12th International Congress on Mathematical Education
Program Name XX-YY-zz (pp. abcde-fghij)
8 July – 15 July, 2012, COEX, Seoul, Korea (This part is for LOC use only. Please do not change this part.)

AN EFFICACY STUDY ON THE USE OF DYNAMIC GEOMETRY SOFTWRAE

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A four-year research project funded by NSF examines the efficacy of an approach to high school geometry that utilizes dynamic geometry (DG) software and supporting instructional materials to supplement ordinary instructional practices. It compares effects of that intervention (the DG approach) with standard instruction that does not make use of computer tools. This paper reports a study conducted during the second year of the project. Student learning is assessed by a geometry test and other tests. Data for answering the research questions of the study are analyzed mainly by appropriate HLM methods. The analysis on the geometry test data is discussed in detail. The experimental group significantly outperformed the control group in geometry performance.
Key words: dynamic geometry, random assignment, HLM methods

INTRODUCTION

The term *Dynamic Geometry (DG)* refers to active geometric explorations carried out with interactive computer software such as the Geometer's Sketchpad (Jackiw, 2009) or Cabri Geometry (Texas Instruments, 1994), which has been available since the early 1990's. Along with the widespread use of DG software, many related research studies have been conducted. A relatively small group of researchers (e.g., Dixon, 1997; Gerretson, 2004; Myers, 2009) used experimental or quasi-experimental designs in their studies. Most of the studies (e.g., Hannafin, Burruss, & Little, 2001; Hollebrands, 2007; Baccaglini-Frank and Mariotti, 2010) used qualitative research methods. Built upon these studies, a four-year research project funded by NSF examines the efficacy of an approach to high school geometry that utilizes DG software and supporting instructional materials to supplement ordinary instructional practices. It compares effects of that intervention (the DG approach) with standard instruction that does not make use of computer investigation/drawing tools. This paper reports a study conducted during the second year of the project.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

An integrative framework (Olive & Makar, 2009) drawing from Constructivism, Instrumentation Theory and Semiotic Mediation was used to guide the study. Central to Instrumentation Theory is the process of *Instrumental Genesis* – How a tool changes from an artifact to an instrument in the hands of a user, and how both the tool and user are transformed in the process (Olive, 2011). The notion of *semiotic mediation* was introduced by Vygotsky (1978). According to this notion, cognitive functioning is closely linked to the use of signs and tools, and affected by it. Olive & Makar (2009) focus on the mathematical knowledge and practices that may result from access to digital technologies. They put forward a new tetrahedral model that integrates aspects of instrumentation theory and the notion of semiotic mediation. “This new model illustrates how interactions among the didactical variables: student, teacher, task and technology (that form the vertices of the tetrahedron) create a space within which new mathematical knowledge and practices may emerge” (Olive, 2011, p.3).

The research questions of the study include: 1) How do students taught in a DG oriented instructional environment perform in comparison with students in the control condition? 2) How does the DG intervention contribute to narrowing the achievement gap between students receiving free or reduced price lunch and other students? 3) How is students' learning related to the fidelity and intensity with which the teachers implement the DG approach in their classrooms? and 4) What characterizes the learning communities in the experimental and control classes?

As the first efficacy study on the DG approach at a moderately large scale in the nation, its answers to these research questions will significantly contribute to the knowledge base of how the dynamic geometry approach can really enhance our students' geometry learning and how we can effectively help our geometry teachers to develop and improve their technological pedagogical content knowledge.

RESEARCH DESIGN

The population from which the participants of this study were sampled are the geometry teachers and their students at the high schools in Central Texas. Based on a power analysis to determine the optimal sample size and taking attrition into consideration, 76 geometry teachers were selected from those who applied to the project with support from their principals.

The research study follows a mixed methods, multi-site randomized cluster design, with teachers as the unit of randomization. The 76 teachers selected were randomly assigned to the Experimental Group and the Control Group. For schools where the selected teachers teach more than one class, only one class per teacher was randomly chosen to participate in the study. Therefore each teacher is represented in the study with measurements from only one classroom of students, and the classroom and teacher unit of analysis overlap, yielding the design where the students are nested within teachers/classrooms, which are nested within schools.

