A Longitudinal Analysis of Children's Unit Iteration Concepts for Length, Area, and Volume (Grades 2-4)

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WHERE DISCOVERIES BEGIN

Introduction

To promote students' understanding of linear and area measurement, we want to help them gain a working definition of the unit concept. Students often have a procedural understanding instead of a more complete, conceptual understanding of measuring and units (e.g., Barrett & Clements, 2003; Clements, Battista, Sarama, Swaminathan, & McMillen, 1997; Kamii, 2006). Yet, unit iteration, at the center of unit concept understanding, is poorly developed in many of the Kindergarten through third grade curricula (Smith, et al., 2008; Smith, Males, Dietiker, Figueras, & Lee, 2008).

In keeping with the cognitive theoretical framework of Hierarchic Interactionalism (Clements & Sarama, 2007), we employ *learning trajectories (LTs)* as research tools to characterize the level of thinking for a child or group of children with similar conceptual knowledge for measurement. Consequently, the LT indicates instructional tasks appropriate to the children's level of thinking. We have used LTs to examine thematic aspects of measurement concepts, especially those related to unit, in teaching experiments with eight individual students. Our analysis involves predicting and then checking for student success or struggle, based on the current model for that student in relation to the LT (Steffe & Thompson, 2000).

Students' progress was consistent with the sequential model of levels in our Learning Trajectory for Length, including end-to-end collection counter, unit repeater, and consistent length measurer. Persistent themes accounting for students' roadblocks included use of discrete models of quantity without noticing continuous quantity, assignment of zero at places other than zero on corresponding number lines, and tendency to dichotomize tick marks and spaces (gaps) for counting length units. Children's difficulty in transferring unit concepts across dimensions to area or volume did not necessarily match the trajectory for length; a few students successfully used units to find volume while struggling with length unit iteration.

Research Questions for Children's Measurement Project

Research Question 1: How do students develop coherent knowledge and integrated strategies for measurement across the pre-K through Grade 5?

Research Question 2: How are students' abilities for perceptual and numerical comparison, for coordinating and discriminating, for deductive logic, and for ordering and nesting sequences related to the development of knowledge and strategies for measurement?

Research Question 3: How are students' abilities for spatial thinking, algebraic reasoning, or proportional reasoning related to their measurement knowledge and strategies?

Three Parts of Learning Trajectory

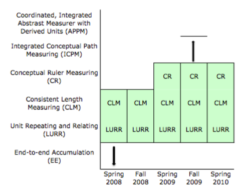
A learning trajectory has three components (3 columns):

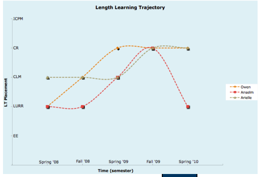
(1) An educational goal, an aspect of a mathematical domain children should learn, such as linear measurement

(2) A cognitive account of processes and relevant schemes for action engaging the relevant objects and concepts in the domain

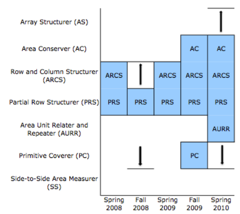
(3) An account of relevant tasks appropriate to each developmental step along the way

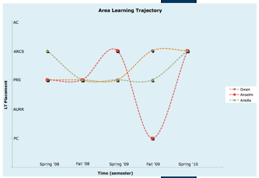
Focus student placement per semester in LENGTH



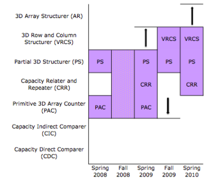


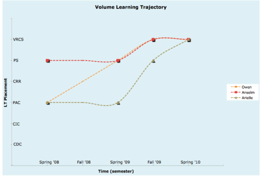
Focus student placement per semester in AREA





Focus student placement per semester in VOLUME





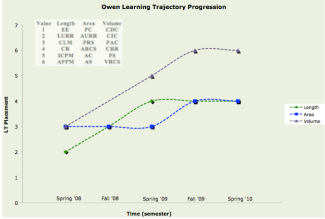
Area Task: Owen

Area (March, 2010)

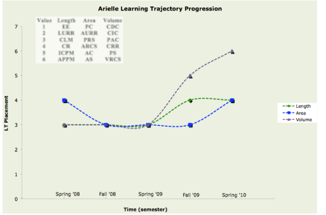
The students were given pictures of four "lakes" and transparency grids.

<u>Ask:</u> *Please put these lakes in order in terms of there area*. Oskar counted quietly to himself and then showed the space he was counting on the lake and called it 35. He did not count all the way from one to 35. Oskar said that used multiplication by doing 5x7. All he needed was the linear dimensions of the rectangle in order to know how many squares there were.

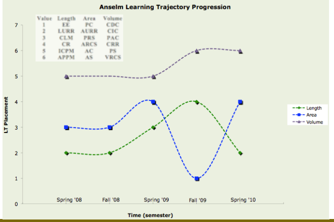
Placement of OWEN for each dimension



Placement of ARIELLE for each dimension

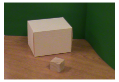


Placement of ANSELM for each dimension



Volume Task: Arielle

Volume (February, 2010)



The students were given a covered solid, the cube, the ruler, and a pencil. They were not allowed to iterate the cube or directly compare the cube and covered solid. <u>Ask</u>: *Compare the volume of this cube to the volume of this figure*.

Arielle explained that there were 12 on a side (geometric face) and the four sides were the same, so she multiplied 12 times four to get 48. When asked about the other two sides Arielle replied, "Since cubes are three dimensional they would fill up that space too." After the solid was unwrapped Arielle built single layers of 12 (3x4x1) and decided it would take 36 to fill the solid.

Results & Conclusions

Column 1:

• We anticipated that it would take six months to progress to the next level, but our results show that it might take longer. There are only two level increases over a two year period.

Column 2:

• In any one dimension, there is variation in student progression. As illustrated in column one, this variation stabilizes when considering progress of groups overall.

Column 3:

• Anselm shows "low" performance in one dimension (length) but "high" performance in another (volume). We think Anselm's struggles to integrate points, intervals, and zero-one issues. This continuous nature of length appeared in area tasks and caused his fallback in area. However, this continuous nature of length was not present in the volume tasks posed.

• Owen and Arielle's progression through the trajectories is representative of a typical student.

Implications for Teaching

Unit identification and iteration are important and prevalent concepts in teaching length, but do not receive as much emphasis in the standard curriculum for area and volume. We suggest focusing on identifying and iterating a unit to help students coordinate and generalize the idea of measurement across length, area and volume.

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Length Task: Anselm

Length (May, 2008)

<u>Ask</u>: *How many 1-inch strips would fit along the 3-inch strip?*

Anselm measured all three strips and reported the 1-inch strip to be one inch, the 2-inch strip to be three inches, and the 3-inch strip to be four inches. Anselm reported that two would fit along the 3-inch strip. The interviewer started to ask Anselm what a 2-inch strip would look like, but Anselm blurted out "actually you should count the spaces" and demonstrated that two of the 1-inch strips would fit along the 2-inch strip. Anselm iterated the 1-inch strip along the 3-inch strip and reported the strip to be three inches "because one entire space is one inch."