Building and Validating a Learning Trajectory for Children's Measurement

Jeffrey E. Barrett
(Illinois State University)

Julie Sarama, and Douglas H. Clements
(University at Buffalo, SUNY)

Presented at the NCTM Research Pre Session in San Diego, April 20, 2010.

We appreciate the support of the National Science Foundation: Funding for the Children's Measurement project DRL-0732217.

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Critical Questions:

- 1. What constitutes a LT? **How stable and linear are LTs**?
- Do LTs developmental progressions represent "natural" development, informal experience, or instruction?
- Are there LT **transition points** in time or in *content*?
- May we expect to generate **multiple LTs in one domain**? Can a **LT be too extensive**, or **too focused** in *grain size*? How might a LT in one **domain relate to LTs** in other domains?
- 5. What kinds of **research data support construction or validation of a** LT? Do particular **assessment tasks indicate** a given level of a LT?

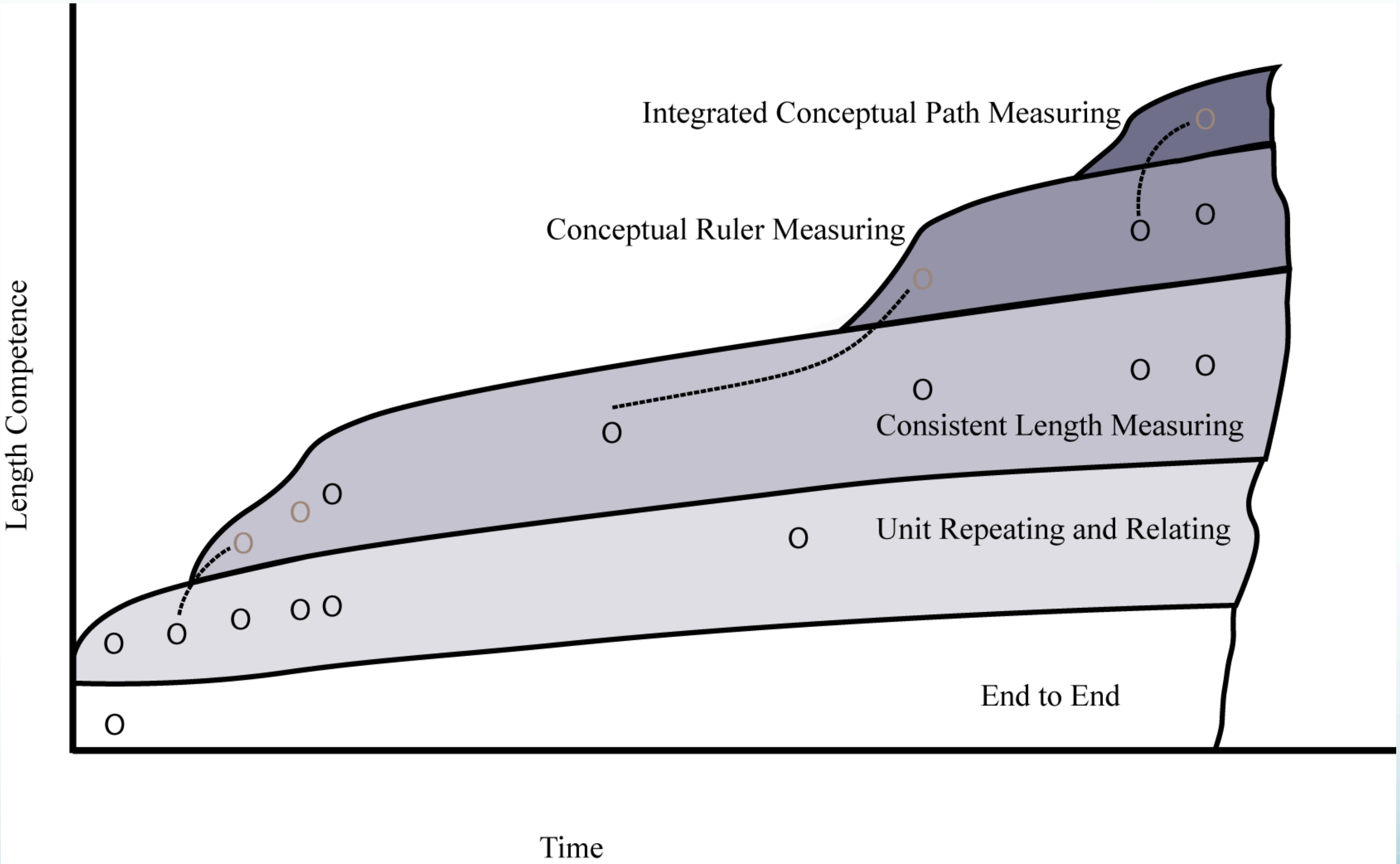
We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

We use a teaching experiment to check for linearity and stability of LTs. We complement this by item analysis of assessment data (Question 1).

- We propose profiles for each case student, across multiple developmental shifts (every six months) occurring over four years.
- We group students based on these profiles to set instructional groups, and to check for the fit of each LT (length, area and volume).
- We note adaptations to the LT with each cycle of implementation (1 to 6 months) at a given grade level (cf. Steffe & Thompson, 2000).

