Designing Math Lessons for English Language Learners

Kathryn B. Chval and Óscar Chávez

A middle school mathematics teacher taught native English speakers for the first fifteen years of her career. As immigrants from other countries moved into the community and student demographics began to change, she realized that she was not prepared to teach mathematics to students who were English language learners (ELLs). The demands that she faced are not unique. The recent growth of the ELL population in the United States has challenged teachers to identify effective strategies to meet the needs of ELL students and their families.

Recent curriculum and instructional recommendations emphasize language-rich environments, cognitively demanding mathematics tasks for all students, and multiple modes of communication (e.g., written, verbal, gestures, and pictorial) (NCTM 2000; Steinbring, Bartolini Bussi, and Sierpinska 1998). In addition, researchers in bilingual education argue that the development of English as a second language in school can be better achieved when students use it while working to understand one another's meanings during discussions or while solving problems (Mohan 1990; Mohan and Slater 2005).

Unfortunately, linguistic minorities in the United States too often sit silently in mathematics classrooms (Brenner 1998). They do not have opportunities to use and experiment with language, particularly the specialized language of mathematics. Typically, teachers use low-level questions that only require simplistic language when presenting lessons to ELLs (Kramsch 1998, 2002). Needless to say, these classroom practices hinder ELLs' learning of mathematics.

PROMOTE PARTICIPATION

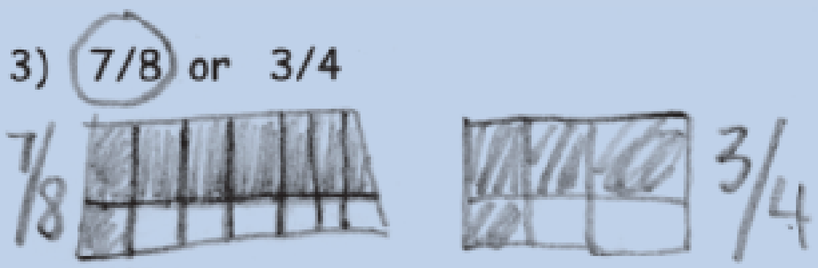
How can mathematics teachers enhance curriculum materials for ELLs who are in their mainstream classrooms? Kathryn Chval has worked with teachers to design lessons that incorporate research-based strategies for ELLs (see **fig. 1**). At times, these recommendations seemed overwhelming, and teachers often felt tempted to try just a few simple strategies, such as displaying pictures on the SMART Board™. This article describes a four-part process that teachers used to plan lessons and make enhancements while incorporating research about teaching math to ELLs. We share examples from research to illustrate these four parts. The research-based strategies within each example are highlighted to illustrate how such research can be turned into practice. Although we discuss the four components separately, they need to be enacted in relation to

Edited by **Debra I. Johanning,** debra.johanning@utoledo.edu. Readers are encouraged to submit manuscripts through **http://mtms.msubmit.net**.

Fig. 1 These 7 research-based strategies are key to supporting ELLs' mathematical proficiency.

1. Connect mathematics with students' life experiences and existing knowledge (Barwell 2003; Secada and De La Cruz 1996).
2. Create classroom environments that are rich in language and mathematics content (Anstrom 1997; Khisty and Chval 2002).
3. Emphasize meaning and the multiple meanings of words. Students may need to communicate meaning by using gestures, drawings, or their first language while they develop command of the English language and mathematics (Moll 1988, 1989; Morales, Khisty, and Chval 2003; Moschkovich 2002).
4. Use visual supports such as concrete objects, videos, illustrations, and gestures in classroom conversations (Moschkovich 2002; Raborn 1995).
5. Connect language with mathematical representations (e.g., pictures, tables, graphs, equations) (Khisty and Chval 2002).
6. Write essential ideas, concepts, representations, and words on the board without erasing so that students can refer to them throughout the lesson (Stigler, Fernandez, and Yoshida 1996).
7. Discuss examples of students' mathematical writing and provide opportunities for students to revise their writing (Chval and Khisty 2009).

Fig. 2 These nonverbal representations of 7/8 and 3/4 allowed the teacher to assess this ELL's prior understanding.



one another. Furthermore, this four-step process is challenging, complex, and takes time to learn. The ability to implement these components will evolve over time and through experience.

1. SUPPORT THE DEVELOPMENT OF MATHEMATICS

When Chval began her work with the teachers, they assumed that she would focus on helping ELLs learn English so that they could learn mathematics. Instead, the work focused on this question, which was posed to the teachers: What mathematics do you want your students to learn this week? Teachers discussed the important mathematical ideas, tools (e.g., manipulatives), and representations that students needed to learn. They also discussed expectations concerning students' prior knowledge and mathematical misconceptions.

