Front. Educ. China 2018, 13(4): 601–632
https://doi.org/10.1007/s11516-018-0031-z

RESEARCH ARTICLE

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Transition from Textbook to Classroom Instruction in Mathematics: The Case of an Expert Chinese Teacher

Abstract This study reports how an expert Chinese teacher implements mathematics textbook lessons in enacted instruction. Our video analysis indicates that both textbook and enacted teaching included only one worked example; however, the teacher engaged students in unpacking the example in great depth. Both the textbook and the enacted teaching showed "concreteness fading" in students' use of representations. However, the Chinese teacher incorporated students' self-generated representations and facilitated students' active modeling of quantitative relationships. Finally, the Chinese teacher asked a greater number of deep questions than were suggested by the textbook. These deep questions often occurred as clusters of follow-up questions that were either concept-specific or promoted comparisons which facilitated connection-making between multiple representations and solutions.

Keywords textbook-instruction transition, expert Chinese teacher, worked example, representation, deep question

Introduction

Prior studies indicate that Chinese students generally demonstrate superior mathematical achievement on international tests (Cai, 1995, 2000; Li, Ding, Capraro, & Capraro, 2008; PISA, 2012) which may be partially attributed to

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effective presentations of Chinese textbooks (Ding, 2016; Ding & Li, 2010; Li et al., 2008) and classroom teaching (Li & Huang, 2013). Consider, for example, the concept of mathematical equality. Li et al. (2008) found that while 98.6 % of Chinese sixth-graders correctly responded to items such as $6 + 9 = \square + 4$, only 28.6 % of U.S. sixth-graders could do so. These authors traced the source of students' understanding difference back to textbook presentation. While Chinese textbooks from the very beginning of elementary school treated the equal sign as a relational sign (e.g., introducing the "=" along with the ">" and "<" signs), the U.S. textbooks treated it as an operational sign (introducing the "=" along with the "+" and "–" signs).

It is important to note that the potential impact of Chinese textbooks on student learning cannot be separated from the fact that Chinese teachers implement textbooks with fidelity (Ding, Li, Li, & Gu, 2013; Ma, 1999). This is quite different from the Western context where teachers do not have a prescribed (or may have no) textbook. In China, instructional design is generally "textbook-centered," meaning that textbooks provide teachers with teaching content, lesson structure, and homework material (Bi & Wan, 2013; Wang, 2011).

However, Chinese teachers' fidelity of textbook implementation does not mean they mechanically follow every word written in their textbooks. Instead, Chinese teachers study their textbooks and make necessary changes to better fit their students' needs (Ding et al., 2013; Ma, 1999). How Chinese teachers actually use textbooks in mathematics classrooms has seldom been studied. Existing studies on Chinese teachers' textbook use are often summaries of teachers' experience (Wang, 2011) without rigorous comparisons between actual changes from textbook to classroom instruction and corresponding instructional decisions made by the teachers. Lacking a clear picture of the detailed textbook-instruction transition process may limit the potential contribution of Chinese mathematical instruction to the field. The current study examines how an expert Chinese elementary teacher implements mathematics textbook lessons in the classroom. What changes does this teacher make during the textbook-instruction transition process, and what rationale lies behind these instructional decisions?

Prior Research on Teachers' Textbook-Instruction Transition

Textbooks present an "intended curriculum" (Remillard, 2005). To impact student learning, teachers need to turn this "intended curriculum" into an "enacted curriculum" in the actual classroom (Remillard, 2005; Stein, Remillard, & Smith, 2007). In this study, we call this transition process "textbook-instruction transition," or, interchangeably, "textbook use." We aim to study the similarities and the differences between the textbook and the enacted lessons and to understand the rationale behind these changes and non-changes. Prior studies report that Chinese teachers uniformly acknowledge the importance of textbooks and use them with fidelity (Ding et al., 2013; Ma, 1999). This uniform style of textbook use with loyalty is different from the varied styles (e.g., adhering, adapting, or creating) reported in other cultures (e.g., Nicol & Crespo, 2006; Remillard & Bryans, 2004). As previously mentioned, Chinese teachers' loyalty to textbooks, however, is not simply blind adherence; rather, Chinese teachers frequently "study" textbooks (Ma, 1999) so as to identify important and difficult learning points, understand the purpose of each worked example and practice problem, and explore best approaches for presenting examples from the perspective of students (Ding et al., 2013).

Prior studies on teachers' textbook use provide insights into the textbook-instruction transition process. However, few studies have documented how Chinese teachers turn the "studied" textbook materials into actual classroom practice, much less provide a side-by-side comparison between Chinese textbook presentations and expert teachers' enacted lessons. Indeed, prior studies on Chinese teachers' textbook use have been based on teacher surveys and interviews rather than actual classroom observations (e.g., Ding et al., 2013). Without fine-grained analysis of how the intended curriculum is turned into enacted lessons, we have little knowledge of what insights may be useful in practice. This study aims to explore an expert Chinese teacher's transition of textbook presentations into classroom teaching.

A Conceptual Framework for Examining the Textbook-Instruction Transition

To examine the expert Chinese teacher's textbook-instruction transition, we follow a conceptual framework that contains three key aspects: (a) interweaving worked examples and practice problems (simply, "worked examples,"), (b) making connections between concrete and abstract representations (simply, "representations"), and (c) asking deep questions to elicit students' self-explanation (simply, “deep questions”). These aspects were drawn from the Institute of Educational Sciences’ (IES) practice guide (Pashler et al., 2007) which was developed from high-quality cognitive and classroom research on organizing instruction to improve student learning. Note that the aspect of “representations” combines two IES recommendations: “Combine graphics with verbal descriptions” and “Connect and integrate abstract and concrete representations of concepts.” We did not select the remaining recommendations (spacing out learning over time, using quizzing to promote learning, and helping students allocate study time efficiently) because these were somewhat distant from our analyzed classroom lesson. In fact, this three-aspect framework was used in prior research on textbook examination (Ding, 2016) and teachers’ lesson planning (Ding & Carlson, 2013). Below are elaborations.

Worked examples (problems with solutions given) help students acquire necessary schema to solve new problems (Sweller & Cooper, 1985; Zhu & Simon, 1987). Despite the value of worked examples, many existing classrooms do not use worked examples or only discuss example tasks in brief (Kirschner, Sweller, & Clark, 2006; Stigler & Hiebert, 1999). In fact, many teachers tended to rush through numerous examples due to a common misconception that “the more examples, the better” (Ding & Carlson, 2013). Aside from findings on worked examples, recent studies have suggested interweaving worked examples and practice problems (Pashler et al., 2007), that is, they have found that an alternation between worked examples and exercise problems enhances student learning.

