What Can We Learn from Correct Answers?

Dig deeper into classroom artifacts using research-based learning progressions to enhance your analysis and response to student work, even when most students solve a problem correctly.

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Assessing student learning traditionally involves determining whether students can solve a certain percentage of problems correctly, under the assumption that this achievement indicates they have the knowledge and understanding they need to progress to new topics. In this article, we explore what teachers can learn from looking closely at student strategies, even when most students in the class obtain the correct answer. The strategies that students use to solve a problem can be quite revealing for making instructional decisions. As teachers, coaches, and teacher educators associated with the Ongoing Assessment Project (OGAP), we have been engaging in an approach to formative assessment that draws on research about student learning to analyze evidence in student work to inform instruction. Through an example from a second-grade classroom, we explore how this approach helps teachers target diverse student needs to move all students forward toward building procedural fluency from conceptual understanding (NCTM 2014).

Formative assessment informed by learning trajectories

Kate Severini is a second-grade teacher who uses formative assessment on a regular basis to guide and improve her instruction. Teachers in her large urban district have been using the OGAP formative assessment system to enhance the use of their math program in relation to core content at each grade level. This meant that Severini and her colleagues were giving open-ended formative assessment problems as exit slips about two times per week and analyzing students' thinking to guide future instruction and deepen student understanding in relation to a progression that identifies levels of student strategies moving from least sophisticated to flexible and efficient (Hulbert and Ebby 2017).

TABLE 1

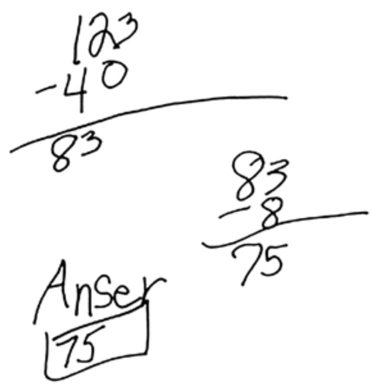
Student strategies for addition and subtraction of multidigit quantities move from counting strategies toward flexible and efficient strategies that are based on numerical reasoning.

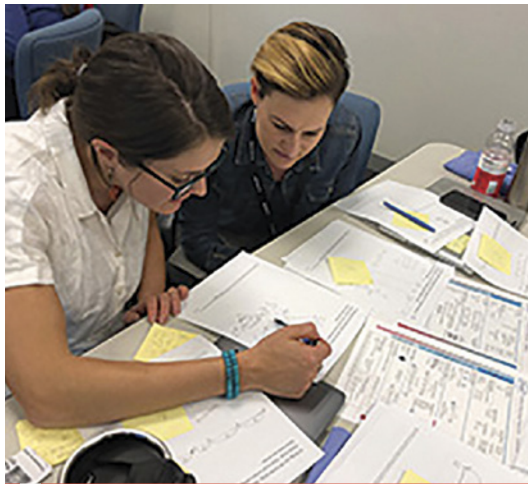
A progression of strategies for addition and subtraction of multidigit quantities (Hulbert and Ebby 2017)

Level of strategy	Examples
Early counting	Directly modeling the problem situation and counting all by ones
Counting	Counting up or back from one quantity by ones, mentally or with a model (e.g., counting with fingers or jumping by ones on a number line)
Early transitional	Adding or subtracting by increments of ten, with or without a model (e.g, base-ten models, number lines, or ten-frames).
Transitional	Efficient use of a visual model to add or subtract (e.g., jumping by multiples of ten on a number line or using number bonds to decompose and recompose)
Additive	Efficient use of standard, alternative, or invented algorithms or strategies that involve decomposition and recomposition by tens and ones and/or the use of properties of operations.

FIGURE 1

The team sorted student work into three stacks. This work has evidence of an additive solution strategy.

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Severini and Tilli sorted student work with the progression.

