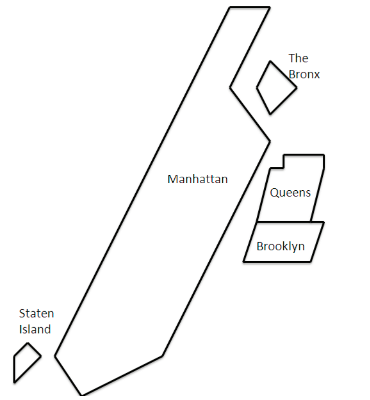
Fig. 2 The U.S. median population center (shown in green) and U.S. mean population center (shown in red) have changed over time.



Borough	Number of Starbucks Locations (2009)
The Bronx	6
Brooklyn	15
Manhattan	184
Queens	19
Staten Island	3

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Fig. 3 In the cartogram, New York City's boroughs are proportional in size to the number of Starbucks coffee shops located in each borough.

ways. Teachers can compare the Mercator, Gall-Peters, and other projections (see Wood, Kaiser and Abramms [2006] for more examples) and become more experienced in critically evaluating maps as mathematical representations.

The National Level

After working with global maps, we zoom in to the national level of the United States. Maps, as scalar representations, readily serve as a context for investigating concepts such as ratio, proportion, scale, transformations, and area or perimeter. Maps are typically used for driving or for comparing land sizes and so are typically scaled according to land area. However, we might be interested in a representation that depicts the country according to a scale other than land area. An array of examples of these cartograms can be found online (see, e.g., www.worldmapper.org).

National maps can also be used as contexts for geometric analysis, such as finding centers of the country according to different definitions of the term *center*. **Figure 2** shows the location of the U.S. median population center over time compared with the location of the mean population center.

National maps are also a useful context for a variety of graph theory investigations. For instance, the Traveling Salesperson problem seeks to identify a route that minimizes travel distance among a group of locations (for more information about this problem, go to www.tsp.gatech.edu). Graph-coloring investigations seek to color a map's regions so that neighboring states are minimally assigned different colors (for more on the four-color theorem, go to www.ccmr.cornell.edu/education/modules/documents/MapsandtheFour-ColorTheorem.pdf).

The City Level

Our next level of scale is the city level. Our particular context is New York City, where people typically travel by subway, so a land area map might not be the most useful representation of traveling time across distance. In fact, we have found that the map most familiar to teachers and students is the city subway map, which does not accurately represent area or distance (for the New York City Transit Map, go to www.mta.info/nyct/maps/subwaymap.pdf.). Teachers compare the transit map to the land area map, noting the significant differences in sizes and orientation.

The teachers also create their own city cartograms, using scales other than land area. For instance, if we use a scale such as median household income, some land areas of the city become larger and others become smaller. This representation gives us a way to compare the city's five counties according to any variable of interest.

Teachers work with a variety of data sets, disaggregated by county, such as human population, number of public parks, number of homicides, or number of Starbucks branches. In groups, they select one of these variables and "redraw" the city's five counties scaled by that variable. For instance, **figure 3** shows a cartogram scaled according to the number of Starbucks locations in 2009. Manhattan, which has the smallest land area of the five counties, appears biggest on this map, since it has many more Starbucks locations.

Individual cartograms help teachers analyze a variable of interest across the city. Putting their cartograms side by side allows teachers to compare the city's five counties across an array of variables.

The Local Level

Finally, we zoom in to the level of local neighborhoods. One way to use local maps is to conduct a community walk in the neighborhood where one's students live, with the goal of learning more about students, their families, and the community itself. Maps of census tracts—geographical areas populated by between 1500 and 8000 people—are particularly appropriate to use in urban settings because the high-population density makes census tracts easily walkable (for more information about U.S. Census Bureau tracts and data, go to www.census.gov).

Teachers investigate a census tract with a partner through one of several approaches:

1. Choose a particular theme in advance (such as access to financial institutions, variety of grocery options, availability of recreational spaces, or types of housing). Then create a route on the map and walk that route to investigate that theme.
2. Find an interesting spot in that neighborhood. Sit silently and observe for half an hour, without interacting with anyone and without taking notes. After this time of silent observation, make detailed notes of what you observed. Then discuss your findings with your partner.
3. Create a route and walk through the neighborhood, taking notes of things of interest as you go along.

After completing one of these three activities, teachers explore a variety of electronic resources, such as factfinder.census.gov or various state or municipal databases that contain information linked to U.S. Census Bureau tracts. They then relate those data findings to their own findings from the walk.