Student learning is assessed by a geometry test, a conjecturing-proving test, and a measure of student beliefs about the nature of geometry. Teachers in both treatment and control groups receive relevant professional development. To determine how to capture the critical features of the DG approach, we have designed measures of fidelity of implementation – a DG implementation questionnaire and a classroom observation instrument. To probe more deeply into the teachers' and students' thinking processes, and to gather evidence about the range and variability of participants’ development of the most important abilities that the

DG approach fosters, this study uses in-depth interviews with selected students and teachers.

DATA ANALYSIS

The project team has completed its year 2 data collection. Some initial data analysis (the analysis on the geometry pretest and posttest data and the psychometric analysis on the project developed instruments) has been conducted. More thorough analysis of the collected data is still on going and will be conducted during project year 3.

For all project-developed measures, the Cronbach's Alpha statistical values are within the acceptable ranges for reliability. Other psychometric properties were examined for some of the instruments and provide evidence supporting the validity of each.

The principal method of data analysis will involve fitting a two-level hierarchical linear model (a linear mixed effects ANCOVA model) to the data. This multilevel approach enables us to address the first two research questions, examine the potential treatment effect and explore the potential of the DG learning environment for reducing the achievement gap while taking into account the nested structure of the data (i.e. students nested within teachers' classrooms).

Qualitative data analysis will use the constant comparative approach (Glaser & Strauss, 1967; Grove, 1988) to answer research question 4. The quantitative data analysis and the qualitative data analysis mentioned above, as a whole, will answer research question 3 that relates to implementation fidelity.

The remainder of the paper will report the analysis of the geometry test, which consisted of a pretest and a posttest. The pretest was the Entering Geometry Test (ENT) used by Usiskin (1982) and his research team at University of Chicago. ENT has been used by numerous studies on students' geometry learning over the past 29 years, and has been considered as a good and easy-to-administer multiple-choice geometry test to assess students' geometric background before entering a full-year high school geometry course. ENT consists of 20 multiple choice items and has a reliability of $\alpha = .77$. The posttest (XGT) was developed by the project team through selecting questions from released items of the California Standards Test: Geometry (CSTG). Paying close attention to the alignment with Texas geometry standards and the geometry curricula of the participating school districts, the project team chose 30 items from CSTG and pilot-tested them in the non-project classes at a participating high school. Based on the pilot-test results and the feedback of the master teachers (who are high school geometry curriculum and instruction experts working for the project), five items were removed. The final version for XGT has 25 multiple-choice items.

RESULTS

Based on the pilot-test results, the instrument has high reliability ($\alpha = .875$). Factor analysis provided strong evidence that XGT corresponded to uni-dimensional scale. Item Response Theory (IRT) scoring routines were applied to the scored posttest to generate examinee 'abilities' and item parameters, which allowed us to determine that collectively the items included on the posttest provided a range of performance that holistically represented a well functioning instrument. The adherence of the data to the three-parameter logistic IRT model provided some evidence for the assessment's construct validity.

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the DG approach on overall student achievement. The models were analyzed using student pretest (ENT) scores included as a covariate. Once the significant predictors of overall achievement were identified, performance on each individual item was investigated in order to better understand the possible effect of the DG treatment on student learning. We used mixed logistic regression with the same predictors used in the HLM to estimate the impact of DG for each item.

The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectation and quality of the students in each of these levels is very different, the factor *Class Level* was included in each model. Additionally, the years of classroom experience of the teachers in the sample varied a lot, ranging from 0 years all the way up to 35 years. Given the emphasis of technology in the study, entering the study the possible effect of teaching experience was unclear. A more experienced teacher may have greater command of the classroom but be less able to implement the technology. For this reason, the covariate *Years Exp* (number of years of classroom experience) was included in the models.