Research shows that concrete representations support initial learning because they provide familiar situations that facilitate students’ sense-making (Resnick, Cauzinille-Marmeche, & Mathieu, 1987). However, overexposing students to concrete representations may hinder their transfer of learned knowledge if they contain distracting information (Kaminski, Sloutsky, & Heckler, 2008). Therefore, to promote learning and transfer, researchers suggest a method called concreteness fading (Goldstone & Son, 2005), that involves a progressive transformation from concrete representations (e.g., word problem contexts, real objects) to semi-concrete representations (e.g., circles, dots) to abstract representations (e.g., symbols, numbers). Recent studies report that concreteness fading is effective in supporting students’ learning of mathematics (Fyfe, McNeil, & Borjas, 2015; McNeil & Fyfe, 2012). To facilitate concreteness fading, schematic diagrams (e.g., number lines, tape diagrams) are recommended as they illustrate key concepts better than photorealistic pictures and may serve as a bridge to link concrete and abstract representations (Pashler et al., 2007).

The final aspect involves deep questions. Students can effectively learn new knowledge through self-explanations (Chi, 2000). However, they themselves usually have little motivation to generate high-quality explanations. It is necessary for teachers to ask deep questions to elicit students' deep explanations (Craig, Sullins, Witherspoon, & Gholson, 2006; Pashler et al., 2007). By deep explanations, the IES recommendation refers to "explanations that appeal to causal mechanisms, planning, well-reasoned arguments, and logic" (Pashler et al., 2007, p. 29). To elicit deep explanations, they recommend that teachers ask questions with the following stems: Why? Why-not? What if? What-if-not? What caused X? What is the evidence for X? Why is X important? How? How did X occur? And, how does X compare to Y? In the current study, given that our focus is on mathematics teaching, we refer to deep questions as those that potentially elicit students' deep explanations of the underlying mathematical concepts, principles, relationships, and structures which are deemed the most important aspects of mathematics learning in elementary school (Kieran, 2018; Shifter, 2018). In other words, even though we refer to the question stems listed in Pashler et al. (2007) to identify deep questions in textbooks and classroom teaching, we do not limit our identification to these linguistic hints.

The above IES recommendations are general guidance for teaching all subjects. It has been found, however, that teachers have difficulty implementing the IES recommendations in lesson planning (Ding & Carlson, 2013), let alone complex classroom teaching. As such, our examination of the expert Chinese teacher's textbook-instruction transition using this conceptual framework holds both practical and theoretical importance.

Prior Findings on Chinese Textbooks and Classroom Instruction

Prior research has studied features of Chinese textbooks and classroom instruction in terms of their use of worked examples, representations, and deep questions. Ding and colleagues, based on comparative textbook analyses between U.S. and Chinese elementary textbook series (Ding, 2016; Ding & Li, 2010, 2014), reported that Chinese textbooks usually situate a discussion of worked examples in concrete word-problem contexts. These textbooks also sequenced representations from concrete to abstract to promote abstract understanding and arranged a few deep questions, including comparison questions (e.g., What are the connections between the two solutions? Which solution is easier? Can the basic laws of arithmetic with whole numbers also be used with fractions?), to facilitate explicit understanding of structural relationships. The use of worked examples, representations, and deep questions in Chinese textbooks appeared to be consistent with the IES recommendations on how to organize instruction to support learning (Pashler et al., 2007) but generally contrasted with the U.S. textbook presentations.

With regard to Chinese classroom instruction, prior studies have revealed similar features. For instance, Chinese teachers value the use of worked examples to develop students' problem-solving skills (Zhu & Simon, 1987). They also attend to connection-making between different representations, with the eventual goal of fostering students' abstract thinking (Cai, 2005). Finally, Chinese teachers tend to ask deep questions about conceptual knowledge that is often situated in concrete contexts which may facilitate students' sophisticated mathematical understanding (Perry, VanderStoep, & Yu, 1993). In particular, when addressing students' mistakes during teaching, Chinese teachers were found to ask follow-up questions more frequently than their U.S. counterparts (Schleppenbach, Flevares, Sims, & Perry, 2007). Chinese teachers' questioning skills were also in contrast with the skills of U.S. teachers in cognitively guided classrooms who were capable of asking initial deep questions but faced challenges asking follow-up questions (Franke et al., 2009). The above findings about Chinese teachers' instructional features were also revealed by a systematic investigation into how Chinese teach and improve teaching (Li & Huang, 2013).

Overall, prior studies on Chinese textbook and classroom instruction reveal the key features emphasized by IES recommendations, which suggest the necessity and the feasibility of studying how static textbook presentations may be enacted as complex and dynamic classroom instruction. In particular, what kinds of changes does an expert Chinese teacher tend to make, and what is the purpose of these changes? In keeping with our conceptual framework, we pose three research questions: (1) How does an expert Chinese teacher implement the textbook's worked examples in enacted lessons? (2) How does an expert Chinese teacher implement the textbook's representations in enacted lessons? and (3) How does an expert Chinese teacher implement the textbook's deep questions in enacted lessons?

Methods

We employ a case study method (Stake, 1995) to obtain deep understanding of the textbook-instruction transition process. The intent of this type of study is not to generalize findings to other settings. Rather, it aims to identify meaningful themes through thick description and comparison in a particular setting (Creswell, 2014; Stake, 1995). In this study, one of the authors is a participating expert Chinese teacher who taught the lessons while the other author is the project's principal investigator. The collaboration between the researcher and the teacher practitioner, or the participatory mode of research (Creswell, 2014), allows us to hear the teacher's own voice on the textbook-instruction transition process and her rationale behind changes from textbook to classroom instruction.

Note that the researcher and the teacher played equally important roles in this study. As elaborated below, the researcher, who is more familiar with the literature, selected the lessons. The teacher taught the lessons based on her own understanding without the researcher's input. When analyzing the data, both authors met to discuss the IES recommendations to achieve a shared understanding of the conceptual framework. To avoid potential bias in data analysis, both authors coded the data independently before comparing their findings. Triangulation among various data (videos, textbooks, teacher interviews and reflections) were also conducted to improve validity and reliability (Creswell, 2014). Moreover, when it comes to the rationale for "changes" from textbook to classroom instruction, the teacher, who was the informant, provided a detailed account which also served to improve the validity of study (Creswell, 2014).

The Participant and the Project

The study is part of a five-year cross-cultural research project supported by the U.S. National Science Foundation (NSF). A total of 17 U.S. and 17 Chinese elementary teachers are involved. The goal of the large-scale project is to identify the necessary knowledge for teaching early algebra in elementary school based on expert U.S. and Chinese teachers' instructional insights. For the current study, the participating expert Chinese teacher is female. By the time of her involvement, she had 16 years of experience in teaching elementary mathematics from grades 1 to 3. This teacher has received numerous teaching awards from mathematics teaching competitions at both local and national levels and frequently taught model lessons for her peers. Due to her teaching expertise, she has served as the director of the Teaching and Research Group (Ma, 1999) at her school and as a member of a teacher qualification evaluation committee in her city. While working as a full-time mathematics teacher, she also is a first-year doctoral student pursuing a Ph.D. at a top-tier normal university in China.

