Longitudinal Learning of Viable Argument in Mathematics for Adolescents (LLAMA)
Assessment Version B
Master Scoring Rubric
7.25.18

Read Correctly Rubric

The “read correctly” score applies to all items. Each item is scored with its own rubric for the argument score. See next pages for those.

The general goal of the argument score (in the rest of the rubric) is to measure the student’s understanding of LLAMA argumentation ideas and practices. However, if the student doesn’t understand some of the mathematical terms in an item, or if they don’t understand what the item is asking them to do, then that can interfere with their ability to display their knowledge of the argumentation concept(s) we are seeking to measure. The purpose of the *read correctly* score is to measure whether the student had access to the prompt. If a student scores a 2 on *reads correctly*, then we are confident that their argument score on the item accurately reflects their understanding of the targeted argumentation concept. They could still get a 0 on the argument score but if they do, it is because of their low argumentation reasoning (for instance, maybe they provide an empirical argument for a general claim). If a student scores a 0 on *reads correctly*, then we have no evidence that they would be able to demonstrate their understanding of the targeted argumentation concept on the item, even if they understood that argumentation concept robustly (for instance, maybe they understand indirect arguments thoroughly, but we can’t tell because they so badly misinterpreted the task instructions). Thus, the *read correctly* score is independent of the student’s understanding of the targeted argumentation concept.

Purpose: The *read correctly* score measures the degree to which the student has shown an understanding of
(a) the mathematical objects and definitions in the item, and
(b) the task’s format, structure, and instructions (e.g., develop an argument, assess the claim, critique the presented argument)
that would enable the student to demonstrate their understanding of (c) the argumentation concept(s) we seek to assess with the item (e.g., what is a counterexample, when is an exhaustion argument appropriate, etc.)

<table>
<tr><th>Category / Rating</th><th>0</th><th colspan="2">1</th><th>2</th></tr>
<tr><td rowspan="2">Read correctly
(understand situation)</td><td>Respondent shows no evidence of understanding either (a) or (b) above.
OR
The understanding of (a) and (b) demonstrated is not sufficient for the respondent to be able to score higher than a 0 on the argument score, even if they understood the argumentation concept in the task.</td><td colspan="2">Respondent demonstrates some understanding of (a) or (b) above, but it is not clear that they understand these fully enough that their argument score accurately reflects their understanding of the argument concept in the task. In other words, you could imagine someone with this understanding of (a) and (b) getting at least a 1 on the overall score, but it is not clear that anyone with this understanding of (a) and (b) would be able to get a 3.</td><td>Respondent appears to understand (a) and (b) fully enough that the argument score accurately reflects their understanding of the argument concept in the task.</td></tr>
<tr><td>Code:
NR, N</td><td>Code:
U</td><td>Code:
P</td><td>Code:
Y</td></tr>
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Problem 6

1. After exploring three examples, Isabella believes that the sum of any five consecutive counting numbers is 5 times the middle counting number. (The "middle counting number" is the third counting number in the list when ranked from smallest to largest.)

Isabella's examples:

$$1+2+3+4+5=15=5(3)$$

$$2+3+4+5+6=20=5(4)$$

$$3+4+5+6+7=25=5(5)$$

Isabella wishes to develop a viable variable-based argument for her claim. Help Isabella by completing the following parts:

a) Let n represent the middle counting number of the five consecutive counting numbers. Write a variable expression for the sum of any five consecutive counting numbers.
b) Using your expression in part a), write a viable **variable-based** argument for the claim: For any five consecutive counting numbers, the sum is 5 times the middle counting number.

Purpose of the item: Generic examples arguments, pictorial arguments, and recursion arguments are considered viable in LLAMA and can be rated as a 3 on Version A task 1. However, this item assesses whether respondents can construct an argument using a variable expression.

In my first iteration of LAMP, I found that some respondents were unable to come up with a variable representation of consecutive number that was useful in crafting an argument. This task was designed to ease that issue in order to assess whether or not a respondent could make a viable argument after a creating a useful variable expression based on a particular conceptual insight that was provided for them.

Rating	Description
3: Viable Argument	If response fully meets the purpose of the item, it should be scored a 2 or a 3. Response includes a variable representation of the conditions (*sum of five consecutive counting numbers*) that is linked to the conclusion through algebraic manipulations (*e.g., n-2+n-1+n+1+n+2=5n*); Variable based argument (Examples of actual respondents' responses): • Claim: The sum of five consecutive counting numbers is 5 times the middle counting number. • Foundation: (n-2)+ (n – 1) + n + (n + 1) +(n+2)= 5n