During the project year 1 professional development workshop, the participating teachers completed a demographic survey that included information about years of teaching experience, the level of the class chosen and gender. From our initial teacher sample (76 participants), six teachers didn't compete project year 2 mainly due to either family/health or job displacement reasons. Additional six teachers submitted incomplete posttest data or failed to submit the data. Therefore, 64 teachers submitted complete posttest data for analysis in the study. Among them, 33 are in the experimental group (DG group), and 31 are in the control group. Table 1 shows the summary statistics for years of experience of these teachers by *Treatment* and *Class Level*.

HLM Results

In development of the HLM models summarized below, full factorial designs were explored and insignificant interactions were discarded. Only the final model is discussed below.

Model 1, shown in Table 2, examines the effect of the DG intervention when taking into account Entering Geometry Test (*ENT)* as well as *Class Level* and *Years Exp*. To simplify interpretation of the other coefficients, *ENT* was centered by subtracting the overall mean. The results of Model 1 indicate that the DG effect was strongly significant ($p = .002$). Table 3 shows the summary statistics for each level of class. Comparing the means, the DG group outperformed the control group in each level of Geometry and the effect was substantially larger at the Regular Geometry level.

Table 1: Summary Statistics for Years of Experience of Teachers

	DG			Control		
	n	M	SD	n	M	SD
Overall	33	7.00	7.18	31	6.48	8.29
Class Level						
Regular	20	6.44	7.89	19	5.63	6.72
Pre-AP	12	8.30	6.25	8	10.75	11.94
Middle School	1	4.00	NA	4	2.00	2.16

Table 2: Model 1 - HLM Results with Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	79.05	6.119	12.919	28	.000
DG Effect	5.62	1.678	3.352	43	.002
Level					
Regular	-20.10	6.119	-3.284	21	.004
Pre-AP	-13.23	6.293	-2.102	20	.049
Level*Years Exp					
Regular*Years Exp	-.4137	.1516	-2.729	53	.009
Pre AP * Years Exp	.4451	.1617	2.753	22	.012
M. School*Years Exp	1.811	1.868	0.969	20	.344
ENT (Mean Centered)	.4114	.0366	11.237	47	.000

Note. XGT is the response variable.

As expected, *ENT* is a significant predictor of student performance on *XGT* (p = .000). However, even controlling for the pretest, compared with Middle School students, on average Pre-AP students scored 13.2 points lower (p =.049) and Regular students scored 20.1 points lower (p =.004). Teaching experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. The effect of experience in the Middle School group was not significant, but an increase in 10 years of experience raised the scores 4.5 points for the Pre-AP group and decreased the scores by 4.1 points for the Regular group.

Table 3: Summary Statistics for XGT by Treatment and Level

	DG			Control			
	n	M	SD	n	M	SD	ES
Overall	501	62.36	19.26	438	59.12	20.40	.16
Class Level							
Regular	276	54.19	17.64	232	46.81	15.10	.45
Pre-AP	210	71.26	16.09	163	69.28	15.50	.13
Middle School	15	88.27	7.01	43	87.07	10.10	.13

Note. Includes only posttest data for subsample with matching pretest results.

Results for Mixed Logistic Regression Models

To better understand the nature of the impact of the DG approach on student learning, each item of the *XGT* was analyzed separately. In this case, the dichotomous outcome for each item (1 = Correct, 0 = Incorrect) was used as the response variable and the predictors were the same as those used in Model 1 above. As an example, the results for the item 1 model are shown in Table 4.

Table 4: Model 2 - Mixed Logistic Regression Results with Pretest as a Covariate

Fixed Effect	β : Coefficient	Exp(β)	p-value
Intercept	-0.065	6.119	.758
DG Effect	0.388	1.475	.078
Level			
Regular	-1.537	6.119	.112
Pre-AP	-1.100	6.293	.200
Level*Years Exp			
Regular*Years Exp	0.020	.1516	.313
Pre AP * Years Exp	0.019	.1617	.375
M. School*Years Exp	0.249	1.868	.478
ENT (Mean Centered)	0.011	.0366	.019

Note. Item1 (1= Correct, 0 = Incorrect) is the response variable. Logit is the link function.