Instructional Tasks

For this study, the teacher taught four lessons selected by the project's principal investigator from a Chinese second-grade textbook to fulfill the goal of the large project. In other words, this teacher did not herself select the lessons; rather, she taught these identified lessons as she normally would. All these lessons were selected from the Chinese second-grade textbook published by Jiangsu Education Publishing House (Su & Wang, 2011); this is one of the three representative textbook series based on the new Chinese curriculum standards (Ding & Li, 2010; Ministry of Education, 2001). These four lessons directly or indirectly involved the inverse relationship between addition and subtraction, a critical early algebra topic that has been emphasized in the field (Common Core State Standards Initiative, 2010; Kieran, 2018). Note that these four lessons were selected based on the literature to cover different types of tasks related to inverse relations (e.g., Barrody, 1999; Carpenter, Franke, & Levi, 2003; Ding, 2016). As such, these lessons came from different volumes of the textbook, with the first and second from volume 1 and the third and fourth from volume 2. In particular, the first two lessons posed comparison problems which together implicitly involved inverse relations. The worked example of Lesson 1 was about how to equalize two strings of beads (8 beads and 12 beads). The goal of Lesson 2 was to find the large quantity (3 more than 11, using addition) or small quantity (3 less than 11, using subtraction) based on the context of making flowers. The worked example of Lesson 3 was to solve a two-step word problem about taking a bus (initially 34 people, 18 on and 15 off). The action of getting "on" or "off" a bus indicated an inverse relation. Finally, the goal of Lesson 4 was to check subtraction using addition through the context of borrowing books (borrowing 93 books from 215). The checking process directly involved inverse relations.

Procedures and Data Collection

As aforementioned, even though the four lessons were selected by the research project, the enacted lessons were designed and taught by the teacher without input from the project researcher. As such, all four lessons were examples of natural classroom teaching. Except for Lesson 2, the worked example used in each enacted lesson was the same as the textbook example. For Lesson 2, the teacher changed the “making flowers” context to “guessing the number.” The teacher explained that this game targeted the same objectives, and her students enjoyed solving mystery problems about their teacher and their peers.

All the lessons were videotaped by NSF project staff. After each lesson, the teacher was interviewed using a structured interview questionnaire focusing on the teacher’s use of worked examples, representations, and deep questions. A copy of the video recordings of the enacted lessons and the teacher interviews was shared with the participating teacher. To ensure the trustworthiness of data analysis and interpretation, the expert teacher herself first transcribed the four lessons. The transcripts were then coded by both authors who documented side-by-side what happened in the textbook lessons and in classroom instruction regarding the use of worked examples, representations, and deep questions. Coding difficulties were discussed and resolved (elaborated upon later).

Coding and Data Analysis

While the coding framework suggested an overall focus, our coding process employed both top-down and bottom-up approaches. For each aspect of the study, we considered several factors based on literature recommendations and the actual data.

Coding Worked Examples

Given that teachers in prior studies either overlooked worked examples (Kirschner et al., 2006; Stigler & Hiebert, 1999) or presented repetitive examples (Ding & Carlson, 2013), we coded the “frequency” of example tasks in each textbook and enacted lesson. Next, we coded the “sequence” in which the worked example and the practice problems appeared in each lesson because an alternation between examples and practice problems was recommended (Pashler et al., 2007). Moreover, we documented the main activities contained in each example task which enabled a comparison of depth between the textbook lesson and the enacted teaching.

Coding Representations

We coded the types of representations involved in the worked examples. These representations ranged from concrete (e.g., word problem) to semiconcrete (e.g., circle/chip, stick, tape diagram, number line diagram, table, flow chart) and to abstract (e.g., number sentence). Since representational sequence is also critical (Fyfe et al., 2015; Goldstone & Son, 2005), for each worked example, we documented the order of the representations which indicated representational sequence. For practice problems in each lesson, we listed additional representations beyond the ones in a worked example. This allowed us to obtain a general sense of the typical types of representations used by the textbook and the teacher.

Coding Deep Questions

Our coding of deep questions was based on the worked example presentations. A bottom-up approach was used during this process. For textbook lessons, there were limited instances, and all were coded. For enacted lessons, we first eliminated all simple questions in the transcripts (e.g., "Right or wrong?" "Do you all agree?"). The remaining questions, which demanded students' articulations or explanations, were classified into three categories based on their purposes: brainstorming, connection, and reflection. Brainstorming questions required students to identify the given information or brainstorm a possible solution which provided an opportunity for students to enter into a dialogue. Example brainstorming questions included: "What do we know about the problem?" "Look at this diagram. Can you explain it to us?" Connection questions prompted students to explain a key knowledge point or make connections between representations and solutions. Example connection questions included: "How is this method different from the one we just discussed?" "Why may we use addition to check subtraction?" Finally, reflection questions prompted students to look back at the solution method or to recall what they had learned. A typical example was: “What did you learn from this problem-solving process?” Given that the connection questions appear to be the closest to deep questions as defined by the literature (Craig et al., 2006), we further analyzed the main features embodied in the “connection” questions.

Coding Difficulties

Difficulties occurred mainly with coding teacher questions. The first difficulties related to the appropriate “unit” to be used. Initially, we considered only the major questions but not the follow-ups. After discussion, both authors agreed that the follow-up questions should be considered due to their importance (Franke et al., 2009). We also encountered difficulties in classifying a few comparison questions that may not have prompted substantial responses. For instance, one coder coded “Which method of student A is similar to which method of student B?” as a connection question whereas the other coder viewed it as brainstorming. After discussion, we decided to code all comparison questions as connection questions because they shared the same purpose of prompting connections.

Verification and Data Analysis

One coder went through four lessons for all aspects. The second coder coded all worked example tasks independently. Reliability for coding of worked examples was 100 %, and reliability for representations and deep questions exceeded 90 % (# of common codes / # of total codes). Disagreements were resolved by ongoing discussions. The finalized codes for the textbook lessons and enacted teaching were compared to identify similarities and differences. To ensure internal validity, both authors were cognizant of their roles and potential bias as declared at the beginning of this section. We also triangulated findings from the video and the interview data. Moreover, the researcher author conducted a second interview during which she prompted the teacher author to explain her thinking regarding why she made changes and what enabled her to make those changes. Finally, we identified typical textbook presentations and video screenshots to illustrate typical changes so as to provide rich, thick, detailed descriptions, which may enhance the external validity of study (Creswell, 2014).