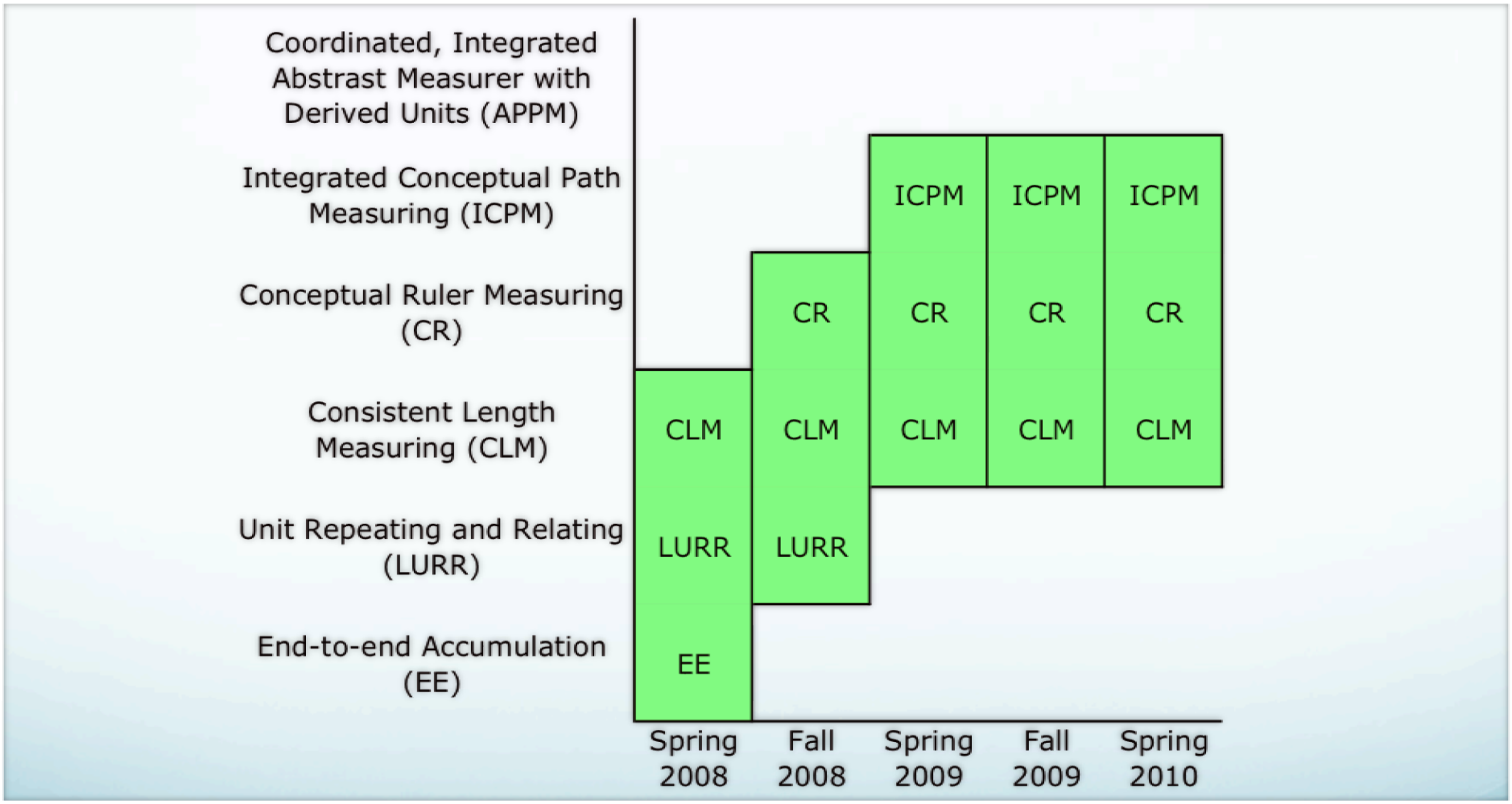
We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

A Profile of one student's performance, showing data on Length LT levels: 20 months from Feb 2008 to Nov 2009



We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Complemented by a *composite profile* of a group of 6 students within a LT across 5 semesters



We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Project Focus and Design (2007-present)

- We use the LT to tie levels of development (first column), mental actions on objects (second column) and instructional interventions (third column) together in each row in an LT.
- We are investigating LTs for Length, Area and Volume with 8 case students as they move through grades 2, 3, 4 and 5 in IL and with 8 case students as they move through grades PreK, K, 1 and 2 in NY.
- We are using Teaching Experiments with individual, group, and classroom settings to form *plausible instructional sequences* to test and improve the LTs through design cycles.

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

What does a Learning Trajectory include (Sarama & Clements, 2009)?