The learning progression is based on research about the development of additive reasoning and also provides instructional guidance. **Table 1** summarizes how student strategies for addition and subtraction of multidigit quantities move from counting strategies toward flexible and efficient strategies that are based on numerical reasoning. The use of structured visual models at the *transitional* level plays a critical role in the transition from concrete models and counting strategies to the development of computational fluency at the *additive* level. As additive reasoning develops, students may employ strategies at various levels on this progression depending on the problem context, complexity, or structure. Through training and professional development on the development of additive reasoning, teachers learn to sort their work by paying attention to evidence of where the strategies are along this progression. Then, within those categories, they look for underlying issues and common errors that may need to be addressed (see **fig. 1**). This approach contrasts with the usual method of sorting student work primarily by correct or incorrect answers and/or the presence of errors.

We explore the process of analyzing evidence from student work with a learning progression to inform instruction through an example of Severini's collaboration with Jessica Tilli, the school-based math coach, in an ongoing cycle of formative assessment: (1) teaching a lesson, (2) administering formative assessment questions, (3) analyzing evidence of student thinking, and (4) responding by planning for next lessons based on evidence. We also highlight how this cycle incorporates many of the Mathematics Teaching Practices from NCTM's (2014) *Principles to Actions: Ensuring Mathematical Success for All.*

Teaching the lesson

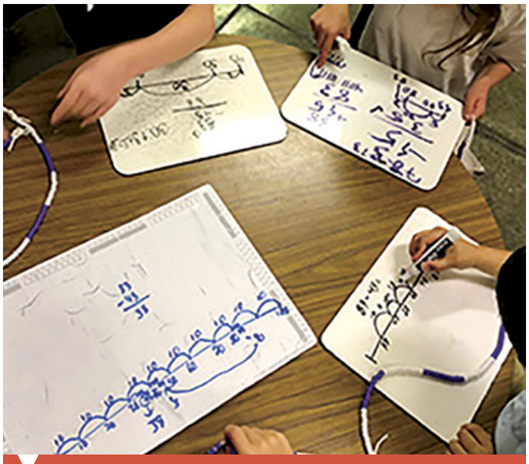
Severini noticed that although her students were proficient with a variety of addition strategies, they struggled with subtraction, often breaking up numbers by place value but then not knowing what to do with those parts, particularly when regrouping was involved. Moreover, many students had learned the standard U.S. subtraction algorithm but made errors that demonstrated a lack of conceptual understanding of the regrouping they were attempting to perform.

Drawing from what she had learned in professional development, Severini decided to pull out ten-structured bead strings to help students model subtraction on the number line (Klein et al. 1998), and then help them transfer those models to written representations to develop understanding of making jumps on an open number line, an example of *using and connecting mathematical representations* (NCTM 2014). After students were comfortable with the open number lines, Severini and Tilli developed a three-act task focused on a real-life subtraction scenario in which 74 Hershey® Kisses had been stolen from a bag of 275 (Fletcher 2016; Meyer 2011).

Assessing student learning

Severini was pleased to see that all of her students were successfully using number lines to solve the take-from-result-unknown subtraction problem that was at the center of the three-act task (CCSSM 2010), so she decided to administer an exit slip that would *elicit student thinking* (NCTM 2014) about subtraction in a slightly different situation. She selected the following take-from-change-unknown problem and gave it to students to solve individually at the end of class. She knew that this would give her valuable instructional evidence about how students were making sense of addition and subtraction in different situations and without the scaffolding she had provided in her lesson.

> There were 123 sandwiches in the cafeteria for lunch. Some of the sandwiches were eaten. After lunch, there were 48 sandwiches left. How many sandwiches were eaten? Show or explain how you know.



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The teacher transitioned students from using the concrete bead string to using the open number line.

