REVEALING TEACHER KNOWLEDGE THROUGH MAKING: A CASE STUDY OF TWO PROSPECTIVE MATHEMATICS TEACHERS

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We describe an experience within mathematics teacher preparation that engages pre-service teachers of mathematics (PMTs) in Making and design practices that we hypothesized would inform their conceptual, curricular, and pedagogical thinking. With a focus on the design of new tools that can generate new possibilities for mathematics teaching and learning, this Learning by Design experience has PMTs exploring at the intersection of content, pedagogy, and Making. We describe the forms of knowledge brought to bear on their experiences through a case study analysis of one pair of PMTs' Making experience. As the advancement of these forms of knowledge is essential to effective mathematics teaching, these findings suggest the promise of a Making experience within mathematics teacher preparation.

Keywords: Instructional Activities and Practices, Teacher Knowledge, Technology

Objectives

Preservice elementary teachers have been characterized as coming to teacher preparation with limited conceptions of mathematics (AMTE, 2013) and a model of mathematics teaching that largely appeals to rules and procedures (Ball, 1990; Ma, 1999; Thompson, 1984). Unfortunately, this model is not consistent with a pedagogy that supports learning mathematics with understanding. Consequently, prospective elementary teachers' preparation must include opportunities that challenge their current models of mathematics teaching and learning. In this proposal, we present one such opportunity that is centered in the activity of *Making*. We draw on Halverson and Sheridan's (2014) conception of Making as *designing, building and innovating with tools and materials to solve practical problems*. We present a novel *Making-oriented experience* within mathematics teacher preparation that tasks prospective mathematics teachers (PMTs) with designing, fabricating, and evaluating new manipulatives (Post, 1981) to promote learners' mathematical thinking and reasoning. In seeking to determine what this experience might offer prospective elementary teachers as they prepare for the work of mathematics teaching, this project addresses the following question: *What forms of knowledge can be brought to bear on prospective elementary teachers' design work as they Make new manipulatives to support the teaching and learning of mathematics?*

Theoretical Framework

Our theoretical framing is organized around the learning theories of *constructivism* and *constructionism*. These theories recognize that knowledge is actively constructed by a learner, with constructionism adding the dimension that the knowledge be constructed during the process of making a shareable object (Harel & Papert, 1991). We drew from the rich scholarship devoted to teacher knowledge to characterize what forms of knowledge might actually be brought to bear on PMTs' design work (Ball, 1990; Borko & Livingston, 1989, 1990; Cochran, DeRuiter, & King, 1993; Hill, Ball, & Schilling, 2008; Koehler & Mishra, 2009; Mishra & Koehler, 2006; Shulman, 1986). In particular, we took a *Learning by Design* approach (Koehler & Mishra, 2005; Koehler, Mishra, Hershey, & Peruski, 2004) to leveraging and potentially advancing this knowledge. Learning by design involves the PMTs in the activity of *designing*, or the purposeful imagining, planning, and intending that precedes and interacts with Making. This approach calls upon PMTs to "actively engage in inquiry, research and design" so that they can make "tangible, meaningful artifacts" that represent "the end products of the learning process" (Koehler and Mishra, 2005; p. 135). This approach provides an opportunity to consider the interplay between the evolving artifact and the application of teacher knowledge domains in the artifact's development. The premise for learning by design honors the proposition that it is productive to develop teacher knowledge within a context that recognizes the interactions and connections among these constituent domains of knowledge.

The artifact that a PMT makes, and the design decisions that go into the Making, provide a rich source of data for understanding these knowledge domains if we consider the artifact and its creation from the perspective of designing for mathematical abstraction (Pratt & Noss, 2010). In the case of manipulatives, the designer aims to embed a concept in its design so that it can be made available to the learner for abstraction through their sensorimotor manipulation of the object (Kamii & Housman, 2000; Piaget, 1970; Vygotsky, 1978). This is the task we set for the PMTs in this project: to design a manipulative that is hypothesized to support learners' abstractions of a mathematical concept from concrete tools. Pratt & Noss's case study (2010) offers a proof of concept that learning by design provides a venue for characterizing the interplay between a participant's beliefs and knowledge domains as they are invoked during the design process.