Another way in which we have worked to explore the local community is to group teachers with a few of their students (on a teacher professional development day, for example) for a community mathematics walk. The goal of this activity is to use a map and a physical experience to investigate a set of four guiding questions:

- Who seems to live here?
- Who seems to work here?
- What kinds of resources does this neighborhood offer its residents?
- What seem to be the challenges facing this neighborhood?

Teams consist of two teachers and four of their students. Each team receives a map and is guided through the neighborhood with a sequence of tasks, some examples of which are presented below.

The community mathematics tasks can include different categories of problem solving. One task focuses on aspects of local architecture as a context

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Fig. 4 An arch like that in Washington Square is a commonly recurring architectural theme in the Greenwich Village neighborhood.

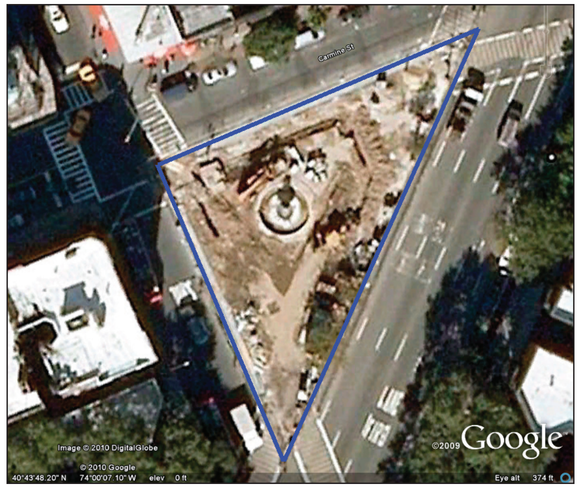

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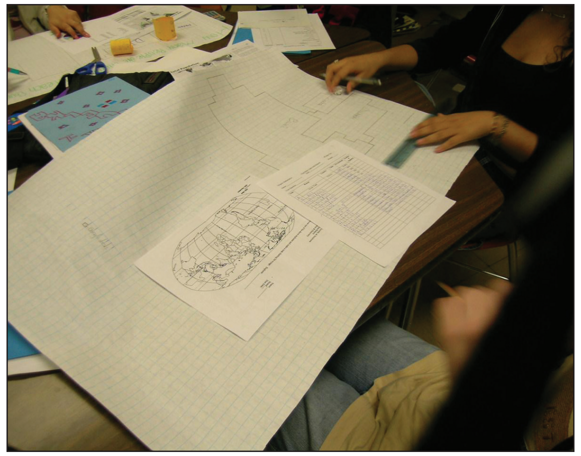
Fig. 5 Use of satellite images, such as this one of Father Demo Square, can provoke mathematical questions.

for mathematical problem posing. On a recent community mathematics walk in the Greenwich Village neighborhood of Manhattan, standing in front of the Washington Square Arch (see **fig. 4**), participating teachers and their students computed the area of its opening. This shape, a rectangle topped by a semicircle, is a recurrent architectural theme in this neighborhood. Participants recognized it as the Norman windows on many neighborhood churches and even as part of the neighborhood basketball courts (the key).

A second task uses mathematics to analyze the geometry of urban space. Teams were led to a triangular public park within that neighborhood, a satellite image of which is shown in **figure 5**. Participants were guided to “walk” various features of the triangle, such as an angle bisector or a perpendicular bisector of a side. This park has a circular fountain within its boundaries, and the teams were asked to consider whether the fountain is in the “center” of the park. To answer this question, they needed to consider the different ways one can define a center of a triangle and how each definition can lend itself to a different location for the fountain, depending on the triangle’s properties.

A third task directs participants to interpret data about the neighborhood’s residents, public health, or quality of life and to use that data to compare the neighborhood with other local neighborhoods.



HAIWEN CHU

Fig. 6 Students in Chu’s class of recent immigrants created a Big World Map.

NEXT STEPS FOR PARTICIPATING TEACHERS

Haiwen Chu and Lauren Shookhoff, two teachers who participated in these teacher education experiences about maps and mapping, have translated these experiences into their teaching in a variety of ways.

Haiwen Chu: My participation in this professional development inspired me to design two different projects for my students. I was teaching ninth- and tenth-grade students at a school for recently arrived immigrants.

In the first project, I organized students’ work around local census tracts. Students researched U.S. Census Bureau data and took community walks around the census tract in which they lived. They then drew cartograms representing places in that census tract according to a “scale of importance” that they chose, such as the number of Spanish speakers there.