	• Narrative Link: There are 5 Ns total. The negative & positive 1's and 2's cancel so you are left with 5n. There is no counterexample. The solutions are infinite.
2: Elements of Viable Argument	Respondent constructs a generic example argument in which if the middle number were replaced with an "n" (or some other variable) then the response would otherwise be scored as a 3.
1: Limited Elements of Viable Argument	Respondent constructs an argument that has a conceptual insight for viable argumentation for the claim; however, a variable expression as suggested is not leveraged in the argument (e.g. recursion, thought experiment, etc.). For instance, they may provide a reasonable variable expression but not use it in an argument or in their response to part (b). Any other viable or near-viable argument that is not based on algebraic manipulation of the intended expression from part (a). **Example argument 1 [Recursive]:** 1+2+3+4+5=15. (1+1)+(2+1)+(3+1)+(4+1)+(5+1) has five extra ones. [Recursive ideas.]
0: No Elements of Viable Argument	Respondent constructs empirical argument. Claim affirmed using one or more empirical examples but general mathematical structure linking the conditions to the conclusion is absent, OR, respondent creates a variable expression for the conditions that does not illustrate "consecutive." **Example response 1 [Empirical]:** • 5+6+7+8+9=35; 7(5)=35. **Example response 2 [Devoid of the Conditions]:** • l+m+n+o+p=5(n) **Example response 3 ["Middle" is position in sum only with no CI (conceptual insight)]:** • 1+2+n+4+5=5n

Problem 7

A student in your class is wondering whether $\sqrt{14}$ is a rational number or an irrational number.

Purpose of the Item: Item assesses respondents' ability to construct a contradiction argument in this context.

<table>
<tr><td>Rating</td><td>Description</td></tr>
<tr><td>3: Viable Argument</td><td>If response fully meets the purpose of the item, it should be scored a 2 or a 3.

Response includes:
A supposition that sqrt(14) is rational, perhaps by saying suppose or assume sqrt(14)=a/b integers.
A resulting equation or sentence equating 14 to the quotient of perfect squares.
An acknowledgement that
14 has an unpaired prime in its prime decomposition (or “every prime factor occurs twice”).
Both a^2 and b^2 have paired primes in their prime decomposition. (or “every prime factor occurs twice”).
An acknowledgement that 14 cannot equal a^2/b^2 because the prime structures are not compatible.
An acknowledgement that this shows us that the supposition that sqrt(14) is rational leads to contradiction (or an absurd statement).
Example Claims:
$\sqrt{14}$ is irrational.
$\sqrt{14}$ cannot be written as the quotient of integers.
Response Characteristics: Must have at least two features:
An acknowledgement that because $14 = a^2 / b^2$ is not solvable with integers, $\sqrt{14}$ is not expressible as the quotient of integers.
Response that demonstrates that $14 = a^2 / b^2$ is not solvable with integers because the right and left hand side of the equation are incompatible: 14 has an unpaired prime in its prime decomposition and both a^2 and b^2 have paired primes in their prime decompositions.</td></tr>
<tr><td>2: Elements of Viable Argument</td><td>Response is similar to that described in a score of a 3, but an important feature is missing. For example, respondent acknowledges that 14 has an unpaired prime but does not explain why this makes 14 not equal to a quotient of perfect squares.</td></tr>
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	Response Characteristics: An acknowledgement that because $14 = a^2 / b^2$ is not solvable with integers, $\sqrt{14}$ is not expressible as the quotient of integers. • Response that demonstrates that $14 = a^2 / b^2$ is not solvable with integers is not viable (missing or incorrect critical inference). ○ Example 1: One of the prime factors is 7. Since there is no way for 7 to equal a^2... ○ Example 2: If the square root of 14 was rational, then it could equal a^2 / b^2, but since 14 is only equal to 7 times 2 and 14 times 1 …
1: Limited Elements of Viable Argument	Student response may not include the key conceptual insight (that 14 has an unpaired prime), but does include some features of the intended argument from the LLAMA lesson, such as recognition of and reasoning with the quotient structure of a^2 / b^2 Response characteristics: Attends to leverage important structure, such as not integer over integer definition or the prime structure of the radicand, but fails to coordinate these facts coherently. **Example response 1**: 14 has an unpaired prime. For a number to have a rational square root, it has to have an even number of “paired” primes…
0: No Elements of Viable Argument	Respondent offers no response or offers a response that has no obvious potential for viable argumentation, such as a calculator approximation. **Example response 1:** $\sqrt{14} = 3.741657387$ There is no way to write this number as an integer over integer. **Example response 2:** Cannot be written as an integer divided by an integer because 3.741653787 is not rational at all.

Problem 8

A student in your class claims that the rule $p = n^2 + n + 41$ *produces* a prime number p when n represents any counting number.

a) Develop a viable argument for or against the student's claim.
b) Explain why your argument in part a) is viable.

Purpose of the item: The item assesses respondents' skepticism of a generalization and whether the respondent finds a counterexample and constructs a viable existence argument against the claim.

