

**Definition of Backward Transfer (BT):** The influences that learning about new concepts has on learners' ways of reasoning about previously learned about concepts

Conceptualization of Backward Transfer:

- Forward transfer (FT) traditionally conceptualized as K acquired in Context A applied to Context B.
- Critics of the traditional conception of FT (e.g., Beach, 1999; Lave, 2002; Lobato 2012).
- Newer conceptions of FT (e.g., Bransford & Schwartz, 1999; Tuomi-Gröhn & Engeström, 2003).
- Actor-oriented transfer (AOT, Lobato, 2012) conceptualized as K acquired in Context A influences reasoning in Context B (i.e., transfer an influence on reasoning rather than an application of K).
- Our conception of BT aligns with AOT; BT is as an influence on rather than an application of K, but BT and AOT opposite directions (i.e., K acquired in Context B influences Context A reasoning).

Mathematical Context for Backward Transfer:

- *Linear functions* (LF) and *quadratic functions* (QF) are somewhat similar concepts (i.e., plausible that teaching students about QFs influences how students reason about LFs, which would be of educational significance). This makes LFs and QFs a good context pair for studying BT.

**MAIN RESEARCH QUESTION FOR STUDIES 1-4:** What kinds of changes in student's ways of reasoning about linear functions are realized, if any, after students learn about quadratic functions (i.e., what kinds of BT effects are realized)?

### Study 1: Backward Transfer Influences on Covariational Reasoning in Two Real Classrooms

10<sup>th</sup> grade students ( $N=57$ ), 2 classes/teachers, Ms. H, Mr. A; business-as-usual quadratics instruction (no emphasis on covariational reasoning); pre-/post-test on linear reasoning; qualitative analysis.

Results:

- Changes pre to post in the number of quantities students reasoned with in a linear context:
  - Ms. H's class, 50% of class; Mr. A's class, 24% of class
- Changes pre to post in the level of covariational reasoning:
  - Ms. H's class, 29% of class; Mr. A's class, 25% of class

Take-aways: (a) many changes (BT) related to covariational reasoning despite no explicit focus on covariational reasoning; and (b) similarities and differences in changes (BT) across the two classes.

### Study 2: Backward Transfer Influences on Reasoning in Summer Math Program

9<sup>th</sup> and 10<sup>th</sup> grade students ( $N=18$ ), 1 teacher; dynamic reps of distance-time quad functions, strong emphasis on covariational reasoning; pre/post test on linear function reasoning; qualitative analysis.

Results:

- Changes pre to post in student responses:
  - Study 2, 24% of responses; Study 1, 46% of responses
- Direction of changes pre to post:
  - Study 2, 16.7% more change in one direction; Study 1, 9% more change in one direction

Take-aways: (a) Study 2 had half as many changes in student responses (BT), and (b) Study 2 changes (BT) tended to be almost twice as many in a single direction.

### Study 3: Backward Transfer Influences on Different Ability Levels of Covariational Reasoning

9<sup>th</sup> and 10<sup>th</sup> grade students ( $N=18$ ), 1 teacher; dynamic reps of distance-time quad functions, strong emphasis on covariational reasoning; pre/post test on linear function reasoning; qualitative analysis.

Results:

- Top-level reasoning productively changed, more sophisticated, but not more correct
- Mid-level reasoning productively changed, more covariational and more correct
- Low-level reasoning productively changed, better set up for being covariational in the future
- Procedural reasoning unproductively changed, stopped relying on procedure, less correct

Take-aways: (a) all reasoning levels can experience productive BT, (b) procedure-grounded reasoning can experience BT influences from conceptual focused instruction that initially looks unproductive.

### Study 4: Backward Transfer Influences on Different Levels of Covariational Reasoning

9<sup>th</sup> and 10<sup>th</sup> grade students ( $N=25$ ), 3 co-teachers; dynamic reps of distance-time quad functions, strong emphasis on covariational reasoning; pre/post test on linear function reasoning; quantitative analysis.

Results: Statistically significant productive changes (BT) pre to post in responses to linear and quadratic problems; BT effect size small.

Take-away: First statistically significant evidence that effective quadratics instruction can also productively influence students' linear function reasoning (BT).

### Main Take-Aways:

- Students' ways of reasoning about concepts learned previously can change when students learn about new concepts. This can happen regardless of whether the teaching is or is not deliberately designed to produce change influences (i.e., BT influences).
- Deliberate efforts to influence students' ways of reasoning about concept learned previously with instruction about new concepts may actually lead to fewer changes in students' prior ways of reasoning, but changes that are more consistent (i.e., more consistent BT influences).
- All ability levels of students' ways of reasoning about concepts learned previously can be productively influenced, but the resulting changes may be different (i.e., different BT effects across ability levels).
- BT influences on high- and low-ability levels may be more difficult to pick up because correctness of performance remains unchanged.
- Although BT effect size of statistically significant productive changes in students' prior ways of reasoning about concepts learned previously is small, the effect is essentially cost free (i.e., the effect sizes of BT may be small, but the cost is low, so its a worthwhile goal).

### Main Implications for Research and Practice:

- Given that our research suggests BT is ubiquitous, generating more teacher awareness of BT is warranted.
- BT is virtually an unexplored research frontier and more research on all aspects of BT is warranted.