- Three parts (3 columns, next slide):

Description of level of reasoning with observable characteristics	**Description of mental actions on objects (cognition)**	**Description of instructional tasks to motivate growth** ***from current level onward***

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Age 4: Length Direct comparer: Physically aligns two objects to determine the longer, or find equality.	A scheme for length as a linear extent from endpoint to endpoint (holistic). Shape can affect scheme use.	Set objects apart further, and then eventually compare objects that are distant and immovable, to prompt use of a longer, movable object to transfer.
Age 5: Indirect Length Comparer (ILC): Compares the length of two objects by representing them with a third object. Some guessing.	A mental image of a particular length can be built, maintained and manipulated.	*To shift toward End to End:* children should talk about numbers for lengths that they can compare indirectly. Use physical or drawn units along objects to compare.
(Serial Orderer to 6+) Orders sets of objects in ascending length, with lengths from 1 to 6 units.	Scheme is organized in a hierarchy, coordinating concept for an ordered series with direct comparison.	N/A
Age 6: End-to-End Length Measurer (EE): Lays units end-to-end. May not recognize the need for equal-length units. Needs a complete set of units to span.	An implicit concept that lengths can be composed as repetitions of shorter lengths underlies a scheme of laying lengths end to end. The scheme improves by attention to splitting and recomposing from parts.	Use "Length Riddles" providing only one unit per child to compare longer items. Have child plan and make a ruler from cardboard and mark it with ticks and numerals to match units (in or cm).
Age 7: Length Unit Relater and Repeater (URR): Measures by repeated use of a unit (initially may be imprecise). Relates size and number of units explicitly, but may use units of varying lengths. Can add parts, and use rulers with slight errors.	**Action schemes iterate a mental unit along a perceptually-available object. The image of each placement can be maintained while the physical unit is moved to the next iterative position. These action schemes allow counting-all addition schemes.**	**Pretend to gap or overlap units as they are repeated to challenge consistent measures. Have students draw objects beginning from a zero point and discuss the end-to-end measures coordination with intervals and numbers along rulers. Measure in different-sized units for the same object.**
Age 8: Consistent Length Measurer (CLM): Considers the length of a bent path as the sum of its parts (not the distance between the endpoints). Uses identical units, relates different units, partitions units, keeps zero point on rulers, accumulates distance. Begins to coordinate units and subunits.	The length scheme has hierarchical components, coordinating an object's length as a total extent and as a composition of units. This scheme adds constraints for equal-length units and, with rulers, on use of a zero point. Units themselves can be partitioned to increase precision.	Use a physical unit and a ruler to measure line segments and objects that require both an iteration and subdivision of the unit. Build sub-units to fourths and eighths. Discuss how to deal with leftover space, to count it as a whole unit or as part of a unit.
Age 9: Conceptual Ruler Measurer (CR): Possesses an “internal” measurement tool. Mentally moves along an object, segmenting it, and counting the segments. Operates arithmetically on measures ("connected lengths"). Estimates with accuracy.	Interiorization of the length scheme allows mental partitioning of a length into a given number of equal-length parts or the mental estimation of length by projecting an imaged until onto present or imagined objects.	In "Missing Measures," students have to figure out the measures of figures using measures for a subset of sides. Prompt students to make explicit strategies for estimating lengths, including developing benchmarks for units and composite units.
Integrated, Coordinated Path Measurer (ICPM): coordinating length attributes, yet with further tendency and ability to relate multiple cases.	Can iterate units of units. Can also integrate whole sides with other sides and with entire perimeter within a figure; coordinates operations on measured objects between several figures in dynamic sequences, even for boundary cases.	

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Fitting a student with the *Unit Relater and Repeater* Level in the LT for Length Measurement

Age 7: **Length Unit Relater and Repeater (URR):** Measures by repeated use of a unit (initially may be imprecise). Relates size and number of units explicitly, but may use units of varying lengths. ***Can add parts, and use rulers with slight errors.***	***Action schemes iterate a mental unit along a perceptually-available object.*** The image of each placement can be maintained while the physical unit is moved to the next iterative position. These action schemes allow counting-all addition schemes.	Pretend to gap or overlap units as they are repeated to challenge consistent measures. ***Have students draw objects beginning from a zero point and discuss the end-to-end measures coordination with intervals and numbers along rulers.*** Measure in different-sized units for the same object.

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

What kinds of **research data support construction or validation of a** LT (Critical Question 5)?

We attend closely to *Problems in the Teaching of Length* out of concern for external validity

- Growth from non-standard informal units like paperclips to the use of inches or centimeters with rulers (motions to sticks, or ruler marks?)
- Growth from counting items in a row, to counting hash-marks or points, to counting intervals or gaps to stand for units (discrete items or continuous space?).
- Growth from counting actions to reading the number labels at endpoints of tools/rulers (zero?)

We engage metaphors of stick counting, step-wise motion, and collection counting from Lakoff & Nunez (2000).

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Problems coordinating indirect comparison with serial ordering and end to end collections

- See notes from UB about the early length sequence of levels (Doug Clements)...

