Supporting Teachers to Teach Mathematics Through Problem Posing

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Project Overview

This 4-year, longitudinal, early-stage design and development project has two major goals:

1. *Support teachers to teach mathematics through engaging their students in mathematical problem posing.*

2. *Longitudinally investigate the promise of supporting teachers to teach with P-PBL for teachers' instructional practice and students' learning.*

Research Questions

1. How do teachers integrate problem posing into their instruction, and how do they develop problem-posing lessons and improve them over time?
2. How does collaboration in networked improvement communities of teachers and researchers help teachers overcome the challenges of integrating and improving the implementation of problem posing in lessons over time?
3. What is the impact of collaborating on the integration and improvement of problem-posing tasks and lessons on teachers' beliefs about problem posing, their beliefs about P-PBL, and their actual instructional practice?
4. What impact does P-PBL ultimately have on students' learning as measured by problem-posing performance, problem-solving performance, and mathematics disposition?

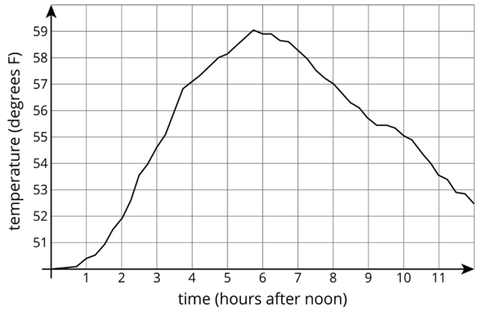
Aim of Research

Widely used curriculum materials fail to incorporate problem posing in a substantial and consistent way, leaving teachers few resources to enact problem-posing-based learning (P-PBL). This project empowers teachers to integrate problem posing into their instruction and to develop with researchers a set of 20-30 P-PBL middle-school algebra lessons and research-supported teaching cases that share key details for the implementation of P-PBL and lessons learned from the development process. These materials will help other teachers and PD providers develop capacity for implementing P-PBL. Our findings add to the field's understanding of the promise of supporting teachers to use P-PBL and of challenges that teachers face when engaging in a networked improvement community that is focused on improving tasks and lessons by integrating P-PBL.

Defining Mathematical Problem Posing

Mathematical problem posing means creating a mathematical task (e.g., a question, conjecture, problem, or exercise) that demands a solution based on a situation. This includes reformulating an existing mathematical problem into another mathematical problem. A *problem-posing task* includes both (1) a *problem situation* that specifies the context and information about which problems may be posed and (2) a *prompt* that communicates to the student what they are expected to do. Problem situations may involve real-life information or be entirely mathematical. Prompts may specify how many problems to pose, how difficult they are, how similar they are to an example, etc.

This Graph shows the temperature between noon and midnight in one day in a certain city



Write down as many questions as you can that can be answered using this graph.

Figure 1. Example of a problem-posing task created by a P-PBL teacher based on a task in their curriculum

Defining P-PBL

P-PBL refers to using problem-posing tasks with students to achieve several goals: (a) create rich opportunities for students to engage in mathematical sensemaking; (b) connect students' life experiences to the mathematics they are learning; (c) develop students' identities as doers of mathematics, engage their creativity, and support their ownership of their learning; and (d) help students learn mathematical content and processes more deeply. The following is an instructional model for using problem-posing tasks in the classroom:

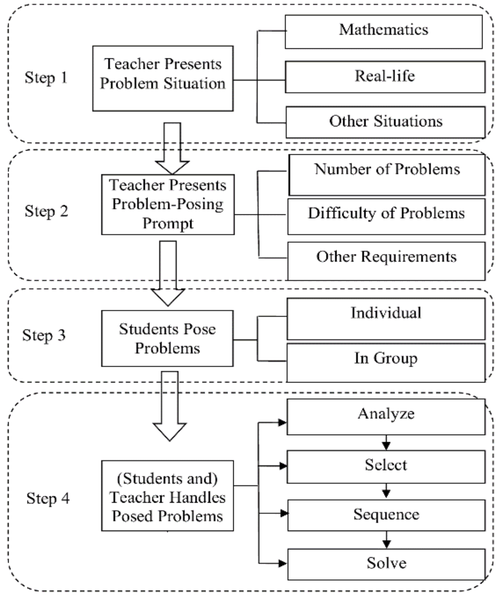


Figure 2. An instructional model for using problem-posing tasks in the classroom

Multiwave (Longitudinal/Repeated Measures) Research Design

Research Context

We worked with students and teachers from two middle schools in Year 1. In Year 2, we have added students and teachers from two more schools. For Year 3, we will keep working with these teachers as well as teachers from at least two more schools. Overall, we will follow a sample of approximately 36 teachers and their approximately 3,600 students from six middle schools for four years.

Project Activities

	Summer Workshop	Integrate P-PBL into daily teaching	Classroom observation	Networked monthly meetings during regular year	Lesson revisions	Implement other lessons	Student assessments	Server as mentors	Craft and finalize P-PBL cases
Year 1									
Cohort 1	●	●	●	●	●		●		●
Cohort 2									
Cohort 3									
Year2									
Cohort 1	●	●	●	●	●	●	●	●	●
Cohort 2	●	●	●	●	●		●		●
Cohort 3									
Year 3									
Cohort 1	●	●	●	●	●	●	●	●	
Cohort 2	●	●	●	●	●	●	●	●	●
Cohort 3	●	●	●	●	●		●		●
Year 4									
Cohort 1									●
Cohort 2					●	●			●
Cohort 3		●	●		●	●			●

Data

- Teacher journals: Each teacher records their experiences designing and using problem-posing tasks and lessons
- Teacher-modified lessons: Archived revisions of lessons with teacher-designed problem-posing tasks
- Classroom observations: Observations of teachers implementing lessons with problem-posing tasks they have designed
- Summer workshop recordings and teacher surveys
- Teacher interviews: Individual interviews with teachers probing their thinking about problem posing, tasks, and challenges they have encountered
- Assessments: Fall and spring student assessments of problem posing, problem solving, and mathematics disposition; DE state test

Initial Findings

- Based on an analysis of 2,164 student-posed problems, a problem-posing prompt that includes more specific requirements, tends to encourage students to engage in more in-depth mathematical thinking and pose more linguistically and semantically complex problems prompts with more relationships or steps required to solve them.
- Based on interviews with participating teachers, teachers do not always favor one type of problem-posing prompt over another, instead choosing a prompt based on the challenges they anticipate their students will face. Overall, teachers choose prompts that they feel will maximize their students' engagement with the task. Teachers attend to the openness or simplicity of a prompt, the degree to which it scaffolds students' posing, and whether it promotes critical thinking.
- Some teachers have begun to incorporate problem posing tasks throughout their regular classroom teaching. Others use problem-posing tasks less frequently and for specific purposes (e.g., reviewing, assessing students' understanding) based on their beliefs about problem posing and about their students.

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