Rating	Description
3: Viable Argument	If response fully meets the purpose of the item, it should be scored a 2 or a 3. Respondent asserts that the claim is false and illustrates or describes a counterexample (e.g., 40, 41, or some multiple of 41); respondent's response is accompanied by an illustration that the proposed counterexamples have the desired properties (i.e., f(41)=a composite number), or offers an explanation for why the candidate input corresponds to a composite output. **Example response 1:** n=41. 41^2+41+41=1763. 41X43=1763. **Example response 2**: 41^2+41+41, each term has 41 that can be factored.
2: Elements of Viable Argument	Respondent asserts that the claim is false and illustrates or describes a correct counterexample (e.g., 40, 41, or some multiple of 41); however the response is not accompanied by an illustration that the proposed counterexamples have the desired properties. Respondent asserts the claim is true and offers a general argument that would be viable if it were based in correct conceptual insights and prior results. In other words, the argument is valid but is not sound.
1: Limited Elements of Viable Argument	Asserts that the claim is true and makes a general but incorrect argument, perhaps by conflating "prime" with "odd." Provides a conceptual or structural insight that might be viable for a different claim. OR Respondent asserts that the claim is false and offers an incorrect counterexample based in incorrect computation.

	Example argument 1: $1^2+2+41=44=11X4$. **Example argument 2:** If n is even, then because n^2 is even, n^2+n+41 is an even plus an even plus an odd, which is odd. If n is odd, then n^2 is odd, so n^2+n+41 is the sum of three odd numbers, which is odd. So the outcome is always odd, which is prime.
0: No Elements of Viable Argument	Respondent asserts that the claim is true based on empirical data from testing cases. **Example response 1**: $2^2+2+41=47$, a prime. $1^2+1+41=43$, a prime.

Problem 9

A student in your class has given the following argument:

Claim: If $3n + 2$ is an odd number, then n is an odd number.

Justification: Suppose $3n + 2$ is an odd number and n is an even number. But if n is even then $3n + 2$ is even because 3 times an even is an even and an even plus an even is an even number. But this would make $3n + 2$ both even and odd, which can't be true.

Can the student's justification be used as a viable argument for the claim? Explain why or why not.

Purpose of the item: This item assesses respondents' abilities to recognize an indirect argument and explain why the indirect argument approach is viable.

Note: Some respondents may address the converse of the claim in addition to addressing the argument presented (e.g., "But 3 times an odd number is odd and odd plus even is odd."). This does not devalue comments addressing the argument presented.

Reading Score: To award a 2 on the "read correctly" score, the rater must have evidence that the student is critiquing the argument, not just the claim. Students who asserts that the claims and develops a similar argument without explicitly addressing the mode of argumentation receives a 1. Understanding the need to critique the argument indicates a reading score of at least a 1.

Rating	Description
3: Viable Argument	If response fully meets the purpose of the item, it should be scored a 2 or a 3. The respondent acknowledges the indirect method of argumentation (not conclusion implies not the conditions, impossible to have counterexamples, etc.) and understands that this method shows the claim to be true (even if this isn't explicitly said in so many words). Student does not need to provide detail about the elements of the presented argument, as long as they do the above. **Example response 1**: Yes, because they clearly state how it would be impossible to have n be even if 3n+2 is odd, because multiplying and adding evens results in another even. **Example response 2**:

	Yes, the response is viable because the respondent shows that if you don't have the conclusion then you cannot have the conditions. **Example response 3**: I think the students' justification can be used as a viable argument. I think this because it proves that n must be an odd number, because if it were even, 3n+2 would be even and odd. Since no number can be even and odd, n can't be even. This proves the claim correct. **Example response 4:** It uses contradiction to show that there are no counterexamples.
2: Elements of Viable Argument	Respondents' response is similar to a 3 response; however, the response is vague. For instance, they are using the elements of the provided justification and understanding its structure, but with some vagueness. Respondent recognizes the indirect argument structure but questions whether this is sufficient or correct in addressing the claim. **Example response 1 [Clear enough to reasonably assume that the indirect approach is understood]**: 3n+2=odd but if it's even then it would only be even and not odd. **Example response 2:** Yes, because 3 times an even number is indeed an even number and an even plus an even is an even. But 3 times an odd number is odd and odd pluss even is odd. **Example response 3**: No it cannot, because it does not fully address the claim. It only proves that if 3n+2 is odd and n is even, that would not make any sense. It does not do anything about 3n+2 being odd and n having to be odd also, for that to work. Although I suppose they could be trying to indirectly argue for that.

1: Limited Elements of Viable Argument	Respondent recognizes that it is not a direct argument and explicitly dismisses indirect argumentation or does not assert it as viable. OR Response is too vague to be sure that they understand or recognize indirect argumentation. **Example response 1:** It's not directly arguing. So I don't think so. It's justifying the opposite. **Example response 2**: No, because he claims that if 3n+2 is odd, then n is odd, then in the justification says 3n+2 is a false statement. **Example response 3 [unclear use of pronouns]:** 3n+2 = odd but if it's even then it would only be even and not odd.
0: No Elements of Viable Argument	Respondent response contains no acknowledgment of the indirect method in the argument presented. Respondent addresses only conforming cases or the converse of the claim but provides no evidence that the argument presented is addressed correctly. **Example response 1:** No because they don't show why it is true. **Example response 2:** Yes. The respondent supports his/her claim with a solid explanation. **Example response 3:** But 3 times an odd number is odd and odd plus even is odd.