Methods

The study took place in a specialized mathematics content course for prospective elementary teachers. Twenty-six participating students comprised twenty-one groups. The PMTs were given the following task: "The purpose of this project is for you to 3D design and print a new physical tool (or "manipulative") that can be used in teaching a mathematical idea. The design of this tool and a corresponding task will reflect a) your knowledge of what it means to do mathematics and how we learn with physical tools, b) your knowledge of elementary-level mathematics content, and c) your perspective on pedagogy and curriculum in mathematics education." This project had three written components, which comprise the corpus of research data: 1) an "Idea Assignment" that describes a group's initial thoughts about a manipulative they want to work on, 2) a "Project Rationale," which is an account of how their design reflects an understanding of what mathematics is and how learning happens, and 3) a "Final Paper" that describes the purpose of the manipulative, the corresponding tasks that were created, and the group's findings from an intended user's manipulative-mediated engagement with those tasks.

Data analysis proceeded in two phases. In Phase I (more fully reported in Greenstein & Seventko, 2017), three researchers analyzed the PMTs' written work and generated codes (Corbin & Strauss, 2008) that identify forms of knowledge in the PMTs' written work, with initial codes derived from the mathematics knowledge for teaching literature. Intercoder reliability on the codes was calculated at .82. This analysis provided a promising foundation for delving more deeply into the PMTs' written work in order to understand how the knowledge we identified was brought to bear in their Maker projects.

In Phase 2, we took a case study approach (Yin, 2009) with purposeful sampling (Patton, 2002) to identify and select design cases whose reflections bear evidence of *mathematical richness*. In these cases, PMTs articulate and express multiple layers of mathematical detail in describing mathematics content, student thinking, or the use of technology during the project. This sampling technique purposefully mirrors the knowledge frameworks so that we could describe the PMTs' constituent forms of knowledge as reflected in their written work. Crafting narratives of the PMTs' design experiences involved a process of moving between the Phase 1 codes and the PMTs' reflections to intuit the data and weave narratives that illuminate the knowledge brought to bear on the PMTs' design activity. Harnessing the case study's virtue for evoking "images of the possible," (Shulman, 2004; p. 147), we present our narrative for Casey and Mia and their tool called *Minute Minis* (see Fig. 1 for digital design).

Results

We viewed the PMTs' written reflections as containing instances of the knowledge they brought to bear in their design work. For Casey and Mia, our analysis identified various dimensions of mathematical knowing in teaching, such as knowledge of mathematical content, pedagogical content knowledge (PCK), curricular knowledge, and knowledge of how learning works in interaction with manipulatives.

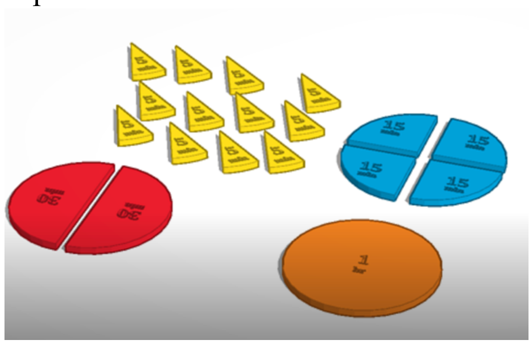

Figure 1: Minute Minis

Casey and Mia were inspired to design a manipulative that could help children reason about the abstract concept of time. In their initial design rationale, they hypothesize about breaking through the ordinarily obscure nature of time to make it more accessible to learners:

> The main goal of this project is to give a concrete representation of the relationship between hours and minutes. Using manipulatives is especially important when exploring new concepts, and sense (sic) time is a very abstract concept, it is especially pertinent that students have something concrete to work with. With these manipulatives, students will be better able to solve addition, subtraction, multiplication and division problems relating to time.