The following year, I took an international perspective and asked small groups of students to create world maps, including the ten to fifteen countries of origin represented in the class. Each small group chose a variable—such as gross domestic product, net migration rate, or number of airports—and scaled the countries on their map according to that variable (see **fig. 6**). Each student individually made another map, with his or her home country at the center, rescaled to an area of one square inch. This unitizing made for ready comparisons, such as how one country has four times as many cell phones as another. I learned about my students’ home countries, while they struggled to faithfully represent the shapes of their countries, using the areas determined by their calculations.

Lauren Shookhoff: I teach an entry-level course to tenth graders; many are English language learning students. Although they have been exposed to these mathematical concepts in previous courses, they struggle nonetheless. Since most or all of the material I teach is not new to them, it is crucial that as their teacher, I find new ways to engage them in the mathematics. As a consequence of my participation in this teacher education program, I now look for ways to make math more “local” for my students.

I find that when I present math in a local context, such as the nearest subway station, the location of our school campus, the proximity of the neighborhood park or local stores, the material is inherently more interesting and engaging for my students because it is literally about their neighborhood. For instance, when we investigated the distance from the school to the nearby train station, I zoomed in on a Google Earth™ image of our school and a typical path to the station. As a class, we discussed the length of this path and compared it with the direct, as-the-crow-flies distance.

Later in the unit, students were asked to calculate a precise estimate of the time it would take to walk the perimeter of the neighborhood and the perimeter of our entire county. **Figure 7** shows a gridded map I created using Google Earth, with which the students could calculate scaled distances and use the Pythagorean theorem to find the perimeter of the polygon. Students raised questions about how neighborhood boundaries get created and how gentrification impacts those boundaries.

In a third lesson, I used neighborhood maps as a context for investigating a triangle’s circumcenter. Each student plotted his or her own address and that of two friends and was asked to calculate a fair meeting location, where *fair* was defined as each person travelling the same distance.

My students were highly engaged in these lessons and the mathematical discussion was on a high level. Best of all, later in the year, they easily recalled how they had applied mathematics to describe aspects of their world.

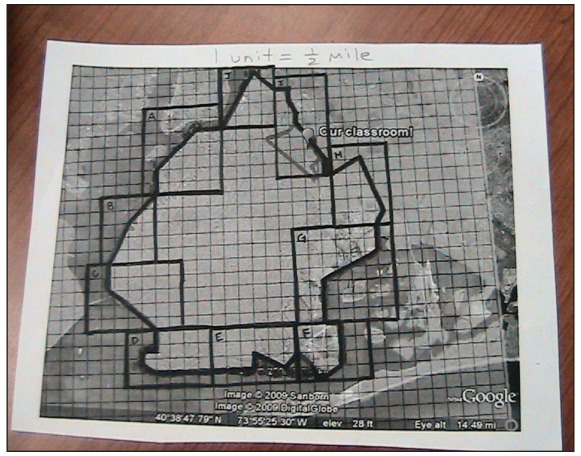


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Fig. 7 Google Earth produces the map that students could use to grid the perimeter of Brooklyn.

The examples from Chu and Shookhoff demonstrate how the teacher education unit on maps and mapping resulted in the development of mathematics projects for high school students. These teachers, along with other teacher participants, found ways to use maps as a context to support the investigation of mathematical ideas. Readers might be interested in trying some or all of the teacher education activities and pursue adapting any of these activities for investigation by high school students. By creating and implementing such projects with students, teachers can demonstrate explicit connections between mathematics and their students’ worlds.

Teachers may find that the context of maps and mapping corresponds well to some of their curricular objectives, but the objective of the teacher education work is more extensive. The deeper idea is for teachers to adopt maps and mapping tools outside the classroom as a way to expand their own knowledge about their students and their school communities. Although these investigations could be conducted in collaboration with students in classroom projects, they can be pursued by teachers as part of their professional development.

For instance, teachers could investigate variables such as community educational attainment rates or employment categories as a way to better discern some characteristics of the adults in the school’s community. Teachers could investigate location and prevalence of community features such as recreation space, grocery resources, religious institutions, or banking or financial outlets as a way to learn more about the opportunities or challenges that students and their families face. In addition to gathering data from electronic sources, teachers could also conduct the types of community walks described in this article.

The essential goal of building knowledge about one’s students, their families, and their communities is a necessary step in the process of learning to contextualize mathematics in ways that are meaningful and relevant for students.

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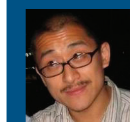


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