Analyzing student strategies

Severini collected the student work, and later that day, she and Tilli sat down to look at it together. They sorted the student work into three stacks on the basis of strategies students had used. The first stack was student work that had evidence of *additive strategies*, including the use of the standard U.S. subtraction algorithm or transparent strategies based on decomposition by place value (see **fig. 1**). The second, and largest, stack was student work that had evidence of *transitional strategies*, where students were efficiently using the open number line by making jumps of multiples of ten, to solve the problem (see **fig. 2**). The third stack was student work that had evidence of *early transitional strategies*, where students were using the number line but jumping by tens rather than by larger, more efficient multiples of ten (see **fig. 3**). After sorting the

FIGURE 2

The second stack was the largest: Students used a transitional solution strategy.

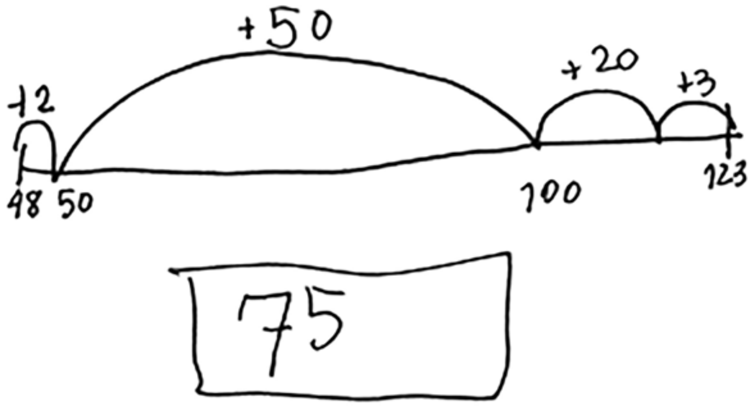


FIGURE 3

The third stack of student work showed early transitional solution strategies: Students used a number line but jumped by tens rather than by larger, more efficient multiples of ten.

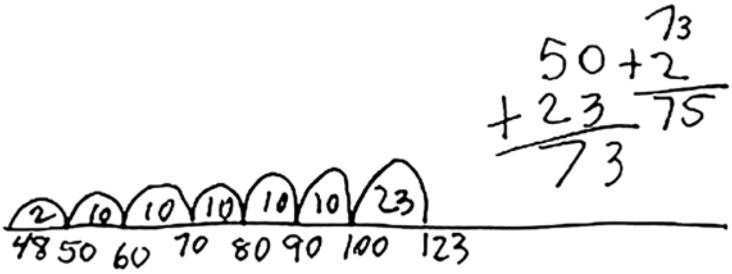
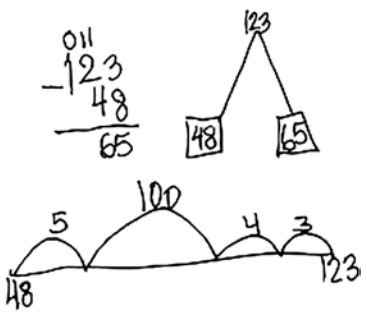


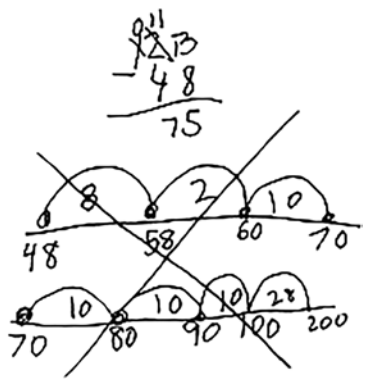
FIGURE 4

Some students correctly used the U.S. standard algorithm without evidence of understanding subtraction.

(a) Chad's number line does not accurately represent the difference between the two numbers.



(b) Unable to make jumps of 10 that were off the decade, Frederic abandoned an attempt from 48 to 123.



student work, Severini and Tilli focused on the three questions about the work that they had learned in professional development:

1. What is the evidence of developing understanding that can be built on?
2. What issues or concerns are evident in the student work?
3. What are instructional next steps based on the evidence (Petit, Hulbert, and Laird 2016)?