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

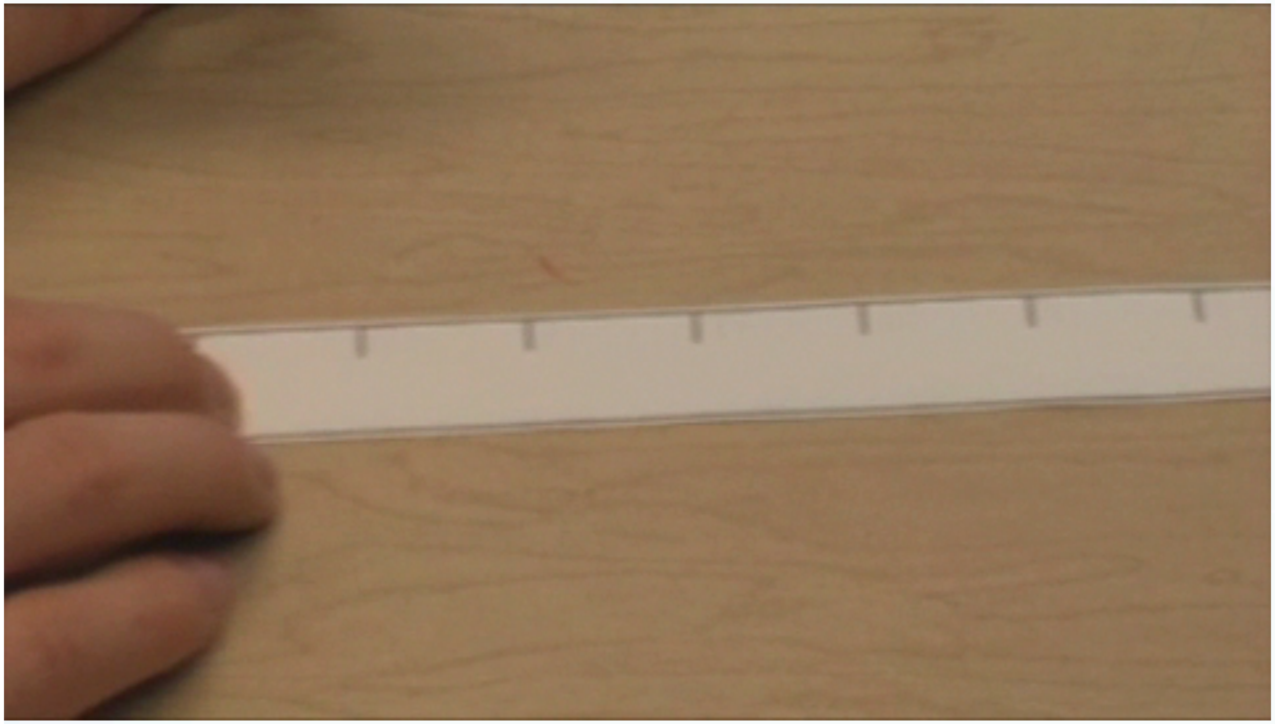
Problems integrating the unit concept, representations and tool meaning

In Grades 2 and 3, students required extensive support to integrate several aspects of linear quantity to overcome mere gap counting (share a storyline):

- hash-mark counting (representing units) in February 2008 and April 2008,
- **Partly coordinated unit scheme: count hash marks, sweeping gaps, & touching sticks (Apr 11 08).**
- reading numbers at endpoints, stick counting to show units, and re-assigning number labels (reports 7.5 then corrects to 4.5; May 7, 2008)
- Coordinate gap counting with arithmetic operations on obscured ruler section and the ribbon (unit concept) (May 15, 2008)
- Re-established meaning for gaps as pairs of "lines" to show sequences of units (Sept 30, 2008).

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

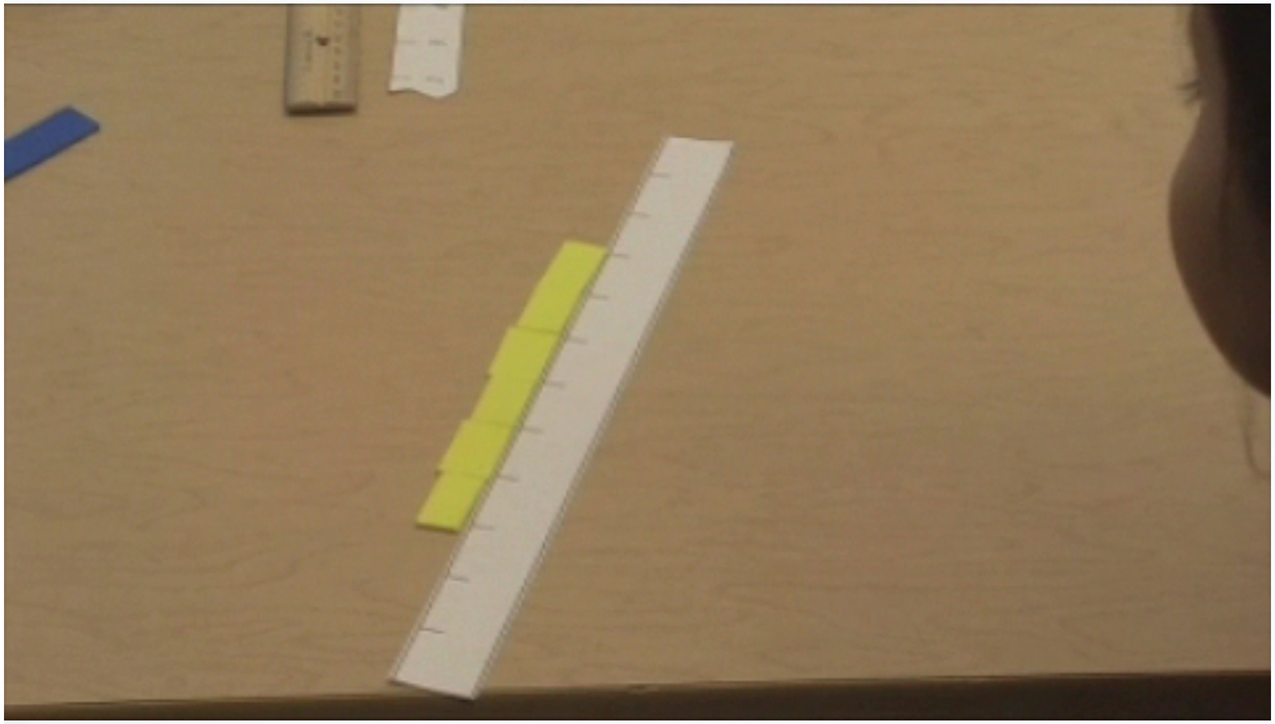
Counting hash marks, Apr 11, 2008 (Gr. 2)