Results

We report findings based on data coding and analysis in terms of three aspects—worked examples, representations, and deep questions—aligned with our research questions. Overall, the teacher made changes from the textbook during the textbook-instruction transition process in all aspects. For each aspect, we first report findings based on the quantification of frequency, sequence, or type. We then illustrate the patterns of findings with narrative descriptions involving typical classroom episodes and images from the textbook and the enacted lessons. Changes that occurred during the textbook-instruction transition, and rationale for these changes, are further elaborated.

Worked Examples: Transition from Textbook to Instruction

All textbook and enacted lessons included only one worked example. In other words, within one 40-minute lesson, the Chinese textbook and the teacher presented relatively few examples. With regard to sequence, there was no alternation between worked examples and practice problems. Instead, instruction flowed from the lone example to practice problems.

Analysis of the main activities contained in each example revealed differences between textbook and enacted lessons. In the textbook lesson, the main activities were "Guide-Solve-Discuss." That is, the teacher was first to guide students to identify the known and the unknown in the worked example problem. Next, the teacher was to ask students to solve the problem on their own. Finally, the teacher was to guide students to discuss their solutions or revisit the problem-solving process. In the enacted teaching, however, the first activity was usually students' report of "pre-learning" products which often included multiple solutions to the same problem. Each of the solutions was discussed and then compared, something that was not explicitly suggested in the textbook. In retrospect, the teacher explained that her rationale for integrating pre-learning with the worked example was to assess student learning and address different learning needs:

> The purpose of using pre-learning is to gain understanding of students' prior knowledge which helps me adjust the design of my lesson plan. Meanwhile, it gives students more time to understand the problem situation, especially for students who have learning

> difficulties. During the actual lesson, some students do not have not enough time to show their solution process through pictorial representations or using manipulatives. (teacher Chen, personal communication, second interview, March 9, 2016)

In addition, the teacher shared that the motivation for her integration of students' self-generated products from pre-learning was to actively engage the students in knowledge construction which is common in current Chinese mathematics classrooms (Ministry of Education, 2001). The teacher pointed out that the textbook lesson only presented three methods for solving the problem; however, her students, through pre-learning, suggested more than three methods. Additionally, her students used their preferred drawings that were different from the textbook representations.

A further comparison between the textbook and the student-generated solutions indicates that they reach different levels of depth. Figure 1 illustrates one example. In this task, students need to make two quantities (eight beads and 12 beads) equivalent. The textbook example presents three solutions: adding four to the small quantity, removing four from the large quantity, and moving two from the large to the small quantity (see Figure 1a). In the enacted lesson, student solutions discussed included not only the same three methods (see Figure 1b), but also new methods. For example, one student suggested adding seven to the small quantity and adding three to the large quantity (see Figure 1c). Another student used a table to sort out kinds of possibilities (see Figure 1d), which facilitated pattern-seeking.

Representations: Transition from Textbook to Instruction

The use of representations is summarized in Table 1, including the representational sequence involved in worked examples and additional representations in practice problems.

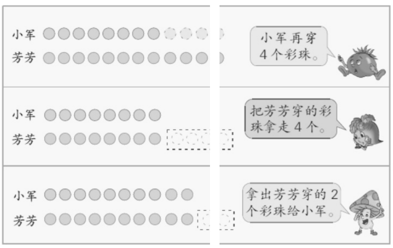
Representational Sequence and Type

Even though the representational sequence in both the textbook and the enacted worked examples generally began with concrete story problem situations and ended up with abstract number sentences, the enacted lesson always contained new types of representations that did not appear in the textbook. For instance, in

Lesson 3, both the textbook and the enacted lessons first presented the "taking the bus" problem, but the reasoning processes used were quite different (see Table 2).

As indicated by Table 2, the textbook guided students to think in three ways (adding those who get on the bus first; subtracting those who get off the bus first; or adding the balance between those getting on and off the bus) with an emphasis on verbal rather than pictorial representation. In contrast, the enacted lesson involved a circle/chip diagram, a number line diagram, and a flow chart representation to facilitate students' thinking. The circle/chip representation and number line diagram were generated by students through pre-learning which covered all three ways of thinking suggested by the textbook (see Table 2). The teacher explained that starting a lesson by discussing students' self-generated representations was a way to teach based on students' existing learning levels. After the discussion of students' representations the teacher presented flow-charts (shown in Table 2) which were initially placed by the textbook in the practice problem section (see Table 1). According to the teacher, the early introduction of the flow-charts was intended to promote students' learning to a relatively abstract level and to make a connection between the worked example and the later practice problem. In the end, while the textbook asked for only one numerical solution, the enacted lesson elicited three different numerical solutions which matched the three ways of thinking.

(a) Textbook: Three solutions

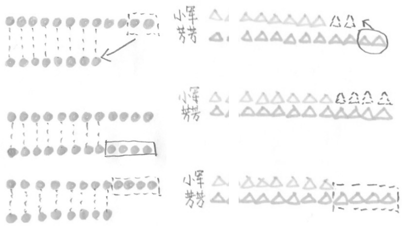


Cartoon (top): Xiaojun makes four more beads

Cartoon (middle): Take four of Fangfang's beads away

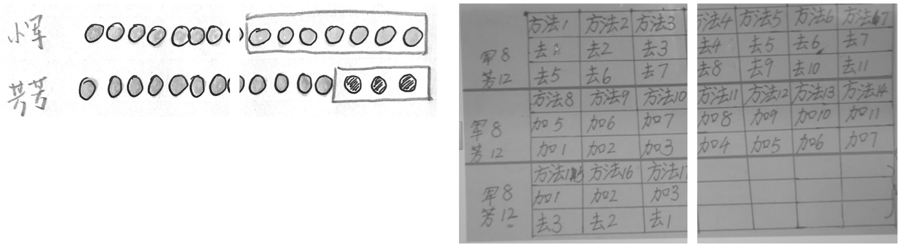
Cartoon (bottom): Take two of Fangfang's beads away and give them to Xiaojun

(c) Instruction: Same solutions



(b) Instruction: A new method

(d) Instruction: Sorting out various methods



Top section (Methods 1–7): Removing beads from both strings
Middle section (Methods 8–14): Adding beads to both strings
Bottom section (Methods 15–17): Adding beads to one string and removing from the other

Figure 1 Types of Solutions in the Textbook and the Instruction in Lesson 1 (with Translation)
Note. The figures in (b) and (c) mimic students' original drawings which were obscured in screenshots of classroom instruction.