The first prompt gives attention to looking across the student work for evidence of "the good news." Severini was pleased with the fact that all her students had represented the problem as subtraction or adding up, and all but five students had obtained the correct answer of 75. Of those who had an incorrect answer, two showed calculation errors, and three had represented the difference correctly on the number line but had failed to provide an answer to the problem. The sort showed that most of her students were using *transitional* or *additive* strategies successfully to solve this problem, and Severini and Tilli knew that this was appropriate given that the second-grade standard is that students can "fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" (CCSSM 2010, 2.NBT.B.5, p. 19). The time spent on understanding and using number lines was apparently helping students move from counting to transitional and additive strategies.

The second prompt focuses on looking for issues or errors in student work. First Severini noticed that of the twenty-eight students, only two had labeled their answer as "75 sandwiches." Together with the fact that three students had not provided an answer after modeling the problem situation, she realized that she needed to emphasize going back to situate the answer within the problem's context.

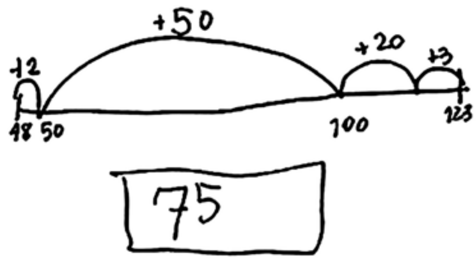
She also noticed that some students who had used the standard algorithm were unable to model the difference on the number line. For example, both Chad and Frederic correctly used the U.S. standard algorithm to solve the problem. However, Chad's number line (see **fig. 4a**) does not accurately represent the difference between the two numbers. Frederic (see **fig. 4b**) seemed to be attempting to jump from 48 up to 123 but was unable to make jumps of ten off the decade (e.g, from 48 to 58) and ultimately abandoned this strategy. Severini realized that some students who were using *additive* strategies needed more work at the *transitional* level to ensure that they were developing deep understanding of place value and subtraction.

The third prompt focuses on instructional implications based on evidence in the student work. Looking at her sorted stacks, Severini recognized that a handful of students could benefit from moving to more efficient jumps on the number lines (e.g., from tens to multiples of ten). She also saw that in the larger group of students who were using the number line correctly and efficiently, many might be ready to move away from the number line to increasingly efficient strategies or algorithms. For example, Hunter's efficient use of the open number line (see **fig. 5a**) showed that he might be ready to transfer this understanding to a written algorithm, and Minh's work (see **fig. 5b**) showed that she could effectively use both the standard algorithm and a number line. Severini wondered if she could help students build connections between the strategies they were using on the number line and algorithms that are more efficient.

FIGURE 5

Severini wanted to help all her students build connections between the strategies they were using on the number line and algorithms that are more efficient.

(a) Hunter's efficient use of an open number line showed readiness to transition to a written algorithm.



(b) Minh's work shows she could effectively use both the standard algorithm and a number line.

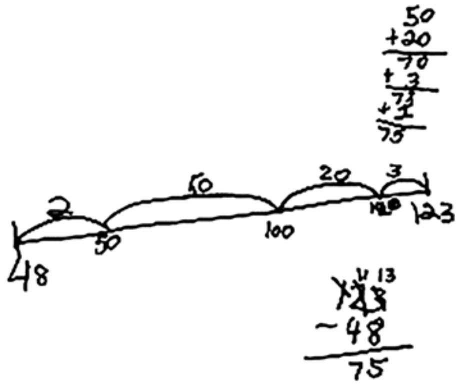
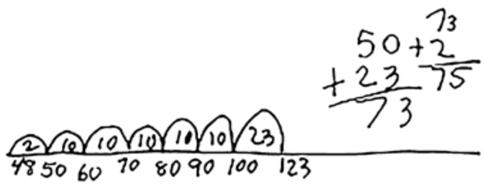


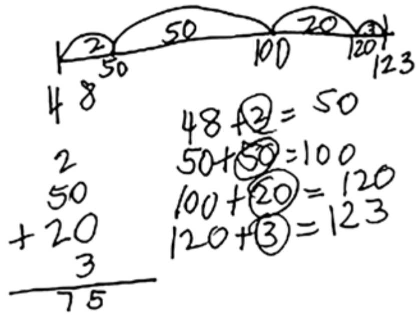
FIGURE 6

Severini and Tilli selected student work for a whole-class discussion and generated questions to focus students' attention on the strategies.