Can you tell me how long it is?
Five. Five inches? Yes. Can you show me how you got that? I counted this as 1, 2, ... (touching the successive hash marks, getting five along a four-inch strip).

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

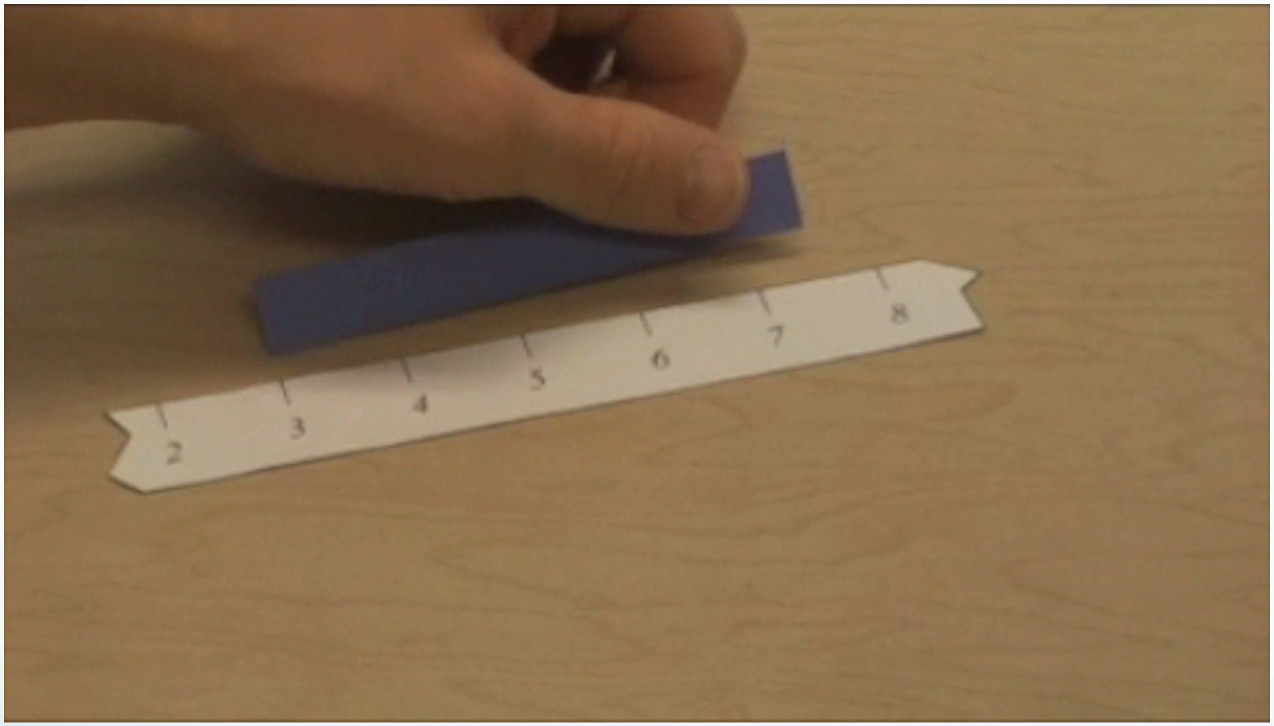
Partly coordinated: count hash marks, sweeping gaps, & touching sticks (Apr 11, 2008).



5 strips is 5 if aligned to end of ruler, but 6 if not aligned; Now we add another strip, and it is 6 in both cases, with a *new way* of coordinating the pointing to stick motion and moving through the ruler gaps between hash marks.