Table 1 The Use of Representations in the Textbook and in Instruction

		Textbook	Instruction
Lesson 1	Worked example	Word problem context Circle/Chip	Word problem context Circle/Chip Table Number sentence
	Practices (additional)	Stick **Tape diagram**	Stick **Tape diagram**
Lesson 2	Worked example	Word problem Circle/Chip Number sentence	Word problem **Tape diagram** Number sentence
	Practices (additional)	**Tape diagram**	**Number line diagram**
Lesson 3	Worked example	Word problem Number sentence	Word problem Circle/Chip **Number line diagram** Flow chart Number sentence
	Practices (additional)	Flow chart **Tape diagram**	**Tape diagram**
Lesson 4	Worked example	Word problem Number sentence	Word problem **Tape diagram** Number sentence
	Practices (additional)		Other types of schema diagrams

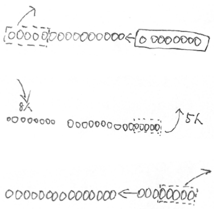
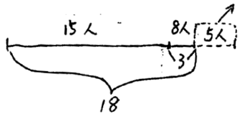
Note. Examples for the representations above (except for the stick diagram) can be found in Figure 2, Figure 3, and Table 2. A stick diagram looks like the following: / / / / / / / / / / .

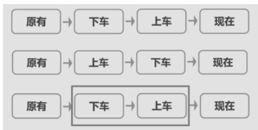
Table 2 Representation Sequence in the Worked Example in Lesson 3 (with Translation)

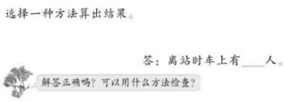
<table>
<tr><th></th><th>Textbook</th><th colspan="2">Instruction</th></tr>
<tr><td>Present</td><td>Present the story problem
(numbers were 34, 15, 18)

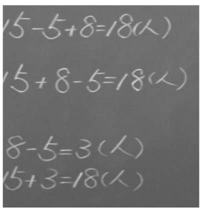
</td><td colspan="2">Present the story problem
(numbers were changed to 15, 5, and 8)

</td></tr>
<tr><td>Guide</td><td>Guide students to think in three ways and suggest verbalizing them


Cartoon (top): What do we already know? What do we need to find out? How can we solve it? Discuss with your classmate.
Cartoons (bottom left): First subtract the number of people that got off the bus, then add the number of people that got on the bus
Cartoon (bottom middle): First add the number of people that got on the bus, then subtract…
Cartoon (bottom right): We can also…</td><td>Circle/chip diagrams that show three methods

</td><td>A number line for method#3


Flow charts for three methods


On the flowchart, it says:
- Original, off bus, on bus, now
- Original, on bus, off bus, now
- Original, on bus, off bus, now</td></tr>
<tr><td>Solve</td><td>Ask students to select one method to compute and find the answer:


Cartoon:
Is it correct? How can we check it?</td><td colspan="2">Number sentences that show three methods discussed

</td></tr>
</table>

Note. Because screenshots of classroom instruction lacked clarity, students' original drawings circle/chip diagrams and a number line) have been mimicked in the figures presented here.

Schematic Diagrams: A Frequently Used Representation

Table 1 shows that among the types of representations, schematic diagrams (e.g., tape diagram, number line diagram) appeared most frequently (see the bold in Table 1; see examples in Figure 2). Three of the four textbook lessons used tape diagrams; in the enacted teaching, all four lessons used tape diagrams and (or) number line diagrams. In fact, tape diagrams only appeared in practice problems in the textbook lessons; yet, in the enacted teaching, the teacher either moved the tape diagram to the earlier worked example instruction (Lesson 2), or added the tape diagram when discussing the worked examples (Lesson 4).

Why did the teacher incorporate schematic representations that were not included in the original textbook presentation of the worked examples? The teacher explained that in prior lessons, the textbook had already presented numerous linear representations with discrete objects (e.g., circle/chip diagrams) which prepared her students for learning the tape diagrams.

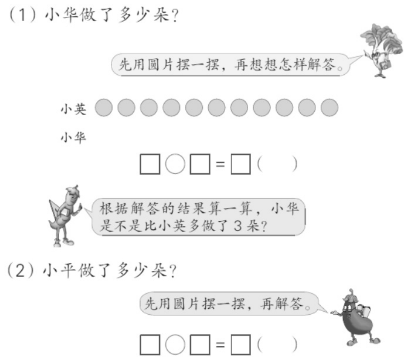
> In this lesson, if I use the circle diagrams, the lesson will be easier for my students because the same representation has been used in previous lessons several times. However, I think that my students are ready for learning more abstract representations. (teacher Chen, personal communication, second interview, March 9, 2016)

The linear circle diagrams, according to Murata (2008), are called “pre-tapes” and have similar structures to the tape diagrams. However, the tape diagrams that are continuous in nature are more abstract than the circle diagrams (Ding, Chen, & Hassler, in press). This is why, in Lesson 2, she replaced the circle/chip diagrams with tape diagrams that were new to her students (see Figure 2).

Moreover, she added a tape diagram in Lesson 4 to stress the quantitative inverse relationships and to help justify the checking procedure. The use of tape diagram is well supported by the literature. According to Duval (2006), students may see different things in concrete situations. For a given story situation in the textbook example, some students may not be able to “see” the problem structure. The addition of a tape diagram potentially addressed this limitation by explicitly illustrating the part-whole structure, facilitating the transition from problem situation to numerical solution (Duval, 2006; English & Halford, 1995). As indicated in Figure 3, the three parts of the tape diagram were labeled “the total, the checked-out, and leftover.” This diagram indicates inverse quantitative relationships (“total – checked – out = leftover” and “checked-out + leftover = total”), potentially helping students understand why addition can be used to check for subtraction. In fact, many students in later practice problems spontaneously drew schematic diagrams to represent the inverse relationships between addition and subtraction based on the part-whole structure.

(a) Textbook: Make flowers

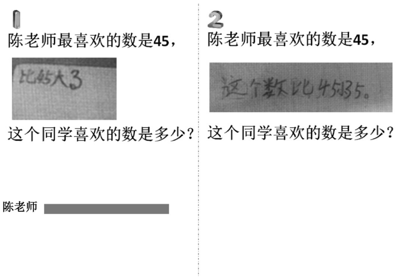
(A pictorial context shows xiaoying made 11 flowers. Xiaohua made three more and Xiaoping made three less followers)



(1) How many flowers did Xiaohua make?
(Cartoon 1: First display the chips, then think about how to solve it.)
(Cartoon 2: Compute again based on the answer, did Xiaohua make three more flowers than Xiaoying?)

(2) How many flowers did Xiaoping make?
(Cartoon 3: First display the chips, and then solve it.)