One participant identified comparing fractions as a topic. The group then discussed discrete, linear, and area models; the role of number lines and fraction strips; and prior knowledge that students needed to compare fractions. To assess prior knowledge, this teacher said that she displayed 7/8 and 3/4 and asked students to work with a partner to "determine which fraction was larger and to explain why using pictures, words, or numbers." We viewed video that captured how Maria, an ELL, created representations of both fractions. We observed that she drew a rectangle, divided it into two rows, used the numerator to determine how many columns to make, and then used the denominator to determine how many parts to shade. Finally, she counted the unshaded parts to determine which fraction in the pair was larger (see **fig. 2**).

The teacher also gave students two other cases to compare:

1. 1 1/2 and 3/2
2. 1/6 and 1/10

Maria struggled to represent 1 1/2, 1/6, and 1/10. She remained quiet, relying completely on the work of her partner. Giving Maria the opportunity to draw a representation to convey her understanding illustrated strategy 3 in **figure 1:** *Students may need to communicate meaning by using gestures, drawings, or their first language.* This assessment also provided teachers with information about what mathematics needed to be supported, which allowed them to think through strategies 3, 4, and 5. It became evident that Maria (and others) would benefit if the mathematical meaning of the numerator and denominator were highlighted. An opportunity occurred to connect these terms to representations that had been developed and discussed in classroom conversations. Most important, Maria's teacher needed to consider how to support Maria's future development of mathematics, since what appeared to be a problem with language was in fact the result of a deeper mathematical misconception.

2. SUPPORT THE DEVELOPMENT OF LANGUAGE

The story of Sara, an effective teacher of Latino ELLs (as reported in Khisty and Chval 2002; Chval 2004; Chval and Khisty 2009), illuminates important aspects of the research-based strategies in **figure 1**. Sara's foundational assumption was that one need not reduce the curriculum's level of complexity, especially its language. She created an environment in which students experienced and used language in the context of joint activity and problem solving.

First, Sara used a sophisticated vocabulary and enacted strategies 2, 6, and 7: *creating language-rich classroom environments, writing essential terminology on the board,* and *discussing and having students revise their work.* In her classroom environment, students experienced new words in context. They not only heard but also saw these words written on the chalkboard and in feedback on their papers. When Sara asked students to produce a written explanation for solving a math problem related to a right triangle, she made various comments on ELLs' drafts:

- Verify your results;
- Combine the areas;
- Clarify this example; and
- What does that number represent?

Sara's students wrote multiple drafts of this assignment and engaged in a written dialogue with her. Sara was immersing students in an environment filled with sophisticated talk. When asked why she used sophisticated language with her ELLs, Sara replied:

> When are they going to learn it? How are they going to learn it? They encounter those words in books. I'm angry with the notion that students are not competent to learn.

Fig. 3 In this conversation, Sara builds understanding of the term *congruent* by using the terms *equal* and *exact* and finally *exact copy*.

1. *Sara*: Why do I divide it by two?
2. *Ana*: You have two triangles.
3. *Sara*: I have two *congruent* triangles here. Two *equal* parts, two *exact* triangles. I want only the area of my original triangle, *ACB*. Then I'm going to divide this by two. And what will my answer be? [Pause]
4. *Sara*: Number three. Would you please read that, Julia?
5. *Julia*: The triangle and its . . .
6. *Sara*: Congruent.
7. *Julia*: Congruent [struggling to say the word] . . .
8. *Sara*: Look at that word everyone. Congruent. What does that mean?
9. *Javier*: Like another copy.
10. *Sara*: An *exact copy.* Because here, look here is the circle. Is this circle congruent to that circle?
11. *Chorus*: No.
12. *Sara*: No, they're not *exact copies*. They're similar, they're both circles, but they're not *exact copies*.
13. *Chorus*: Yes.
14. *Sara*: How about this one and this one?
15. *Chorus*: Yes.
16. *Sara*: They appear to be congruent to each other. I agree. They appear to be congruent. But this one and this one are not congruent, are they?
17. *Sara*: So, congruent means an *exact copy*. Javier, you are super right. So read again, Julia.

Sara did not avoid using mathematical vocabulary when discussing problems. Instead, she built meaning for each of the words. For example, students were unfamiliar with the word *congruent*. They had trouble pronouncing it, resisted saying it, and did not understand it. The transcript in **figure 3** illustrates how Sara used this term in a mathematical context to guide students in understanding the term and developing a meaning for it.

In the dialogue, Sara repeated the students' answer from line 2, but added the word "congruent" (line 3). She used "equal" and "exact" to describe congruent (line 3). However, Javier described it as "another copy" (line 9). Sara refined Javier's answer by combining her description with Javier's description to create "exact copy" (lines 10, 12, and 17) and gave Javier credit for the usage (line 17). From this point forward, Sara consistently referred to "congruent" as a "copy." Over several lessons that followed, Sara combined these two words as "congruent copy." As students became comfortable using the term, she removed the word "copy" and began to use more precise language, such as "congruent triangle." Eventually, congruent became a word that appeared in the writing and speaking of every student in the classroom.