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

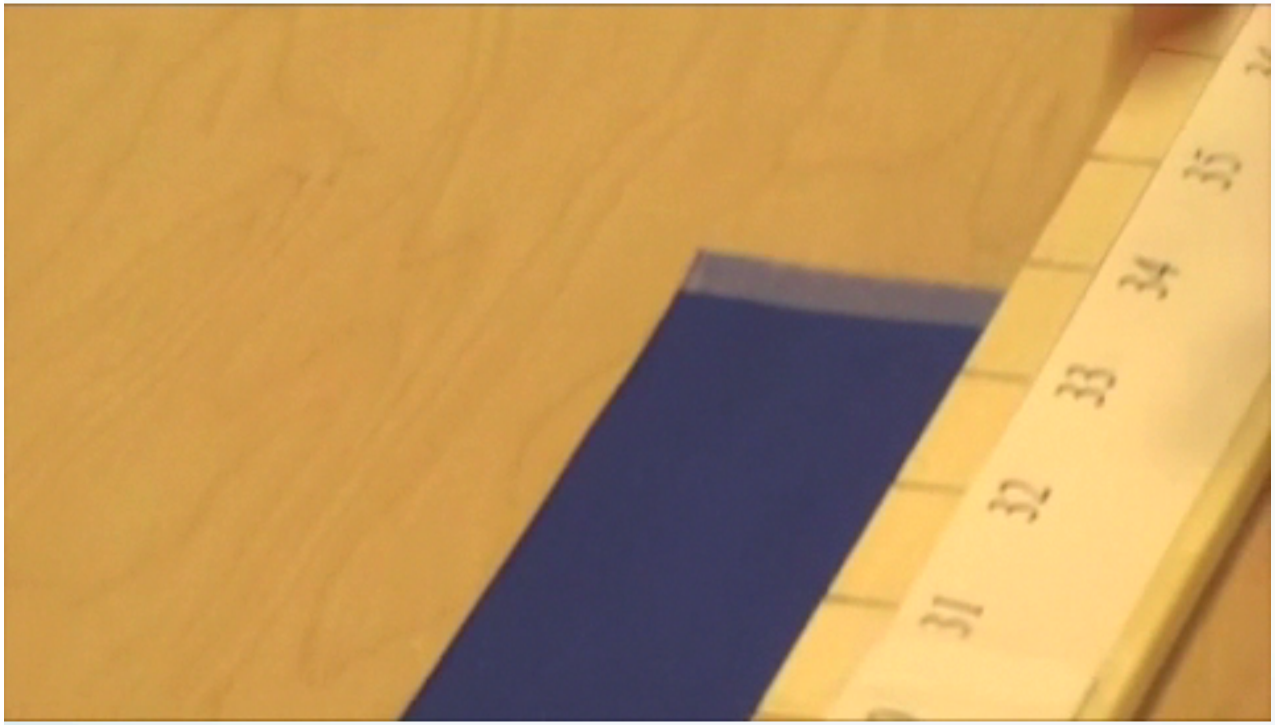
reading numbers at endpoints, sweeping to show units, and re-assigning number labels (May 7, ‘08)



She reads the endpoint number label to report a length of 7 ½ in., But she wants to change. Beginning at 3, she calls it zero, 4 is one... Re-assigning number values and sweeping through gaps to find 4 ½ in.

We appreciate the support of the National Science Foundation:
Funding for the Children’s Measurement project DRL-0732217.

Coordinates arithmetic operations with number labels at endpoints and stick counting (May 15, ‘08).



Tries to count gaps, but covered section prevents this approach. She is trying to get rid of the 8 at the front, and then says she can figure it out: 8 + 2 is 10, … (she is treating 34 as 8 + x1 + x2 + x3). She wants to subtract 8 from 34.

We appreciate the support of the National Science Foundation:
Funding for the Children’s Measurement project DRL-0732217.

Summary of shift from URR into CLM in the Learning Trajectory (supporting linearity and stability):

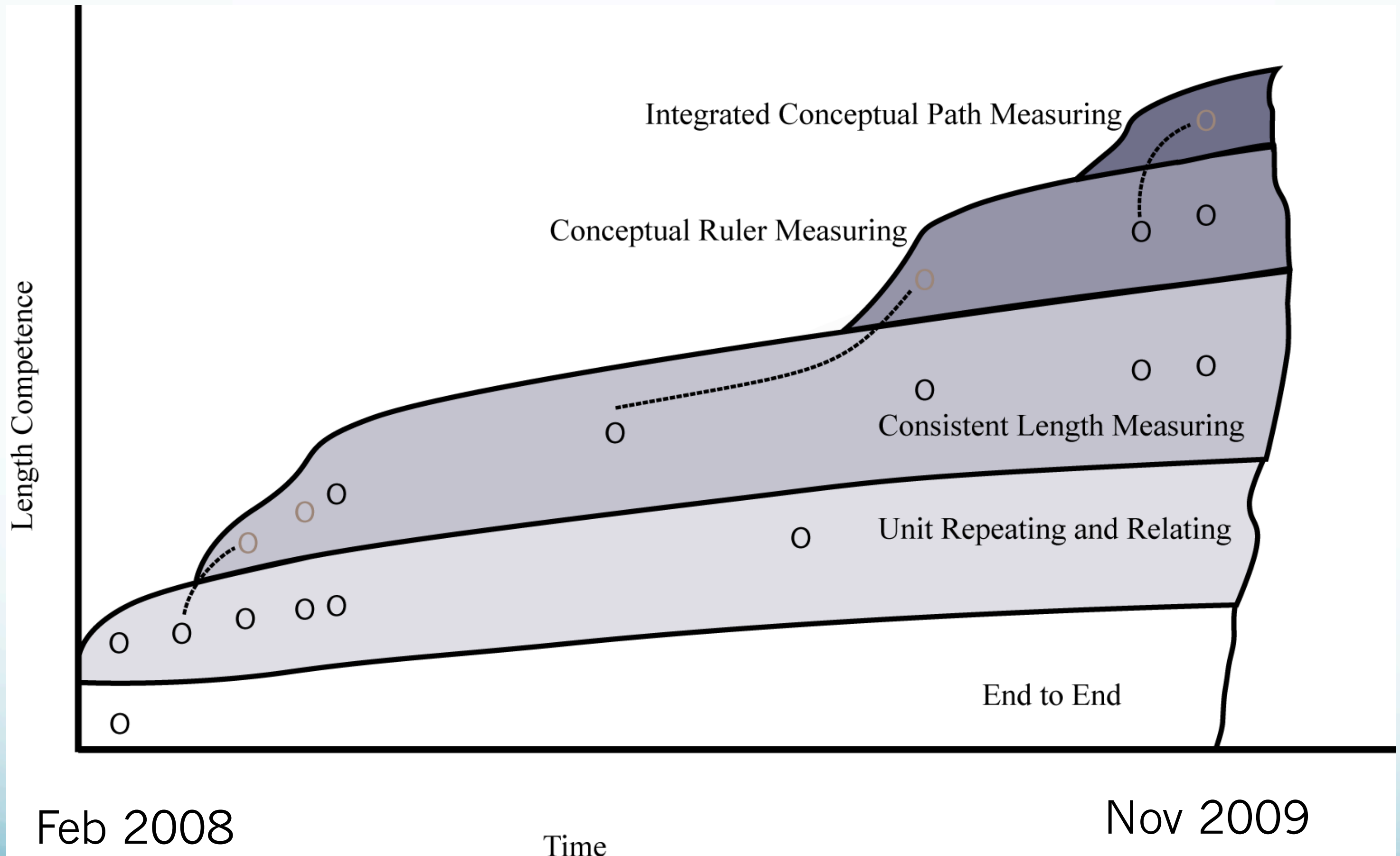
Students progress as they coordinate gaps, marks and numbers with arithmetic operations and unit iteration