(b) Instruction: Guess my number

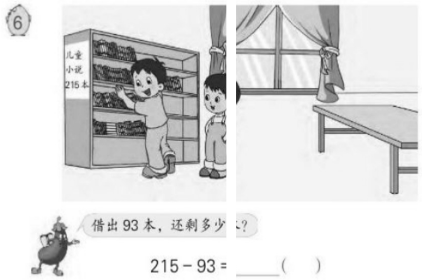


(1) Teacher Chen's favorite number is 45.
This student's favorite number is three bigger than 45.
What is this student's favorite number?

(2) Teacher Chen's favorite number is 45.
This student's favorite number is 35 smaller than 45.
What is this student's favorite number?

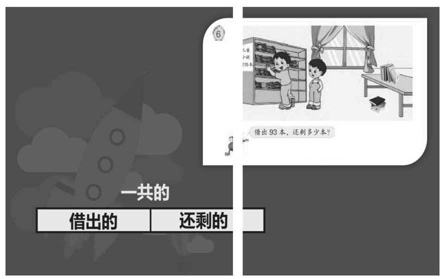
Figure 2 A Change to a Worked Example Context in Lesson 2 (with translation)

Textbook



On the book shelf: "There are 215 children's books."
Cartoon: "Checked out 93 books. How many books are left?"

Instruction



On the tape diagram:
(Top) "total," (Bottom) checked-out, leftover

Figure 3 Schematic Diagrams Used in the Textbook and the Enacted Teaching in Lesson 4

Deep Questions: Transition from Textbook to Instruction

Questions that called for substantial answers were coded as worked examples and classified into brainstorming, connection, and reflection questions. Table 3 summarizes the frequency of each type of question.

Table 3 Types of Questions in the Worked Examples across Lessons

	Lesson 1		Lesson 2		Lesson 3		Lesson 4	
	Textbook	Instruction	Textbook	Instruction	Textbook	Instruction	Textbook	Instruction
Brainstorming	1	3	2	1	2	8	2	5
Connections	0	17	1	12	1	22	1	6
Reflections	1	1	1	0	1	1	1	1
Total	2	21	4	13	4	31	4	12

As indicated in Table 3, the connection questions in the enacted lessons were far more frequent than in the textbook. We noticed that the connection questions often worked together with the other questions to promote understanding. For instance, within a conversation unit, the teacher often started with a brainstorming question (e.g., “Can you explain this diagram or method to us?”) which was usually followed by a cluster of connection questions to further unpack the brainstorming question (“What is the difference between this method and the previous ones?” “Why do you say it is similar to the previous method?”). On some occasions, the conversation concluded with a reflection question encouraging students to recall the key points discussed. Overall, the clusters of connection questions, along with the other two types of questions, played an important role in unpacking the worked example discussions. The teacher explained that the textbooks only suggest what students need to learn. A teacher needs to unpack the textbook questions to help students make connections between different types and levels of representations (e.g., concrete and abstract). A closer inspection of this teacher’s connection questions revealed two main features—(a) promoting comparisons and (b) being concept-specific—which were often interconnected in the enacted lessons.

Questions Promoting Comparison

In the enacted lessons, the teacher constantly asked students to compare different representations and different solutions. According to the teacher, comparison is a common practice in Chinese classrooms, used to help students understand core concepts. Excerpt 1 illustrates typical discussions from Lesson 1. In this excerpt, the teacher shared two pre-learning products, each illustrating three methods similar to the textbook solutions (see Figure 1b). The teacher then asked students to compare the solutions and to match the solutions that were the same in nature.

Excerpt 1

T: Which method in Student A's work is the same as which method in student B's work?

S1: The first method in both students' work is the same because both moved some part.

T: What do you mean by "both moved some part?"

(At this time, another student pointed out that there was another pair of similar methods across both students' work.)

T: Let's go back to what we just discussed. Why were they the same?

S2: Fangfang's beads in both students' first method are moved.

S3: They both move the more to the less.

In this excerpt, the teacher's initial comparison question, "Which method in Student A's work is the same as which method in student B's work?" elicited different student observations but with surface explanations (e.g., "both moved"). With the teacher's follow-ups (e.g., "Why were they the same?") and several students' continued attempts, the class grasped the structural similarity between different solutions (e.g., both moved the more to the less). Thus, through asking a cluster of comparison questions, the teacher was able to guide students to identify the conceptual underpinnings of these solutions. In comparison, the textbook presented an open-ended question after the three given methods—"Review the problem-solving process. What have you learned?"—which may elicit interesting, but not necessarily deep, responses. The teacher explained that her rationale for asking these comparison questions was to "draw students' attention to the essence of mathematical concepts and to discover the underlying principles of mathematics."

On other occasions, comparison questions were used to make connections between different types of representations. For instance, in Lesson 3 the teacher discussed the circle/chip representation, number line diagram, and flow chart representations for the task of getting on and off the bus (see Table 2). To prompt connection-making among these representations, the teacher asked a series of comparison questions: "Which circle diagram has the same meaning as this number line diagram?" "Which one of these three flow charts is the same as which one of these circle diagrams?" "What is the meaning of this number? Can you point out which part of the diagram represents this quantity?" These comparison questions were specific, and they oriented student thinking toward a structural level. In contrast, the textbook guided students to think in three ways (see Table 2). In the end, it asked, "Is your solution correct? How may you check it?" This reflection question may not necessarily facilitate connection-making between different solutions.

Concept-Specificity of Questions

Another feature of this teacher's deep questions was concept-specificity. Excerpt 2 illustrates an example from Lesson 2. In this excerpt, the teacher guided the class to work on drawings for the two sub-problems: find three more than 45 (Figure 2b, left) and find 35 less than 45 (Figure 2b, right).

Excerpt 2

T: I will use the computer to draw this. If you say "stop," I will stop drawing.

(The teacher moves the mouse slowly on the screen without actually drawing it out.)

T: Think about when I should stop. (Before the mouse reaches 45). Can I stop?

Ss: No!

T: (Continues drawing, but does not reach 45). Still not stop? Why?

S: Go further.

T: (Continues drawing, but does not reach 45). Oh, I am here now. Still not stop?

S: No!

T: Why can't I stop?

S: Because it shows the number is three more than 45, which means that the second tape should be longer than Teacher Chen's tape. …

T: Okay, let's continue. (Now the mouse reaches 45). Stop?

S: No!

T: It has reached 45! Why can't I stop?

Ss: Draw a little big longer.

T: I am not going to draw. I will let you draw. (Shows the first tape for the second sub-problem.) Can you also draw this second problem as well?

