REASONING LANGUAGE FOR TEACHING SECONDARY ALGEBRA (ReLaTe-SA)

Cody L. Patterson, Texas State University, PI **Priya V. Prasad, University of Texas at San Antonio, Co-PI** **Emily Bonner, University of Texas at San Antonio, Co-PI** **Elizabeth Wrightsman, Texas State University, Co-Author**

Project Description

Reasoning Language for Teaching Secondary Algebra (ReLaTe-SA) is a three-year, Level I Exploratory project in the Teaching Strand of the Discovery Research PreK-12 (DRK-12) program. This project is led by an interdisciplinary team of faculty at the Texas State University and University of Texas at San Antonio in collaboration with academic directors and coaches at the San Antonio Independent School District.

Goals:

- ❖ Investigate middle and high school teachers' algebraic discourse through written assessments and analysis of classroom observations
  - Understand the discourse currently used by teachers when presenting algebraic concepts to students
- ❖ Design and implement a collaborative professional development (PD) program with middle and high school teachers that:
  - Addresses and enriches mathematical meanings for algebra teaching;
  - Defines, operationalizes, and helps teachers develop reasoning language for teaching algebraic concepts and procedures;
  - Makes reasoning-rich discourse for algebraic problem solving explicit and accessible
  - Identifies pedagogical practices that support learners' algebraic reasoning and discourse; and
  - Illuminates the importance of attending to students' cultural and knowledge assets thus allowing for richer engagement in algebraic content.

Hypotheses:

- ❖ Teachers' algebraic discourse is influenced by their mathematical meanings for algebraic concepts.
- ❖ Teachers' discourse influences the meanings that their students develop for these concepts.
- ❖ Addressing the discourse and related mathematical meanings that teachers use will enhance students' opportunities to develop robust understandings of algebra.

Progress:

- ❖ Designed and pilot-tested our *Survey of Algebraic Language and Reasoning* (SALR).
- ❖ Recruited a diverse group of middle and high school teachers.
- ❖ Administered the SALR for our 2021 cohort of middle and high school teachers.
- ❖ Collected and analyzed the responses to SALR according to discourse framework.
- ❖ Conducted and examined classroom observations of 2021 teacher participants.
- ❖ Developing a curriculum for a professional development (PD) course.

2021 Cohort:

- ❖ 8 participants (4 MS teachers, 3 HS teachers, 1 instructional coach)
- ❖ 6 schools
- ❖ Diverse levels of mathematics taught (7th & 8th grade math, Algebra I)

Theoretical Background

Our research aligns with Sfard's (2007, 2008) commognitive perspective on mathematical teaching and learning which assumes:

- ❖ All thinking is a form of communication
- ❖ To learn mathematics is to expand one's discourse or to participate in a new discourse

Our framework for algebraic discourse is based upon the *arithmetical discourse profile* of Ben-Yehuda, Lavy, Linchevski, and Sfard (2005).

Ben-Yehuda et al. divide learners' discursive actions into two dimensions:

- ❖ Subjective Dimension (utterances describing or reporting on the speaker)
- ❖ Object Dimension (utterances about objects of mathematical actions) – **Our Focus**
  - Analyze word use, uses of mediators, discursive routines, and endorsed narratives
  - Include procedures for solving problems
  - Include actions that are repeated so often as to become routinized
  - Endorsed narrative as statements accepted as true

Research Questions

1. What **language** do secondary (middle and high school) mathematics teachers use to describe and explain routines commonly used in algebra? To what extent does this discourse contain explicit descriptions of algebraic objects and their properties and relationships?
2. What changes in algebraic **discourse** - in general, and with respect to specific concepts and procedures - occur in teachers who participate in an intensive, content-focused professional development program? To what extent are these changes visible in classroom practice?
3. What **opportunities** can teacher-designed lessons, created in the context of such a professional development program, offer for the development of students' algebraic reasoning and discourse?