(a) The teacher showed Lily's work to the class first and asked questions from her work with the math coach the previous day.



(b) Students made connections between Cara's equations and jumps on the number line.



Developing an instructional response

Although her students were largely successful with this problem, by looking closely at their work, Severini recognized that she could provide some targeted instructional support to help all students develop deeper understanding and strategies that are more sophisticated. Working with Tilli, she decided to select student work for discussion in the next day's lesson, and together they generated a series of questions to focus students' attention on the strategies (Smith and Stein 2011). They decided to begin by projecting Lily's work (see **fig. 6a**) because it would likely be accessible to most students in the class after their work with the bead strings. As students gathered on the carpet and looked at Lily's work, Severini *posed purposeful questions* (NCTM 2014) to help them make sense of this strategy, knowing that a few students had not modeled the problem successfully on a number line:

- What did this student do to solve the problem?
- Why did she go to 50?
- How many jumps of 10 did she make? Why?
- Why did she go to 100?
- Why did she stop at 123? How did she know to make a big jump of 23?
- How can you figure out the answer from her number line?
- What is the answer? How do you know?

Severini then projected Cara's work (see **fig. 6b**) alongside of Lily's and again asked students to talk about the solution and how the two strategies were the same or different. When students noticed that Cara had made a jump of fifty, Severini asked, "Where is the fifty in the other solution?" Finally, she asked students to identify the answer to the problem and asked "seventy-five what?" to highlight that the problem was asking for the number of sandwiches.

Next Severini asked students how Cara had determined the answer to the problem. As students drew connections between Cara's equations and the jumps on the number line, Severini highlighted how the circled numbers were jumps. She wanted students to see that they could write equations to show what was happening on a number line. Knowing that some students in the class were ready for the next step, she had them talk through how to use this strategy of adding up with another subtraction problem, 164 – 85, this time imagining but not actually drawing the number line. She then asked all students to go back to their seats and sketch this solution on a number line to ensure that all students could connect this solution to their understanding of difference on a number line.

Implications for equity and access

This example illustrates ongoing formative assessment that draws on research about student learning to target diverse student needs. Even though most students were successful in solving the problem, Severini considered the evidence in their work when thinking about how to move them forward on the progression to using strategies that are more efficient and more sophisticated. She knew that eventually learning to use efficient algorithms to solve addition and subtraction problems was important for her students but that efficient use of algorithms must be built on strong conceptual understanding. She focused on helping students who were using algorithms without understanding move back to using the number line to anchor their thinking. She thought about how to help students who were using the number line inefficiently develop strategies based on number sense that would be effective with larger and more complex problems. And she thought about ways to help students who were already using the number line efficiently transition to using equations and transparent algorithms to represent their thinking.

Eliciting and using evidence of student thinking to develop targeted instructional responses in this way addresses diverse student needs and provides access to important mathematics for all children. In this case, selecting and sequencing the student work and using purposeful questions to focus on the identified needs provided access through *facilitating meaningful discourse* in a whole-group setting. Often the response to diverse needs is to break into small-group instruction. This classroom vignette offers an alternative and less intrusive approach to instructionally meeting all students' needs.

Research-based learning progressions can help teachers identify levels of students thinking along the developmental progression and consider the appropriate next instructional step. Teaching all children the most advanced and sophisticated strategies, such as the standard U.S. algorithm, before they have built conceptual understanding is not an equitable solution. By focusing on transitional strategies, such as the open number line, Severini helped build a link between students' concrete understanding and the more abstract algorithm, opening up access to procedural fluency built on conceptual understanding for *all* students (NCTM 2014).

Common Core Connections

2.OA.1
2.NBT.5
2.NBT.9 E

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