- Counting gaps is inadequate for longer measures and for mixed units.
- Counting hash marks does not correspond directly to units of length, allowing errors.
- Using number labels on rulers at endpoints assumes alignment to zero and arithmetic.

Thus, by integrating schemes for reading number labels, arithmetic, and counting intervals or hash marks, students build flexible strategies and abstract linear quantity.

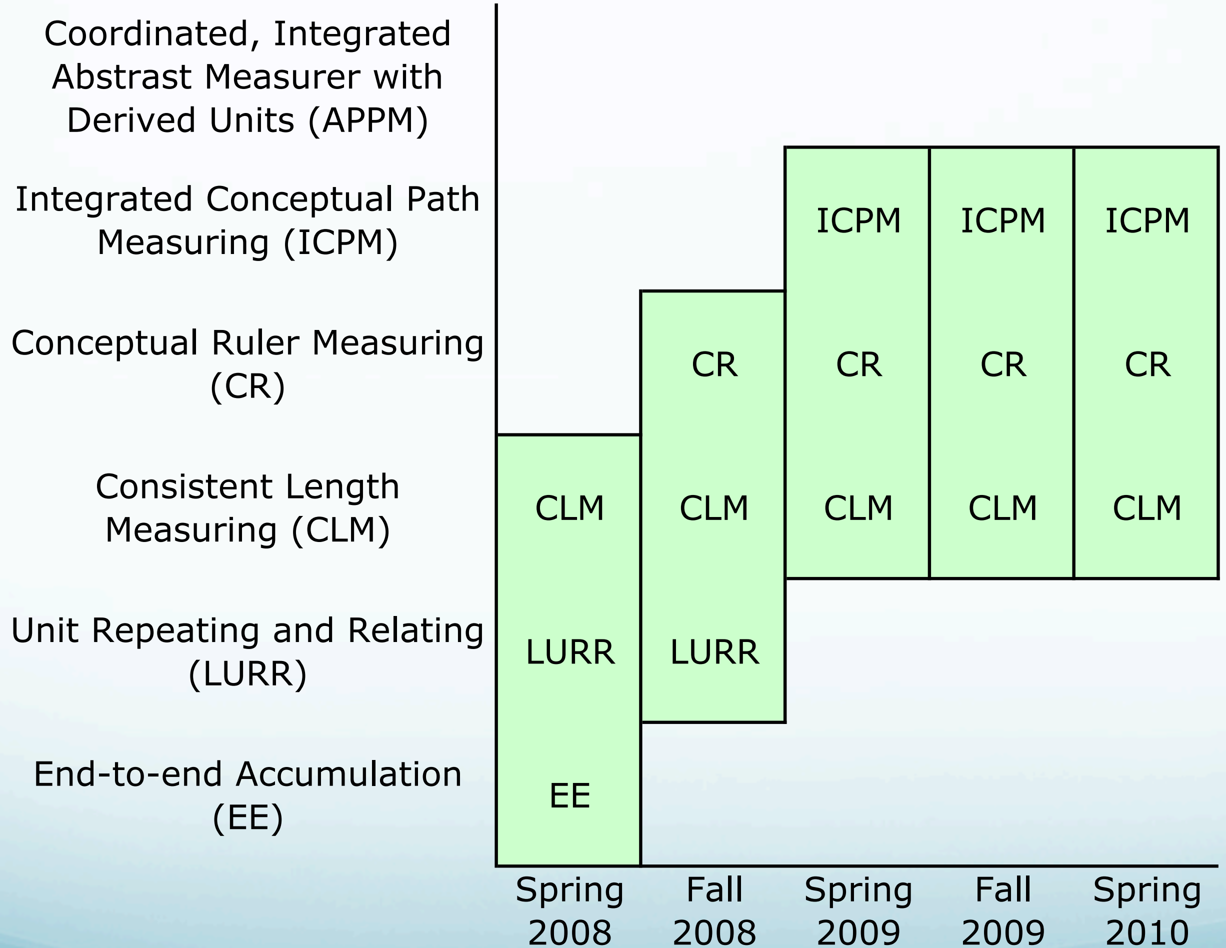
We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

