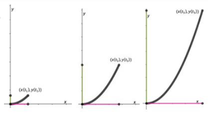
Task Design Principles to Support Middle Schoolers’ Graphical Reasoning

Teo Paoletti, Allison L. Gantt, Srujana Vinaykumar Acharya, & Claudine Margolis

University of Delaware

Introduction



- **Covariational reasoning** entails conceptualizing two quantities changing together (Thompson & Carlson, 2017). **We conjectured building middle-school students’ covariational reasoning would support their algebraic knowledge for graphs**.
- Prior research has argued :
  1. Students should **connect reasoning** with covarying quantities in real-world situations and graphs (Paoletti et al., 2023).
  2. Student should consider the **amounts of change** of one quantity with respect to another to describe quantitative relationships (Carlson et al., 2002).
- We describe **two new principles of task design** to support students’ graphical meanings via covariational reasoning:
  1. Embed **recursive opportunities** for students to bridge their reasoning across situational contexts and graphical representations.
  2. Initially emphasize relationships that **increase by smaller amounts** (i.e., “grow by less”).

Research Question

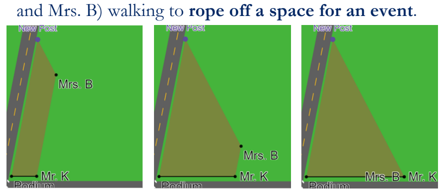
- How can middle-school students’ covariational reasoning serve as a **foundation** for their development of algebraic reasoning and knowledge related to graphs?

Methods

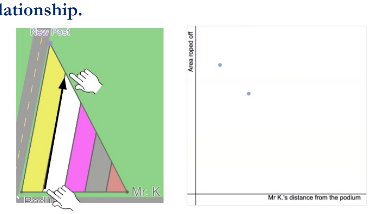
- We developed our conjectures through **four small-group teaching experiments** focused on supporting students’ graphical reasoning via covariation.
- We designed tasks in the Desmos platform. We built models of students’ mathematics using conceptual analysis to understand their activity and **develop task design principles**.
- We present a **case study** from one teaching experiment where two sixth-grade students (Sebastian and Tom) engaged with the *Growing Trapezoid Task*.

Finding: Principle 1

- The *Growing Trapezoid Task* involved two teachers (Mr. K and Mrs. B) walking to **rope off a space for an event**.



- We asked Sebastian and Tom to coordinate **Mr. K’s distance from the podium** and the **area of the space.**
- As the students moved **back and forth** between their situational and graphical meanings, they built connections toward **stronger graphical reasoning**.
- **Situationally**, Sebastian and Tom described that, as Mr. K’s distance from the podium increased, the **total area would also increase.**
- **Graphically**, however, they did not explicitly refer to the quantity of area. Tom pointed to several heights of the shape and **plotted points showing a decreasing relationship.**

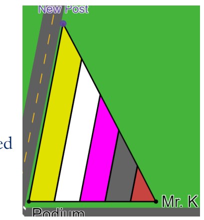


- We **revisited the situation again**, focusing on describing a segment that could represent the total area (in purple at right). The students used this to **revise their graph to show an increasing relationship** between both quantities.

> Sebastian: As Mr. K’s distance is increasing and the jumps are getting smaller, [the total area]’s still getting bigger [*places hands orthogonally*], no matter how far Mr. K walks.

- When we prompted Sebastian and Tom to describe connections between situational and graphical quantities **iteratively**, they provided stronger justifications for their graphs.

Finding: Principle 2

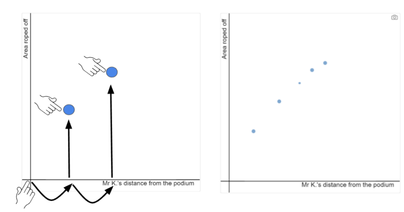


- We note as Mr. K’s distance increased by equal amounts, the total area **increased by smaller amounts**.
- We found that this “growing by less” relationship presented **productive complexity** for Tom and Sebastian as they differentiated between an **increasing total and decreasing amounts of change**.
- When first engaging with the situation, Tom noticed that “each jump is consecutively getting **smaller**.” Similarly, Sebastian said he knew the graph would “go big first” and then “go down **smaller and smaller**.”
- These descriptions both emphasize the **decreasing amounts of change** but do not draw connections to increasing total area.
- When the students re-examined a segment to match the quantity of total area, Sebastian explained the **growing by less relationship** in greater detail.

> Sebastian: This is kind of like reverse psychology to me because we start off with big jumps and started to decrease with smaller jumps, but the area starts to get bigger as those jumps go on.

- Tom created a **revised graph** to show the growing by less relationship, plotting a series of points one at a time.

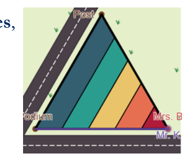
> Tom: First I am gonna start out with a big jump [*plotting first point*] like that, and then we’re going to go here and then it’s a smaller jump [*plotting second point*]…



- The growing by less relationship presented an **opportunity** for Tom and Sebastian to actively reflect on the **relationships between total area and amounts of change of area** as they created their graph.

Conclusions & Directions

- Our findings support two task design principles for supporting students to graph via covariation:
  1. Embed **recursive opportunities** for students to bridge their reasoning across situational contexts and graphical representations.
  2. Initially emphasize relationships that **increase by smaller amounts** (i.e., “grow by less”).
- From our case study, we saw that these principles led to **productive shifts** in students’ activity and in how students reflected on their own thinking (e.g., “This is kind of like reverse psychology”).
- To **further test these principles**, we redesigned the *Growing Trapezoid Task* and piloted it through **three whole-class teaching experiments** in late May 2023.



- We are engaged in **ongoing analysis** to understand the impact of these principles on students’ activity.

For **more updates** about our project’s progress and new findings, we invite you to visit our website: sites.udel.edu/projectcare

Selected References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352. https://doi.org/10.2307/4149958

Paoletti, T., Gantt, A. L. & Corven, J. (2023). A local instructional theory for middle school students’ emergent reasoning. *Journal for Research in Mathematics Education, 54*(3). https://doi.org/10.5951/jresematheduc-2021-0066

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.

Acknowledgements

- This material is based upon work supported by the NSF under Grant No. DRL-2142000