CAREER: Proof in Secondary Classrooms: Decomposing a Central Mathematical Practice
Partnering with Secondary Classroom Teachers to Improve the Teaching and Learning of Mathematical Proof

Michelle Cirillo

Department of Mathematical Sciences, University of Delaware

Introduction

This design and development research study focuses on secondary students' success with mathematical proof. The goal of the *Proof in Secondary Classrooms (PISC)* project is to develop an innovative intervention to support the teaching and learning of mathematical proof.

Research Questions

1. How do teachers introduce proof in geometry?
2. When engaging in lesson study based on introducing proof by first teaching particular sub-goals of proof, how do teachers respond to and execute the lesson plans?
3. How do students respond to these lessons?
4. How do students in the control and experimental groups think about proof and perform on a set of proof tasks?

Aim of Research

The **intellectual merit** of this project is in its contribution of new and important insights about teachers' conceptions of proof, student thinking about proof, and the nature of knowledge that is useful for teaching and learning proof.

This project promises **broader impact** on the field of mathematics education. Classroom videos and resources will provide teachers with a vision of what this approach to teaching proof looks like.

Methods

Project Timeline

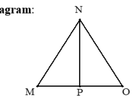
Planning Year	Baseline Data Collection & Lesson	Professional Development & Summer Lesson Study	Pilot Lessons (Core Teachers)	Pilot Lessons (Again)	Publication & Dissemination
Phase I 2015-2016	Phase II 2016-2017	Phase III Spring & Summer, 2017	Phase IV 2017-2018	Phase V 2018-2019	Phase VI 2019-2020

Given: $\overline{NM} \cong \overline{NO}$

P is the midpoint of $\overline{MO}$

Prove: $\triangle MNP \cong \triangle ONP$

Diagram:

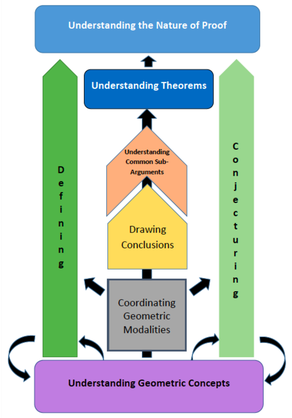


Statements	Reasons
1. $\overline{NM} \cong \overline{NO}$, P is the midpoint of $\overline{MO}$	1. Given
2. $\overline{MP} \cong \overline{OP}$	2. Definition of Midpoint
3. $\overline{NP} \cong \overline{NP}$	3. Reflexive Axiom
4. $\triangle MNP \cong \triangle ONP$	4. SSS ≅ SSS

Decomposition of Practice

- Grossman and colleagues (2009) described "decomposition of practice" as the breaking down of a practice into its constituent parts for the purposes of teaching and learning.
- For students to learn to engage in a complex practice, they may need opportunities to practice the different parts before integrating them fully.

Geometry Proof Scaffold

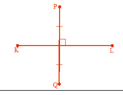


Coordinating Geometric Modalities Task

- Write a statement for the following geometric notation. Then sketch, label, and mark a diagram with the given features.
- $KL \perp \overline{PQ}$ and $\overline{KL}$ bisects $\overline{PQ}$

Verbal statement: Line segment KL is perpendicular to line segment PQ, and line segment KL bisects line segment PQ.

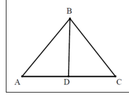
Diagram:



Drawing Conclusions Task

Given: $\overline{BD}$ bisects ∠ABC

- What conclusion(s) can you draw based on the "**Given**" statement?
- How do you know the conclusion(s) is/are true? Justify your reasoning.



Conclusion: ∠ABD ≅ ∠CBD
Justification: Definition of angle bisector.

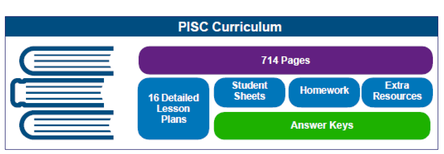
Data Collected

Assessments	Interviews	Classroom Observations
1,550 Pre-Tests Administered (EGT) 1,278 Post-Tests Administered (SGT)	24 Teacher Interviews 31 Student Interviews	294 Classroom Observations

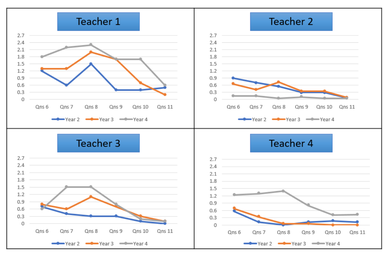
Results

- Four Core teachers implemented 16 special PISC lessons for two years.
- Fifteen non-Core teachers' students functioned as a control group.
- Analyses of 1161 students' pre-test and post-test scores indicated a positive effect from the PISC curriculum.
- After controlling for grade level and pre-test scores, Core students scored 6.61 NCE points higher on post-test.

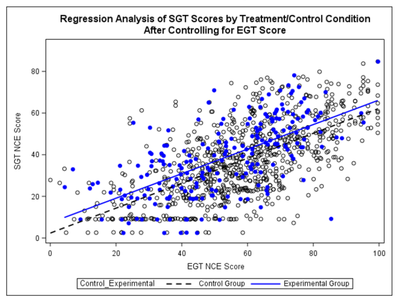
PISC Curriculum



Student Scores on CDASSG by Teacher & Year



What is the estimated impact of the PISC curriculum on students' post-test scores?



Conclusions

- Introducing proof by decomposing the proof process is a promising strategy for teaching proof in secondary mathematics.
- The PISC curriculum materials were educative for teachers, particularly in terms of math content knowledge.
- Teachers' attention to student thinking supported their efforts to improve the teaching of proof.
- Professional development on classroom discourse, particularly eliciting and responding to student thinking, played a critical role in the outcomes.
- When interviewed, teachers stated that the PISC PD model was extremely effective.

Dissemination & Future Work

- The Core teachers presented at a national research conference, and they presented two sessions at a national teachers' conference.
- One Core teacher co-presented at a research colloquium at an R1 university.
- A future study will explore under what conditions the treatment is most effective and what supports are needed for teachers.

Acknowledgements

I appreciate the teachers and their students for allowing me to conduct this research. I also thank Henry May, Jenifer Hummer, and Amanda Seiwell for supporting the data collection and analysis. The research reported in this poster was supported with funding from the National Science Foundation (NSF; Award #1453493, PI: Cirillo). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the NSF.