Research Design

To investigate the **language** that secondary mathematics teachers use to describe and explain algebraic concepts, we have developed a *Survey of Algebraic Language and Reasoning* (SALR) to be completed by teachers both before and after participation in the PD program. In developing the SALR, we focused on key algebraic concepts in the Texas Essential Knowledge and Skills (TEKS) for Grade 7 Math, Grade 8 Math, and Algebra I and created 15 items that present realistic scenarios that arise in the teaching of these concepts, often including hypothetical student work based on known conceptions that algebra learners exhibit.

In analyzing teachers' responses to the SALR, we attend to respondents' algebraic language and discourse at three levels:

1. Respondents' use of **words** and **mediators** (symbols and visual representations) to refer to mathematical objects. Specifically, we observe whether participants' responses to questions tend more toward descriptions of mathematical objects and their relationships (*e.g.*, "Because $2x + 3$ is equal to 17, we know that $2x$ must be equal to 14") or stories about personified actions on symbols (*e.g.*, "We move the 3 to the right side of the equation").
2. **Narratives** about mathematical objects and respondents' meta-rules for endorsing or rejecting these narratives. Specifically, we consider whether participants endorse narratives based on mathematical definitions and deductive reasoning or based on textual consistency with other endorsed narratives.
3. Respondents' use of discursive **routines** (such as problem-solving procedures) and their descriptions of these routines. Specifically, we consider the extent to which participants analyze routines as chains of deductive reasoning that generate endorsed narratives about mathematical objects.

To investigate the changes in **discourse** that can occur in a PD program focused on algebraic reasoning and discourse, we plan to capture Zoom video recordings of teacher participants as they work on a variety of activities, including:

- ❖ Mathematical problem-solving activities that involve algebraic reasoning
- ❖ Reflections on the algebraic reasoning opportunities inherent in these mathematics tasks
- ❖ Analysis and critique of available curricular materials (such as interactive lessons and worksheets) based on the opportunities they afford for students to engage in algebraic discourse
- ❖ Design and planning of lessons for the 2021–2022 school year that provide rich opportunities for development of students' algebraic reasoning

We are interested in the algebraic language that teachers use to explain and justify their own mathematical thinking as well as the language they use to describe opportunities for algebraic reasoning for their students.

To investigate the **opportunities** for algebraic reasoning and discourse that occur in lessons collaboratively planned by teacher participants, we plan to observe classes in which teachers implement these lessons and take field notes and transcripts to capture the language that teachers use as they orchestrate these lessons (specifically, how this language invites students into mathematical inquiry or performance of pre-established routines). We will also capture the language that students use as they explore problems, explain their reasoning, and justify findings so that we can compare the algebraic reasoning opportunities as hypothesized by teachers during the planning process with these opportunities as enacted by teachers and students in the classroom.

Research Results

The SALR has now undergone one round of implementation with our 2021 teacher cohort. At the end of the SALR, we ask questions about teachers' perceptions of the relevance of the questions to algebra teaching and the degree of authenticity of the hypothetical student responses. We summarize teachers' responses to these questions here:

- Question 1: On a scale of 0 to 5, with 0 meaning "totally irrelevant" and 5 meaning "extremely relevant," how relevant did the topics of the questions feel to your teaching of algebra content to students? (Mean: 4.6)
- Question 2: On a scale of 0 to 5, with 0 meaning "totally unrealistic" and 5 meaning "I have seen exactly these student conceptions in my own classes," how authentic did these items feel? (Mean: 4.5)

We interpret these results as evidence of the content validity of the survey as a research tool for investigating the reasoning and discourse that teachers have at their disposal as they explain concepts and procedures in algebra. We are still investigating the ways in which teachers' written responses contrast with their discourse in classroom practice.