Another aspect of developing language in context was derived from strategies 1 and 3: *connecting with students' life experiences* and *exploring the multiple meanings of words*. For example, one teacher inquired about the meaning of *round* and was surprised to hear responses related to rounding up cattle, a round in a boxing match, as well as the common reference to a circle. This conversation helped distinguish multiple meanings of the term *round* and introduced another meaning in relation to rounding to the nearest tenth of a centimeter. As teachers began to emphasize language and assist conversations about multiple meanings of mathematical terms, all the students began to pose more questions about the meanings of certain terms.

3. ENHANCE MATHEMATICAL TASKS

According to Gutstein, "No single curriculum will be relevant to all students, and a real-life context is not necessarily a meaningful one" (2003, p. 63). Therefore, discussions about supporting the development of mathematics and language should also involve examining the mathematical tasks in curriculum materials. Teachers of ELLs are in a position that requires them to analyze and enhance or revise mathematical tasks. Following strategy 4, *use visual supports*, some of the teachers decided to display short YouTube videos or produce an image on the SMART Board to help students build contextual meaning for problems in the curriculum. In addition, when teachers found lessons employing a variety of contexts, they recreated problems using only one context.

The teachers began to wonder, "When does a context get in the way of students learning the mathematics?" The experienced teacher discussed earlier could not always anticipate when contexts would be problematic for her students. When introducing a context involving postage stamps, she brought in letters and bills containing postage stamps and sheets of stamps to show students what they looked like. She also showed pictures of a post office. However, her students had never used postage stamps or visited a post office. They asked, "Are they like food stamps?" This incident gave her new insights into preparing and using contexts for mathematical problems with her students.

In some cases, the teachers created contexts related to events or activities at the school because the students were familiar with them. They also considered ways to enhance and build meaning for language that was used in the curriculum materials. More important, teachers recognized that adapting problems did not mean making them easier but rather making the context accessible while preserving the mathematical integrity of the task.

4. ESTABLISH, FACILITATE, AND MAINTAIN PRODUCTIVE CLASSROOM INTERACTIONS

During our work with mathematics teachers, they watched video segments of the ELLs working individually and in groups and taking part in whole-class discussions. The teachers recognized that it would be insufficient to consider the mathematics, language, and curriculum materials only. They also had to work strategically to build a classroom community that valued ELLs' contributions. At times, teachers observed unproductive interactions that hindered ELLs' learning, such as when native-English-speaking peers disrespected them, ignored their questions or requests, did their work for them, or did not understand their mathematical misconceptions. Too often, ELLs were allowed to work in isolation or participate as a spectator (Brenner 1998). Therefore, teachers should carefully monitor a classroom environment to enhance ELLs' participation in mathematical activities.

The teachers used a variety of strategies to help ELLs participate in different settings. They carefully selected partnerships, recognizing that some students who dominated partnerships would not help ELLs gain confidence in group activities. Although the teachers ultimately wanted the ELLs to work well with all their peers, in the beginning they needed to identify partners who would be the most productive.

The teachers also looked for opportunities for ELLs to share their work during whole-class discussions—for example, by displaying tasks on the SMART Board while the ELLs shared their solution strategies. This approach was consistent with **figure 1**'s strategy 5. The ELL was able to refer to the written work using verbal descriptions and gestures.

To implement this specific strategy, the teachers needed to change their beliefs that such experiences would make ELLs feel uncomfortable. In the past, they did not ask ELLs to answer questions during whole-class discussions because they did not want to put them on the spot. When the teachers adjusted their practice to take into account the highlighted research-based strategies, they changed their expectations. For example, one of the teachers Chval worked with revealed the following:

> At the beginning, I didn't really expect a lot from Mario just because he was so quiet and I wasn't really sure if he knew what was going on for the most part. I never imagined that he would be able to be one of the students that I am going to ask to help other students.

She said that she had learned to "give it more time and look at what they can actually do and push them to be involved. Push them because they can do it. There is no reason they can't."

CONCLUSION

We have discussed and illustrated the use of seven research-based strategies that support ELLs in mathematics classrooms. Although the literature suggests that these strategies can support ELLs, it also suggests that they are not typically or consistently implemented in classrooms where ELLs are mainstreamed. Teachers need to consider how to support the development of mathematics and language; enhance mathematical tasks;

and establish, facilitate, and maintain productive classroom interactions specifically for ELLs so that equity can be achieved in the classroom. For example, when teachers use visual aids or gestures, they must use them in purposeful ways that target the development of language and mathematics for the ELLs in their classroom. This work is challenging and complex. It also requires thoughtful conversations and planning with other teachers. However, the investment will be worth it when teachers witness the success of ELLs in learning mathematics.