What kinds of **research data support construction or validation of a** LT? We argue for profiles of each student showing both regressive and progressive levels



We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Complemented by *composite profiles* of groups of students within a LT across substantive periods of development (6 months)



We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

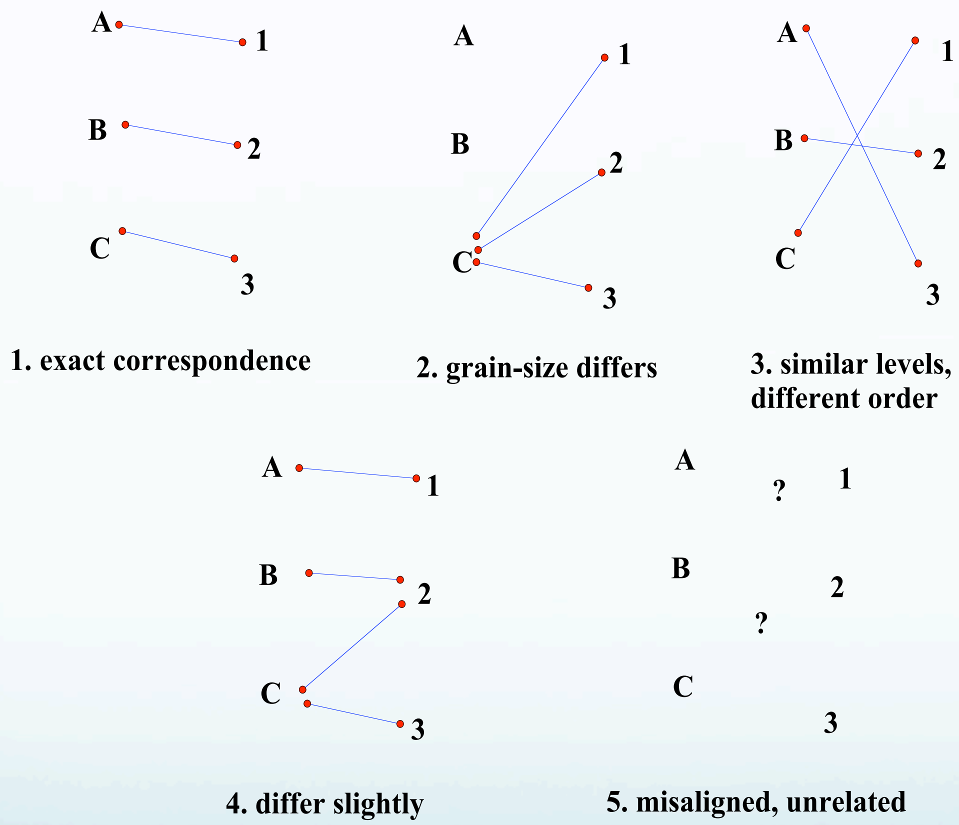
Linearity and stability

1. What constitutes a LT? **How stable and linear are LTs**?

- Some researchers argue that stability only occurs with high-performing students (Steedle & Shavelson, 2009), but "buggy" strategies are unstable (cf. van Lehn, 1996).
  - **We argue that longer periods of development are stable even for students using "buggy" strategies, given monthly or semi-annual checkpoints.**
- Also, we recently conducted a comparative analysis of the generalizability of our LT for Length, involving a different LT for length from another research team.
  - **This requires a rubric for comparing different trajectory schemes.**
  - We propose such a rubric (Barrett & Battista, forthcoming) in a pending report of the *Consortium for Policy Research in Education* report on Learning Progressions.

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Rubric for comparing similar LTs on a common domain



We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.

Three Scales for using LTs

- Issues of scale- An educational research perspective typically runs across three scales of analysis:
- (1) macro-reasoning, generalization and theorizing,
- (2) school-curriculum topics about typical content with analytic thought,
- (3) componential aspects of thinking underneath that are supporting topical content domains.
- Mathematics education research addresses reasoning writ large (scale 1): macro-level argumentation, theory-testing, and reasoning; these are overarching, *rhetorical* aspects of mathematics or scientific thinking.
- Mathematics Education research addresses reasoning at a “natural” curricular scale (2): topics are broken out into number, algebra, geometry, statistics and probability, and measurement, and these are spread across grade levels (e.g. Curriculum Focal Points, 2006).
- Mathematics Education research addresses components of thought that undergird mathematical reasoning at the prior scale. This scale (3) is a micro-cognitive analysis (this might be considered learning, but it may also be a developmental shift along these narrow, cognitive competencies (e.g., comparison, perception of numberline values, discrimination, coordination, or sequencing).

We appreciate the support of the National Science Foundation:
Funding for the Children’s Measurement project DRL-0732217.

Thanks!

Jbarrett@ilstu.edu

Jeffrey E. Barrett

Illinois State University

We appreciate the support of the National Science Foundation:
Funding for the Children's Measurement project DRL-0732217.