REASONING LANGUAGE FOR TEACHING SECONDARY ALGEBRA (ReLaTe-SA)

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Project Description

Reasoning Language for Teaching Secondary Algebra (ReLaTe-SA) is a three-year, Level I Exploratory project in the Teaching Strand of the Discovery Research PreK-12 (DRK-12) program. This project is led by an interdisciplinary team of faculty at the Texas State University and University of Texas at San Antonio in collaboration with academic directors and coaches at the San Antonio Independent School District.

Goals:

- Investigate middle and high school teachers' algebraic discourse through written assessments and analysis of classroom observations
  - Understand the discourse currently used by teachers when presenting algebraic concepts to students
- Design and implement a collaborative professional development (PD) program with middle and high school teachers that:
  - Addresses and enriches mathematical meanings for algebra teaching;
  - Defines, operationalizes, and helps teachers develop reasoning language for teaching algebraic concepts and procedures;
  - Makes reasoning-rich discourse for algebraic problem solving explicit and accessible
  - Identifies pedagogical practices that support learners' algebraic reasoning and discourse; and
  - Illuminates the importance of attending to students' cultural and knowledge assets thus allowing for richer engagement in algebraic content.

Hypotheses:

- Teachers' algebraic discourse is influenced by their mathematical meanings for algebraic concepts.
- Teachers' discourse influences the meanings that their students develop for these concepts.
- Addressing the discourse and related mathematical meanings that teachers use will enhance students' opportunities to develop robust understandings of algebra.

Progress:

- Designed our *Survey of Algebraic Language and Reasoning* (SALR)
- Recruited a diverse group of middle and high school teachers over three years
- Administered the SALR for three cohorts of middle and high school teachers
- Collected and analyzed the responses to SALR according to discourse framework
- Developed a curriculum for a professional development (PD) course
- Analyzed significant video/audio data collected during PD

Theoretical Background

Our research aligns with Sfard's (2007, 2020) ***commognitive perspective*** on mathematical teaching and learning, which assumes:

- All thinking is a form of communication
- To learn mathematics is to change one's participation in a discourse community

Our framework for algebraic discourse is based on the ***arithmetical discourse profile*** of Ben-Yehuda, Lavy, Linchevski, and Sfard (2005). Ben-Yehuda et al. analyze learners' discourse about concepts and problems in arithmetic along several key dimensions:

- Their uses of ***words*** and the extent to which these explicitly describe mathematical objects
- Their uses of ***mediators*** (symbols and visuals that represent mathematical objects)
- Their ***endorsed narratives*** and apparent meta-rules for accepting and rejecting narratives, and their uses of routines.

For our purposes, we condense the ***words*** and ***mediators*** dimensions into a single dimension.

Definition: Deductive Discourse for Equation Solving

- Words and mediators frequently serve to make the objects of the discourse (e.g., values of expressions, operations, equality) and their properties explicit
- Narratives about equations are endorsed/rejected by deduction from assumptions and other endorsed narratives rather than by appeal to authority
- Routines are flexible tools for generating new narratives about mathematical objects.

Research Questions

What beliefs do teachers have about teaching students a deductive discourse for solving linear equations?
What challenges might teacher beliefs pose for teaching students how to reason deductively about equation solving?

Method

Informed by teachers' responses to survey items dealing with equations and solution processes, we developed an activity titled Linear Equation Talk-Throughs implemented as part of an 80-hour PD for 7 middle/high school algebra teachers in 2022.

1. Teachers privately watched three different video talk-throughs (recorded by researcher) designed to exemplify different possible discourse features of explanations of solving linear equations.
2. Teacher-participants independently watched the researcher's talk-throughs; they divided into small groups to compare the three talk-throughs and discuss the affordances and drawbacks of each.
3. Discussions were video/audio recorded, transcripts electronically generated and verified for accuracy, and coded for any implicit beliefs about teaching and learning of equation solving that were evident.
4. Researchers came together to discuss the independent themes to agree on broad themes related to teacher beliefs (Our analysis of teachers' beliefs is informed by Leatham's ***sensible systems theory***, which suggests that rather than focusing on apparent contradictions among beliefs held by an individual teacher, we should view beliefs as occupying an interconnected network in which some beliefs may take precedence over others at specific times (2006).

Group	Participant	Level
Green	Danielle	High school
	Pablo	High school
	Frances	High school
Pink	Benjamin	High school
	Viola	Middle school (K–8 academy)
Yellow	Denise	High school
	Felipe	Middle school (K–8 academy)

Researcher Talk-Through Video Samples

Video	Description	Sample from Researcher Talk-Through (emphasis ours)
Video 1	Focus on actions-on-symbols *Duration: 1:18*	"My approach here is I'm going to try to **get all my *x*'s on one side** of the equation and **put all the constants on the other side** of the equation. Because there's already a 3*x* on the right side of the equation, **I think I want to move the term 7*x*, so it's over on the right side**, and I'll leave the -20 on the left. I'm going to take this 7*x* here, and I'm going to **change sides and change signs**, so I'm going to move it over to the right and **put a negative sign on it**."
Video 2	Focus on deductions about equal values *Duration: 2:14*	"So, I'm going to start by saying **because 7*x* minus 20 has the same value as 3*x*, if I add -7*x* to both of those values, I should get the same result**. So, in other words, -7*x* plus 7*x* minus 20, that should be equal to -7*x* plus 3*x*, **'cause I took two equal values, 7*x* minus 20 and 3*x*, and I added the same thing to each**. Now, if I look at the left side of the equation, I have -7*x* plus 7*x* minus 20. -7*x* plus 7*x*, those are additive inverses of each other, so they add to zero. That means I'm left with 0 minus 20 equals -7*x* plus 3*x*."
Video 3	Using structure and number sense to solve by inspection *Duration: 0:58*	"**Well, one thing I notice about this equation** is I'm starting with 7*x* and I'm subtracting 20, and that leaves me with 3*x*. One thing that I know is that if I start with 7*x* and subtract 4*x*, that leaves 3*x*. So that means that if I'm subtracting 7*x* minus 20 and getting 3*x*, **that means that 20 has to be equal to 4*x***. And so now I have an equation that says that 20 is equal to 4 times *x*, 4 times my number *x*. **So I think what number multiplied by 4 gives me 20? Well, I know that 4 times 5 is 20**, so that indicates that *x* is equal to 5."