In our preliminary analysis of the completed surveys, we have identified distinct approaches to explaining processes for solving equations, inequalities, and systems, which we call *extractive* and *inferential* discourses for equation-solving. Extractive discourse tends to treat equation-solving as a process of manipulating symbols in order to isolate a set of values for a variable (or variables), while inferential discourse tends to treat an equation-solving process as a sequence of steps, starting with an assumption that a solution exists, that deductively reveal information about the solution(s). We have found that teachers' discourse about equation-solving tends to contain elements of both approaches. Figures 1 through 3 show examples of teacher responses to a question about an equation-solving process that illustrate how these discourses can intermingle.

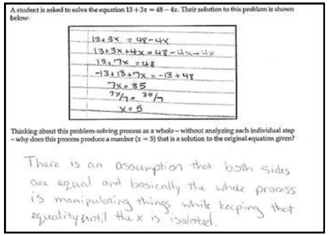

Figure 1. Response indicates that the routine starts with an assumption that two values are equal (inferential), but then describes a process of "manipulating things" (extractive).

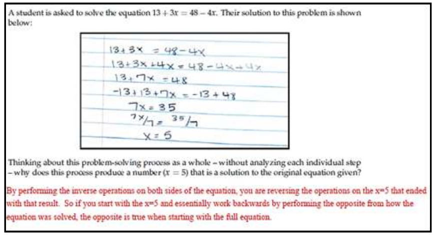

Figure 2. The solution is framed in terms of human actions such as "performing" and "reversing" (extractive), but also invokes structural ideas such as "inverse operations" and makes claims about equality statements being true (inferential).

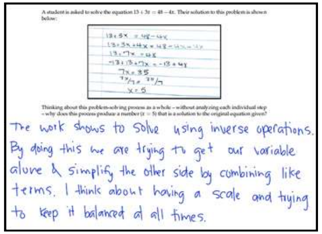

Figure 3. Describes mediators and their locations ("get our variable alone," extractive), but also invokes a metaphor for equality persisting through each step (inferential).

Implications

Rather than seeing inferential discourse about equation-solving as a superior alternative to extractive discourse, we see the two as distinct approaches to equation-solving, each with unique benefits. We hypothesize that inferential discourse allows a person to investigate and explain conceptual "wrinkles" that occur in the process of equation-solving, such as degenerate cases and extraneous solutions. On the other hand, extractive discourse seems better suited to talking about one's strategy for organizing the steps of an equation-solving process (*e.g.*, "isolate the variable *x*"). Our goal is to illuminate benefits of both approaches in algebra teaching and learning.

We hope, through our PD program, to set the stage for inferential discourse about equation-solving (as well as conceptually grounded discourse about other algebraic topics) in the classroom so that students can learn to use routines with greater awareness of their underlying assumptions and meanings of individual steps.

Impact

This project will:

- ❖ Increase our understanding of reasoning language in secondary algebra
- ❖ Expand literature on how algebraic reasoning language might promote conceptual understanding and procedural fluency among middle and high school students
- ❖ Increase mathematics education field's understanding of obstacles to students' conceptual learning of algebra
- ❖ Develop an approach for addressing two current challenges in algebra teaching practice:
  - Difficulty of providing conceptually coherent narratives that explain or justify common procedures and problem-solving strategies
  - Scarcity of opportunities for students to practice explaining algebraic thinking from a conceptually grounded point of view.
- ❖ Create and distribute products such as the SALR and the PD curriculum
- ❖ Present in nationally and internationally recognized journals and at national conferences
- ❖ Develop expertise in qualitative research on mathematical discourse in graduate students

References

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Sfard, A. (2005). Doing wrong with words: What bars students' access to arithmetical discourses. *Journal for research in Mathematics Education, 36*(3), 176-247.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the learning sciences*, *16*(4), 565-613.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing.* Cambridge, UK: Cambridge University Press.

Acknowledgement

This work is supported by the National Science Foundation through the Discovery Research PreK-12 (DRK-12) program (Award #1908825). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

TEXAS STATE UNIVERSITY®

The rising STAR of Texas

MEMBER THE TEXAS STATE UNIVERSITY SYSTEM