Making such a tool for the shared purpose of learning and teaching was initially and genuinely influenced by questions Casey had about how children were thinking of time in the classroom where she was doing her student teaching:

> Currently, most of the 2nd graders in my class can tell time to the nearest half hour, yet I am unsure of how they know how to do this. Is it just because they know that when the minute hand is pointing at the 6 I say _:30 and when it's pointing at the 12 I say _:00? Or do they have a more (sic) deeper understanding of time and how a clock works?

These considerations reflect how Casey's PCK (wondering about students' current conceptions of the topic of time) inform her design. Over the course of the project, these questions develop into other strands of knowledge that she and Mia use to investigate these issues. That is, the Making of Minute Minis was driven by a desire to transform potentially limited conceptions of time from memorized models into deeper mathematical meanings. Drawing upon other aspects of PCK and of mathematical and curricular knowledge, Casey and Mia take an existing design of fraction circles and use concepts from geometry to amend it for their objectives:

> [We] will be using the same concept of fraction circles, yet instead of labeling them with a fraction, they will be labeled with minutes. For instance, a whole circle will be labeled "1 hour," while two half circles will be labeled "30 minutes." [We] will also have [fraction] circles for 15 and 5 minutes. (see Fig. 1)

One of their key design issues focused on being able to "visually illustrate the concept of minutes as fractions of an hour." Additionally, the circular shape was important to them in ensuring "that students would be able to use the Minute Minis directly on the face of a clock. This would aid [students] in exploring the relationship between where the minute hand is pointing and the number of minutes past the hour." Such a design component was seen as essential in supporting student inquiry of the fractional ideas embedded in the tool so that the child could assemble the fractional pieces to compute time.

We continue to see PCK emerge in Casey and Mia's reflections about tasks teachers can pose to students using their tool. They describe teachers familiarizing students with their tools by asking questions like, "How many 30 minute pieces make an hour? How many 5?" Then they go on to consider and describe how a child might use their tool to investigate how many hours it takes to do four homework assignments each of which takes 45 minutes to finish:

> one 30 minute piece and a 15 minute piece to show 45 minutes, then replicating this 3 other times to show 4 × 45. A child might then notice that they can make 2 wholes – hours in this case – using 4 of the 30 minute pieces and 1 whole/hour using the four 15 minute pieces, leading them to an answer of 3 hours.

Concluding Discussion

We hypothesized that an iterative design experience centered on the task of Making and evaluating a physical manipulative for learning mathematics would provide our PMTs an opportunity to leverage and deepen their understandings (Schön, 1992) of mathematics content, curriculum, and pedagogy. Accordingly, we introduced a pedagogically genuine and authentically open-ended task into a Making context, inviting the interplay between the iterative design of a shareable artifact and the application of teacher knowledge in the artifact's development. Casey and Mia, a pair of PMTs who participated in that experience, shared reflections that illustrate their design in development, and leveraged their knowledge of fractions and area to help mediate a bridge between the abstract and concrete representations of time. By supplementing the traditional focus of instruction about time with a concrete representation that facilitates conceptual connections between a clock face and its underlying area properties, Casey and Mia were able to draw on this knowledge to articulate the mathematical richness underlying this manipulative and its possible uses by a child.

As researchers exploring how design experiences might catalyze new possibilities for pedagogical and curricular change, we positioned PMTs as knowledgeable designers of instruction in a space of technological possibilities. As PMTs assumed the multi-faceted role of teachers designing with technology, they created powerful and innovative tools, and their work demonstrated a rich and mature repertoire of knowledge domains that we are not typically afforded opportunities to see (AMTE, 2013). We propose that the identification and advancement of this knowledge, which is essential to effective mathematics teaching, suggest the promise of a Making experience within mathematics teacher preparation. Future research seeks to illuminate the particular features of the configured world (Holland, Lachicotte Jr., Skinner, &

Cain, 1998) of the design environment that contributed to these outcomes, and to assess the impact of the experience on PMTs' identities (Sfard & Prusak, 2005) as designers of mathematical instruction.

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