Research Findings

Three themes surfaced in our analysis of teachers' discussion of the researcher talk-throughs.

- **The role/importance of understanding that solving linear equations is a deductive process**
  - Notably, there seemed to be conflicting perspectives on when mathematical properties in the equation-solving process should be made explicit to students.

[Regarding Video 1, which focused on actions-on-symbols rather than deduction]

Viola: *It's that part that if I was new to algebra, I would not understand, "Why am I changing sides?" I'm assuming that a student who'd do this is well-versed in why I'm changing sides and why I'm changing signs. That statement assumes understanding is what I'm just saying.*
Benjamin: *Especially with negative numbers, and that's where they get confused. My experience, they get confused a lot.*
Viola: *I'm going to tell you straight up; sixth grade is where it's introduced. If it's not introduced with concrete [models], they will struggle for a long, long time. Otherwise, you're going to have to rely on rules and they don't know why it works. So, this is key, right? …**So, the question, "Why does it work?" needs to be happening way down before you... Yeah, because you're too far. You're advanced.***

Counter to the belief that students should be exposed to the deductive reasoning behind the algebraic steps in equation solving as they learn the steps to solving, Danielle posited that introducing properties too early may confuse learners.
Danielle: *But that's after they already have learned to solve equations in ninth grade, in algebra one. Then we're doing it in geometry, we're saying, "Okay, these are what these properties are called now to practice those justifications." So, from that standpoint, but again, doing that not on the first time they're learning this. It's like, the second time. **So, I love the use of properties, but I agree, I think it would be confusing to the people learning for the first time, and that's what I thought, too.***

- **The estimation of students' capacity to understand solving linear equations as a deductive process**
  - Two of the groups (Green and Yellow), comprising of five teacher participants, suggested that a deductive approach would not be suitable for all students
  - Some teachers distinguished those students who would be confused by "too many steps" and students who would benefit from an explicit development of the deductive reasoning behind the problem-solving process

[Regarding Video 2, which explains the algebraic properties underpinning the deductive view]

Frances: ***It's too many steps.*** *And then, I would have simplified the right side instead of taking it to the next step. I would have simplified as I went to the next step on both sides. And he would simplify one side, then bring down to another one, another step, and then simplify on the first side. He wouldn't simplify it as he would go along; he would wait, go to the next step, next step, next step.*
Danielle: *Yeah.*
Frances: *Like, step one, step two, step three. My kids would get confused.*
Danielle: *Yeah.*
Frances: ***Yeah, my kids would get confused; too many steps.*** *I already know that, too many steps. Now, the ones that are real bright, they would catch on real easily. But you have to realize you have to accommodate everybody in the class…*

- **The perception of a difference between those more experienced in algebraic reasoning (teachers) and novices to algebraic reasoning (students) in terms of the potential to understand and engage in deductive algebraic discourse.**

[Regarding Video 3, which relies on structure and number sense]

Felipe: *That one* [Video 3], ***I think is the more complex of them all. Well, no, not for us.*** *For them to rationalize and understand because to them, when they see 5x, they generally, I think would see it as two units, 5 and x. Whereas we can see it as one thing that we can manipulate.*

Discussion/Future Research

- Participants' analyses of researcher talk-throughs suggested that they saw potential benefits in the deductive explanation for the standard solution process given in Video 2 and the structure-oriented approach described in Video 3.
  - Felipe and Denise suggested that they would use an explanation like that in Video 2 to introduce students to the solution process before showing them an "easier" approach
  - Danielle suggested that she would defer the in-depth explanation in Video 2 until her students began grappling with deductive reasoning and formal proof in geometry.
  - While Viola and Benjamin stated that they found the "solving by inspection" approach in Video 3 to be "a fabulous tool," Denise and Felipe hypothesized that this method would be harder for students to understand and suggested offering it to students only as a "fun challenge."

If there is strong consensus that deductive explanations and structure-oriented approaches for solving equations are potentially useful for students, ***why are actions-on-symbols explanations of solution processes so prevalent in teaching,*** *as evidenced by reviews of curricular materials and our own teachers' recorded talk-throughs?*

In this study we have discovered two such families of beliefs:

1. A deductive perspective on equation solving is likely to prove difficult for students (especially those who have been the target of deficit attributions, such as students in an intervention course)
2. Explanations that teachers find approachable (and in fact elegant or efficient) might nevertheless be beyond students' reach.

Given that many teachers feel a strong sense of commitment to engaging all learners in successful mathematical practice, it is understandable that an explanation or approach that appears likely to confuse or frustrate learners might be disfavored in instruction.

One goal of our project is to persuade teachers that it is feasible and worthwhile to engage all learners in deep and conceptually coherent algebraic reasoning.

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