

“I would say we’re trying to learn communication and... coordinates”: Supporting students’ meanings for spatial coordinate systems

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Introduction

Students are **not provided sufficient opportunities** to develop rich meanings for graphs (Lai et al., 2016).

We examine ways students organize space in their development of meanings for coordinate systems, which underlie sophisticated graphing activity.

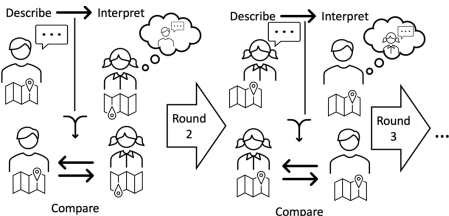
A **spatial coordinate system** is a system of measurements through which an individual organizes spatial elements in conceived or experienced space (e.g., a GPS showing a car’s path on a map); to conceive a spatial coordinate system, students must construct and coordinate **spatial frames of reference (FoR; Lee et al., 2020)**

There are several useful features of spatial coordinate systems:

1. Systematic coordination allows for precise description of spatial locations;
2. A reference point and FoRs constituting the system can be established at any place in the space;
3. They support consistent and efficient communication between individuals.

Conjecture and Research Question

We conjectured **coordinating and negotiating FoRs can support students in developing rich meanings for coordinate systems**, while understanding that any particular system is one of several options.



Exploring this conjecture, we address the question: How do middle-school students establish and coordinate FoRs to identify and describe locations in a particular space?

Methods

We conducted a 10-session **teaching experiment** (Steffe & Thompson, 2000) with 3 students, **Nena, Tara, and Jacobi**, focused on supporting their meanings for spatial coordinate systems and FoR.

We built models of each student’s mathematics using conceptual analysis (Thompson, 2008).

Session 2 – Establishing Spatial Frames of Reference

Students constructed many FoRs and communicated them with increased locational specificity.

Consider Nena’s first attempt describing the green X in the image below (originally given without the circles):

Nena: [after overlaying circles] It’s close to the first circle, but it’s in the second circle. On the right.

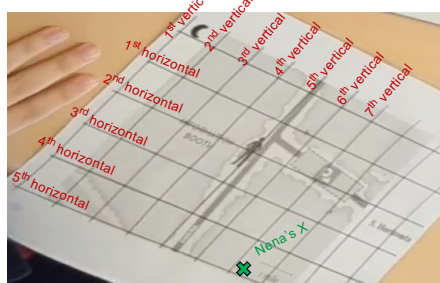


We show Nena’s FoRs in red. She used the Circle overlay to create a FoR of ordinal circles (e.g., 1st circle, 2nd circle) and a vertical line as a FoR cutting the map in half.

While Tara marked an X that fit Nena’s description, it did not correspond to Nena’s exact location, which led to a region but not a unique location.

In her third attempt, Nena used multiple FoRs. Using an image like the one below, she described:

Nena: The fifth line of the horizontal and seventh line of vertical...on the right.



Nena realized she initially described a region, adding “on the right” to be more precise. **Nena re-evaluated her coordination of FoRs to increase the specificity of the location she was describing.**

Sessions 3 & 4 – Providing more precise descriptions

In Session 3, Nena & Tara interpreted multiple hypothetical classmates’ descriptions (e.g., “Imagine the star is a clock with the line going straight up being 12 o’clock... The X is 1.25 miles from the star halfway between 10 and 11 o’clock.”).

In Session 4, Nena adopted some of this language in her first description. For example, giving instructions for Jacobi to find the blue X in the image below she said:

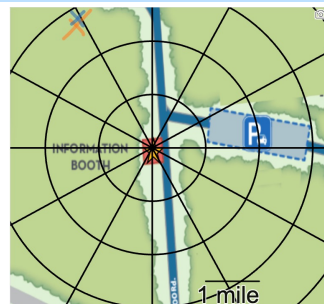
Nena: Click star and circle. Imagine the star as a clock, the top being the 12 and the bottom being the 6. The x is in between the 10 and 11. Inside the third circle.



Nena explored other configurations. Generally, **Nena first coordinated two FoR to determine a region then increased the specificity of the possible location by describing a location in that region.**

Jacobi was absent in Sessions 2 & 3. In Session 4, he first mainly used ordinal descriptions. Then Jacobi adopted Nena’s language in providing his own descriptions. For example, he provided this description for the orange X:

Jacobi: [typed] using the star and circle feature looking at 11 like about close but not right on the line that’s where the x is. [Adding verbally] Near the third circle.

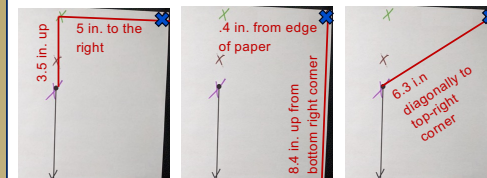


Session 6 – Jacobi generating spatial coordinate systems

In Sessions 4 & 5, students had repeated opportunities to describe and interpret different spatial coordinate systems.

In Session 6, we gave Jacobi the *North Pole* (Lee, 2020) task to explore how he may spontaneously describe locations of X’s in a new context without landmarks beyond a road and a point.

When prompted, **Jacobi provided 3 different descriptions of how to find the blue X** (his descriptions in red).



When the TR asked “What are you taking away from this? What are your thoughts on the activity so far?” Jacobi responded:

Jacobi: I would say we’re trying to learn communication and... coordinates...You can measure [the blue X] from the North Pole, the green X, the bottom right corner. There’s multiple different ways you can measure it. You can even measure it from the peach X.

Conclusions

Students established and used numerous FoRs to describe locations (e.g., directions, ordinal shapes); each student re-evaluated their coordination of FoRs to **allow for increased specificity** of locations within the space.

The goal of communicating with classmates was useful to support their establishing, coordinating, and refining spatial FoR and coordinate systems.

Jacobi provided **evidence he understood that establishing coordinate systems involves choices** that need to be made rather than rules that need to be unquestionably followed.

Selected References

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Acknowledgements

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