This method uses a linear model to estimate the log-odds of a correct response. A positive coefficient, β, indicates that the predictor has a positive effect on the performance when controlling for the other predictors in the model. To facilitate interpretation, e^{β} which represents the effect on the odds of a correct response is also shown. For example, for item 1 the odds of a student in the DG group correctly answering item 1 are 1.475 times that of a student in the control group with same pre-test score when both students are in the same level of class with a teacher with same number of years of experience.

A summary of the results for all 25 items on the posttest is presented in Table 5. The first column gives a brief description of the item. Three of the items will be discussed further below. The next two columns are the percentage of students in each group who answered the item correctly. The values from the final two columns were computed using mixed logistic models like that shown in Table 4. The *P-value* column represents the statistical significance of the *DG* effect in the model, and *Exp(β)* represents the effect of DG on the odds of a correct response.

Table 5: Summary Statistics of the Results for All 25 items on the Posttest

Item Description	DG (%)	Control (%)	P-value	Exp(β)
1. Congruent Triangles	66	57	0.078	1.48
2. Congruent Triangles	86	81	0.313	1.24
3. Triangle Inequality	48	43	0.143	1.33
4. Congruent Triangles	82	76	0.157	1.30
5. Similar Triangles	71	66	0.333	1.21
6. Similar Triangles	56	51	0.990	1.00
7. Properties of a Parallelogram	89	86	0.738	1.07
8. Point on a Circle	55	46	0.023	1.60
9. Area of Figure Composed of Right Triangles	65	63	0.917	1.03
10. Area of a Circle	64	58	0.404	1.18
11. Volume Using a Net	59	51	0.042	1.55
12. Area of Figure (Rectangle – Right Triangles)	62	60	0.528	1.13
13. Area of Trapezoid	55	50	0.507	1.17
14. Exterior Angle of Triangle	59	51	0.125	1.31
15. Exterior Angle of Regular Hexagon	48	41	0.063	1.40
16. Pythagorean Theorem	63	54	0.456	1.23
17. Construct Bisector of Angle	33	29	0.112	1.51
18. Pythagorean Theorem	59	55	0.833	1.04
19. Angles Inscribed in Circle	68	67	0.618	1.08
20. Transformation (Rotation)	61	60	0.906	0.98
21. Transformation (Translation)	79	73	0.333	1.29
22. Properties of a Trapezoid	49	44	0.388	1.19
23. Similar Triangles	47	38	0.123	1.35
24. Parallel Lines with Transversal	54	47	0.085	1.36
25. Parallel Lines with Transversal (Proof)	50	46	0.859	1.04

Note that the odds of a correct response are greater for the DG group for every item except item 20. For item 20, Exp(β) = .98 and is very close to 1. For five of the items (1,8,11,15, 24), the Exp(β) is significantly larger than 1 at the $\alpha = .10$ level.

Although the impact of the DG treatment on performance appeared to be very broad with increases in student performance on 24 of the 25 items, it is informative to examine items where that increase was greatest. Looking at the content in each item description, we see that the 5 items with a significant DG effect cover a broad spectrum of geometry topics: congruent triangles, volumes, hexagons, circles, and parallel lines with a transversal. Looking at the first three of the five specific items we notice that the nature of each item is quite different as well. In Items 1 and 11 a figure is given, while in item 8 no figure is shown (see Figure 1). Item 1 is a straightforward application of the standard side-side-side triangle congruence theorem. Item 8 involves the definition of a circle. In item 11, students need to visualize the box formed by the net before calculating its volume. This further indicates that the impacts observed here do not appear to be connected to a particular task or topic. Rather, working with dynamic geometry improves overall knowledge of geometry. This may be due to the fact that working with DG software increases student engagement with geometry, making the topic more exciting for the students. Alternatively, a different explanation involves the student's ability to make sense of basic geometric objects. When given the chance to dynamically interact with the geometry, students may be better able to internalize the basic definitions and theorems.