REFERENCES

Anstrom, Kris. 1997. *Academic Achievement for Secondary Language Minority Students: Standards, Measures, and Promising Practices*. Washington, DC: National Clearinghouse for Bilingual Education. http://www.ncela.gwu.edu/pubs/reports/acadach.htm.

Barwell, Richard. 2003. "Patterns of Attention in the Interaction of a Primary School Mathematics Student with English as an Additional Language." *Educational Studies in Mathematics* 53 (1): 35–59.

Brenner, Mary. 1998. "Development of Mathematical Communication in Problem Solving Groups by Language Minority Students." *Bilingual Research Journal* 22 (2–4): 103–28.

Chval, Kathryn B. 2004. "Tools for Thought and Communication." In *Proceedings of the Twenty-Sixth Annual Meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education*, edited by Douglas McDougall, vol. 3, pp. 1473–79. Toronto: OISE/UT.

Chval, Kathryn B., and Lena L. Khisty. 2009. "Latino Students, Writing, and Mathematics: A Case Study of Successful Teaching and Learning." In *Multilingualism in Mathematics Classrooms: Global Perspectives,* edited by Richard Barwell, pp. 128–44. Clevedon, UK: Multilingual Matters.

Gutstein, Eric. 2003. "Teaching and Learning Mathematics for Social Justice in an Urban, Latino School." *Journal for Research in Mathematics Education* 34 (1): 37–73.

Khisty, Lena L., and Kathryn Chval. 2002. "Pedagogic Discourse and Equity in Mathematics: When Teachers' Talk Matters." *Mathematics Education Research Journal* 14 (3): 154–68.

Kramsch, Claire. 1998. *Language and Culture.* Oxford, UK: Oxford University Press.

———. 2002. *Language Acquisition and Language Socialization: Ecological Perspectives.* London: Continuum.

Mohan, Bernard. 1990. "LEP Students and the Integration of Language and Content: Knowledge Structures and Tasks." In *Proceedings of the First Research Symposium on Limited English Proficient Student Issues*, pp. 113–60. Washington, DC: Office for Bilingual Education and Minority Language Affairs.

Mohan, Bernard, and Tammy Slater. 2005. "A Functional Perspective on the Critical 'Theory/Practice' Relation in Teaching Language and Science." *Linguistics and Education* 16: 151–72.

Moll, Luis. 1988. "Key Issues in Teaching Latino Students." *Language Arts* 65 (5): 465–72.

———. 1989. "Teaching Second-Language Students: A Vygotskian Perspective." In *Richness in Writing: Empowering ESL Students*, edited by Donna M. Johnson and Duane H. Roen, pp. 55–69. New York: Longman.

Morales, Hector, Lena L. Khisty, and Kathryn Chval. July 2003. "Beyond Discourse: A Multimodal Perspective of Learning Mathematics in a Multilingual Context." Paper presented at the 27th Annual Meeting of the International Group for the Psychology of Mathematics Education, Honolulu, HI.

Moschkovich, Judit N. 2002. "A Situated and Sociocultural Perspective on Bilingual Mathematics Learners." *Mathematical Thinking and Learning* 4 (2–3): 189–212.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Raborn, Diane T. 1995. "Mathematics for Students with Learning Disabilities from Language-Minority Backgrounds: Recommendations for Teaching." *New York State Association for Bilingual Education Journal* 10: 25–33.

Secada, Walter G., and Yolanda De La Cruz. 1996. "Teaching Mathematics for Understanding to Bilingual Students." In *Children of la Frontera*, edited by Judith LeBlanc Flores, pp. 285–308. Charleston, WV: ERIC Clearinghouse on Rural Education and Small Schools.

Steinbring, Heinz, Maria G. Bartolini Bussi, and Anna Sierpinska. 1998. *Language and Communication in the Mathematics Classroom*. Reston, VA: National Council of Teachers of Mathematics.

Stigler, James W., Clea Fernandez, and Makoto Yoshida. 1996. "Traditions of School Mathematics in Japanese and American Elementary Classrooms." In *Theories of Mathematical Learning*, edited by Leslie P. Steffe, Pearla Nesher, Paul Cobb, Gerald A. Goldin, and Brian Greer, pp. 149–75. Mahwah, NJ: Lawrence Erlbaum Associates.

Kathryn B. Chval, chvalkb@missouri.edu, is an associate professor of mathematics education at the University of Missouri–Columbia. She researches how to improve the participation of ELLs in mathematics classrooms.

Óscar Chávez, chavezo@missouri.edu, is an assistant professor at the University of Missouri–Columbia. He was born in Mexico City, where he taught mathematics in grades 7–12. He is interested in curriculum research, the use of technology in mathematics education, and teaching strategies for ELLs that are based on high expectations for all students.