In Excerpt 2, the teacher guided students to represent two comparison sub-problems that implicitly involved inverse relations. Comparison word problems are challenging (Greeno & Riley, 1987). To tackle the difficult concept through the drawing of a diagram, the teacher asked a cluster of deep questions that were concept-specific. During her moving of the mouse, the teacher asked whether she could stop drawing and why she could not stop. When the second tape reached the same length as the first tape, she persisted in asking why she still could not stop. These questions focused students' attention to the core concept of "the same as" which is a key path to understanding that the large quantity contains the "same as" part and the "more than" part (Greeno & Riley, 1987). Later, when discussing students' drawings, the teacher continued asking deep questions: "Why did you draw your second tape longer/shorter?" and "Why only a little bit (longer)?" These seemingly trivial but important follow-up questions may have engaged students in deep thinking about the concepts of "more than" and "less than" and thus promoted implicit understanding of inverse relationships. Note that the textbook did not suggest any questions during the discussion of representations. Instead, it only suggested that students check back with the story context to see if the identified answer was correct. As such, the teacher added concept-specific questions during the enacted lesson. The rationale for adding these deep questions is indicated in her self-reflection:

> Deviations from students often appear in the classroom discussion. Once it happens, the teacher should provide the right guidance and ask the right questions so that students attend to the essence of mathematics rather than focusing on other irrelevant aspects (teacher Chen, personal communication, second interview, March 9, 2016).

Discussion

This study reports, in fine detail, how an expert Chinese teacher transitions textbook lessons into actual classroom instruction in terms of the use of worked examples, representations, and deep questions (Pashler et al., 2007). We acknowledge that this study has limitations. First, this study involves only one expert teacher implementing four lessons. Findings in this study thus should not be over-generalized. We also acknowledge that our findings from Chinese classrooms may not be directly applicable to other countries due to differences in curriculum, instruction, and culture. Nevertheless, our study focuses on three key aspects of classroom instruction and examines what happens during the textbook-instruction process which has not been carefully examined in the literature. As such, findings from this study contribute insights to the fields of both cognitive learning science and mathematics education. Overall, our findings show that the expert Chinese teacher was able to make improvements to even a well-designed Chinese mathematics textbook (Ding, 2016; Ding & Li, 2010, 2014). Across the changes made between textbook and classroom instruction, there seems to be a common consideration, that is, the changes were made to address students' actual learning needs and to increase the cognitive demand of the problems. Below, we discuss our findings in light of the research questions.

Unpacking One Worked Example Sufficiently

There is a long history of worked example research, as a well-studied worked example can help students construct a schema to enable subsequent problem solving (Renkl, Atkinson, & Grobe, 2004; Sweller & Cooper, 1985). In this study the Chinese teacher essentially discussed the same worked examples as the textbook, which echoes prior findings on the fidelity of Chinese teachers' textbook use (Ding et al., 2013; Ma, 1999). However, the Chinese teacher's implementation of the worked example in enacted teaching shows flexibility and depth. For instance, the teacher consistently applied pre-learning as part of the worked example discussion. The content of the pre-learning was partially taken from the textbook example; however, the teacher invited students to study it beforehand with resulting products elicited for classroom discussion. This treatment of worked examples indicates teaching flexibility. As indicated by the teacher's self-reflection, this pre-learning activity enables teachers to assess students' prior knowledge and helps to resolve the dilemma of students' different learning needs (e.g., length of time) as well as actively engages students in construction of knowledge, based on their existing conception.

In fact, prior research on worked examples was mainly conducted in laboratory settings with worked-out solutions directly shown to students (e.g., Sweller & Cooper, 1985). This is a relatively passive process, which may be a reason that the use of worked examples was classified as traditional instruction and appeared infrequently in classrooms (Kirschner et al., 2006). However, our findings suggest that when pre-learning is integrated into the study of worked examples, students can generate various representations and solutions, offering rich opportunities for follow-up class discussion. Students' active input in turn enables the teacher to unpack the worked examples. Of course, whether these rich opportunities can be strategically utilized by teachers in the classroom is another story. Nevertheless, it might be safe to conclude that students' pre-learning product provides likelihood and serves as a platform for the teacher-student joint activity of unpacking the worked examples (as opposed to teachers' simply showing the examples). The engagement of students in co-constructing the worked example solutions is well aligned with cognitive learning sciences (Brousseau, 2002; Schank, 2011) and recent semiotic perspectives (Radford & Roth, 2011; Radford, Schubring, & Seeger, 2011) where teaching and learning are viewed as two sides of a coin and students should be involved in the process of interpreting mathematical meanings.

As previously reviewed, the IES recommendation (Pashler et al., 2007) also suggests that alternating worked examples and practice problems produces better student performance than simply providing practice problems (Sweller & Cooper, 1985; Zhu & Simon, 1987). In this study, neither the Chinese textbook nor the enacted lessons show this pattern of alternation. Rather, each Chinese mathematics lesson contains only one worked example which was sufficiently unpacked. By "sufficient," we refer primarily to the "length" of the discussion of worked examples which our research indicates is critical. We noticed that the expert Chinese teacher purposefully used various representations and asked clusters of deep questions during the course of unpacking a worked example. Of course, we are cognizant that our study only included one teacher and four lessons. As such, findings should not be over-generalized. Nevertheless, our findings indicate that this expert Chinese teacher used worked examples differently from what has been reported in the literature. Stigler and Hiebert (1999) noted that some U.S. teachers spent little time on a worked example before asking students to solve problems on their own. Ding and Carlson (2013), in their study of lesson planning, reported that some teachers planned to teach multiple, repetitive worked examples with each discussed at a rapid pace. In both cases, worked examples were not sufficiently unpacked. Thus, the "worked example effect" (Sweller & Cooper, 1985) might be hard to produce. Interestingly, the Chinese teacher's use of worked examples in this study is more like what takes place in Japanese classrooms where a teacher may spend the entire lesson, or even two consecutive lessons, on an example task (Stigler & Hiebert, 1999). The aforementioned different ways to use worked examples may reflect cross-cultural differences in teaching methods and values (Stigler & Hiebert, 1999). An example may be used to show procedures for finding an answer, which would only take a short amount of time to demonstrate. In contrast, an example may be used to illustrate the underlying concepts, relationships, and structures of mathematics. Consequently, this demands more class time and deeper conversations that enable students to make sense of interconnected ideas. In this respect, this Chinese teacher's sufficient unpacking of one worked example provides insights into classroom practice in terms of teaching in depth and calls for a rethink on effective ways to use worked examples.