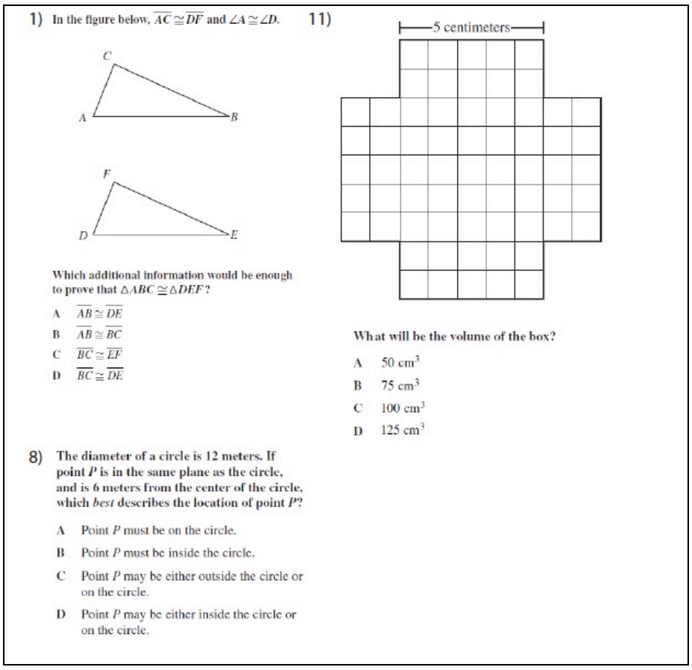

Figure 1. Three items of the geometry posttest

DISCUSSION

The HLM model taking pretest, class level, and teaching experience into account showed that the Dynamic Geometry group significantly outperformed the Control group in geometry achievement. Given that teachers were randomly assigned to the two groups and that both groups received professional development of the same duration on the same topics, the results from this study provide strong evidence to support the finding that the DG approach did make a difference – it did cause the improved geometry achievement observed in the study.

One "unusual" result of the HLM analysis of the data is the effect of teaching experience on student achievement on the geometry posttest – greater experience of the teacher had a positive impact on achievement at the Pre-AP level and a negative impact at the Regular level. This finding was considered "unusual" because we generally assume that experience promotes effectiveness. However, "Over 40 years of teacher productivity research suggests that the simple assumption that 'more is better' requires greater nuance; experience effects are complex and depend on a number of factors … Studies have also documented some evidence that effectiveness *declines* after some point, particularly among high school teachers" (Rice, 2010, pp.1-2). In fact, evidence suggests that the most experienced (25+ years) high school mathematics teachers may be less effective than their less experienced counterparts (Ladd, 2008). So, further research is necessary to fully understand why the negative impact of teaching experience occurred. Our initial explanation is the expectations of teachers for the two different levels. Based on years of interaction with middle and high school teachers, the project researchers have noticed the tendency for experienced teachers to have very different and very rigid beliefs about students' ability to achieve. Pre-AP classes are composed of mostly middle to high achieving students, and hence teachers have high expectations of what those students can learn. Meanwhile, since students in Regular classes have a record of low to middle achievement, teachers have very low expectations of what students can learn. In contrast, as the project team has also noticed, more novice teachers are willing to believe that all students can learn.

With the results described above, we have partially answered the first research question of the project. To thoroughly address all research questions of the project and to better understand issues such as the effect of teaching experience on geometry achievement, during project years 3 and 4, while continuing to use the HLM models to analyse quantitative data, we will conduct in-depth qualitative analysis on the data collected in the classroom observations and student and teacher interviews. Guided by the integrative framework represented by the new tetrahedral model (Olive & Makar, 2009), in the data analysis processes, we will focus mainly on the interactions among teachers, tasks (conjecturing/proving tasks presented to the participating teachers or students), technology (DG tools), and students, so as to develop new insights related to the ways of thinking and communicating that are characteristic of the DG environments.

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This material is based upon work supported by the National Science Foundation under Grant No. 0918744. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.