Using Representations to Model Quantitative Relationships

In this study, the overall representational sequences in both the Chinese textbook example and the enacted lessons indicate concreteness fading (Goldstone & Son, 2005). The worked examples have always been situated in story problem contexts which are likely to elicit students' personal experiences for sense-making (Ding & Li, 2014; Koedinger & Nathan, 2004; Resnick et al., 1987). In addition, while the textbook lessons tended to provide verbal suggestions for multiple solutions, the teacher incorporated multiple concrete and semi-concrete representations. Note that the teacher always treated concrete representations as tools for understanding the quantitative relationships, with the formal solutions always presented in numerical formats. This is consistent with Cai's finding about Chinese teachers' views on representation use, but different from a common practice wherein concrete representations are used to find answers or to serve as solutions (Cai, 2005; Ma, 1999).

In this study, schematic diagrams (e.g., tape diagrams) were widely used to model the quantitative relationships. Given that tape diagrams, as semi-concrete representations, can effectively show problem structures (Murata, 2008; Ng & Lee, 2009), it is important to help students understand this type of representation. In this study, we observed that, consistent with textbook recommendations, the Chinese teacher engaged students in the process of co-construction of a tape diagram (Ding & Li, 2014). Indeed, the Chinese teacher went further by asking a cluster of deep questions related to this diagram which may have boosted students' understanding of this powerful, but sometimes opaque, representation (Ding & Li, 2010). The process of involving students in understanding the tape diagrams once again aligns with perspectives on how students learn mathematics effectively (Brousseau, 2002; Radford & Roth, 2011; Radford, Schubring, & Seeger, 2011; Schank, 2011). Since schematic representations such as tape diagrams are emphasized by the Common Core State Standards Initiative (2010) and have been adopted by many new textbooks, our findings on how the skillful use of schematic diagrams to unpack a worked example by an expert Chinese teacher have practical importance for teachers in the US and beyond.

Asking Clusters of Deep Questions to Promote Connection-Making

Deep questions may elicit students' self-explanations (Craig et al., 2006; Pashler et al., 2007). However, many teachers in current classrooms struggle to ask deep questions (Ding et al., 2007). In this study the Chinese teacher unpacked the textbook question in various ways to elicit students' deep explanations. She often started with brainstorming questions to introduce students to the given information or to voice their initial thinking. These brainstorming questions were always followed by a cluster of deep questions to prompt higher-order thinking. These discussions often ended with a reflection question to highlight key points. As such, we conclude that the cluster of questions (as opposed to a single deep question) function together to elicit students' deep responses. This finding has theoretical and practical implications: On the one hand, while the IES recommendations (Pashler et al., 2007) list a set of sample deep questions (e.g., why, what if, what if not), our findings indicate that in many cases, it may take a cluster of questions to elicit students' deep explanations. This may be because some deep questions are too broad (e.g., "Can you explain why?") to stimulate students' thinking given their diverse learning needs. This finding contributes new information to the community of instructional knowledge. Therefore, we urge teachers in the classroom to follow up on students' responses with a cluster of deep questions before shifting away from the conversation. In this sense, our findings contribute new insights for developing pedagogical strategies to better support student learning.

Franke et al. (2009) reported that many teachers in cognitively guided classrooms struggled to ask deep follow-up questions. In this study, the two features of the Chinese expert teacher's connection questions—comparison and concept specificity—contribute new insights into teacher questioning.

Comparison has drawn renewed interest because it facilitates connection-making and analogical reasoning (Richland, Zur, & Holyoak, 2007; Rittle-Johnson & Star, 2007). In this study, the Chinese teacher asked comparison questions about different representations, solutions, and even problem tasks which functioned together to elicit students' deep explanations. The "concept-specific" feature of the Chinese teacher's deep questions also sheds light on classroom instruction. It is easy to ask "why" questions; however, it may not be easy to ask "why" questions that effectively target key underlying concepts. In this study, these concept-specific questions either prompted students' articulation of representational meanings or made explicit the underlying concepts and relationships. Although prior studies (e.g., Mok, Cai, & Fung, 2008) found that a well-structured Chinese lesson could contain too much guidance, and that teachers may ask overly-specific questions, the teacher in the current study did not ask low-level questions to restrict student thinking. Rather, her comparison and concept-specific questions focused students' understanding in increasingly clear and deep ways. These findings are consistent with prior reports (Perry et al., 1993) and detail how Chinese teachers turn textbook lessons into classroom instruction through questioning (Ding, 2016; Ding et al., 2013).

Conclusion and Future Direction

This case study systematically examined the textbook-instruction transition process in a Chinese expert teacher's classroom based on selected IES recommendations (Pashler et al., 2007). The purpose of a case study is not to generalize findings to other settings. Rather, it aims to identify meaningful themes from a particular, rich context, contributing to deep, practical, and theoretical reflections. Indeed, while the IES recommendations serve as a practice guide for organizing instruction to improve learning, many classroom teachers lack the ability to utilize them in daily work (Ding & Carlson, 2013). The instructional insights gained from our in-depth observation of a Chinese expert teacher, in alignment with the IES recommendations, offer ways to implement these guidelines. Consider, for example, the Chinese expert teacher's integration of pre-learning with the example task. Even if this practice is not feasible for other teachers in other countries, they could use an initial chunk of class time to ask students to explore the example task presented by their textbook, which could generate typical student representations and solutions to be used for comparison or discussion in later instruction. In addition, this Chinese teacher's purposeful choice of representation and her use of clusters of comparison and content-specific questions to promote connection-making are also learnable for other teachers. Beyond practical implementations in mathematics education, the Chinese expert teacher's instructional insights may be informative for cognitive research. For example, it has been found that integrating student explanations into worked examples is more effective than presenting worked examples only. The Chinese expert teacher in this study, however, used a different approach. Instead of first presenting the complete solutions to example tasks followed by explanations, this Chinese teacher engaged students in the process of co-constructing the worked example solutions with requests for student explanations throughout. Such an approach is different from the literature assertion and may be worthy of investigation. Likewise, the Chinese teacher's use of clusters of comparison questions seems to be an important instructional aid which may also deserve exploration by cognitive researchers. Of course, given our study only examined one expert teacher without exploring the associated factors that contributed to observations, future studies may explore further questions based on this line of research. For instance, what does a larger teacher sample reveal about the ways in which expert Chinese teachers make textbook-instruction transition? How do the methods of other Chinese expert teachers compare with those of the participant in the current study? What are the main types of teaching strategies for using worked examples, representations, and deep questions in China? What factors contribute to varied teacher strategies during the textbook-instruction transition? With continued research we believe more instructional insights will be identified, which will benefit students' mathematical thinking and learning.

Acknowledgements This study is supported by the National Science Foundation CAREER program under Grant No. DRL-1350068 at Temple University. Any opinions, findings, and conclusions in this study are those of the authors and do not necessarily reflect the views of the National Science Foundation. We are grateful to the editors and three anonymous reviewers' insightful feedback as well as Laurie Shirley Esposito's editorial assistance with the manuscript revision. Both authors contributed equally to the article.

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