Differentiating linear function instruction for 8th grade students

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This presentation is based upon work supported by the National Science Foundation under Grant No. DRL-1252575

ationale: Why differentiate?

racking:

Traditional way to deal with student diversity in the US

Has negative effects on students and create opportunity gaps (Flores, 2007; Oakes, 2005) that can result in achievement gaps

ifferentiation:

An alternative way to address students' diverse learning needs, but largely untested in Math Ed

Rare (or absent) in secondary math classrooms (Gamoran & Weinstein, 1998)

esearch on Differentiation

earhart & Saxe (2014):

- 4th & 5th grade intervention focused on developing students' understanding of number lines
- Flexible class structure, problems with multiple entry points, partner work tiered for difficulty
- Significantly greater learning gains than students in comparison classrooms

ieso (2006):

- Quasi-experimental study with 31 4th & 5th grade teachers and 645 students, focused on data representation & analysis unit
- Statistically significant higher scores for regularly-achieving and high-achieving students; low-achieving students' increased scores but not significantly

ther findings in other fields: positive influence on various spects of reading (fluency, comprehension) and on students' elf-concepts

)R^{2}eAM Project

vestigating **D**ifferentiated Instruction and elationships between **R**ational Number Knowl**e**dge d **A**lgebraic Reasoning in **M**iddle School

urposes:

- Study how to differentiate instruction for cognitively diverse middle school students
- Study how students' rational number knowledge and algebraic reasoning are related

ifferentiation, to us

efinition: **proactively tailoring instruction to students' mathematical inking while aiming to develop a cohesive classroom community (cf. omlinson, 2005)**

- **mplement on-going assessment to et to know students' thinking**
- **ontinually explore and clarify earning goals for students**
- **rovide choices**
- **se flexible grouping for different urposes**
- **stablish norms** (e.g., re-think fairness)
- **Interact responsively during class meetings** (Dyer & Sherin, 2015; Jacobs & Empson, 2015)
- **Use thinking from individuals and small groups to shape whole classroom discussions** (Fennema et al., 1996; Jacobs & Empson, 2015; Leatham et al., 2015; Tomlinson, 2005)

verview of IDR^{2}eAM Project

hase I (2 yrs): Three design experiments after school
- 6-9 7th and 8th grade students selected for cognitive diversity
- 18 episodes each

hase II (1 yr): Teacher Study Group (TSG) with 15 iddle school teachers from around Indiana
- Summer workshop, 8 monthly meetings, summer workshop

hase III (2 yrs +): Two design experiments co-taught ith teachers who participated in the TSG
- 26-27 day units using Connected Mathematics Project materials (CMP3)
- 20-21 students in each class

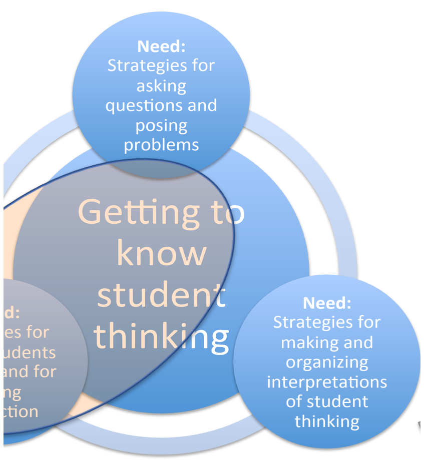
urpose of the talk

search Question: **What cognitive and affective**
luences did tiering instruction have on a class of 20
ghth grade pre-algebra students during a 4-day
structional segment focused on linear functions?

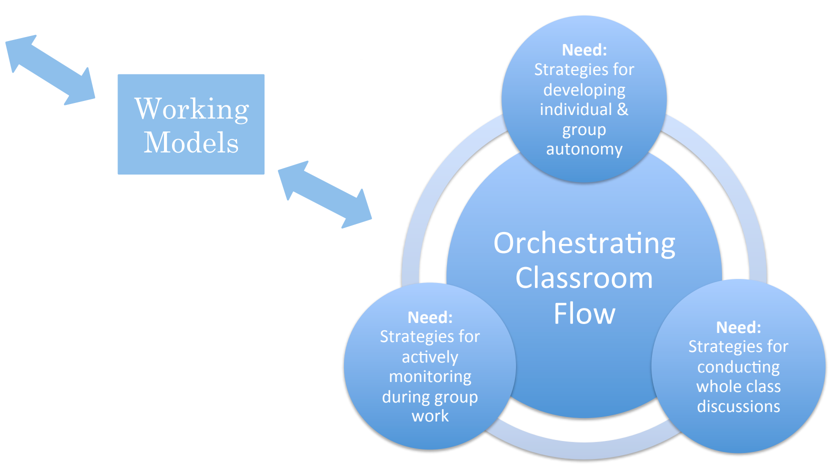
iering Instruction: providing different activities/
roblems to groups of students based on formative
ssessment of students' ways of thinking

his segment occurred on Days 20-23 of a 27-day unit on
quivalence, using the "Say It With Symbols" materials
om 8th grade CMP3

Our emerging theory of differentiating mathematics instruction



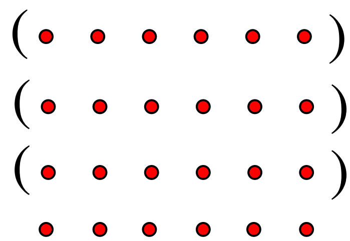
re here today



Jnits Coordination

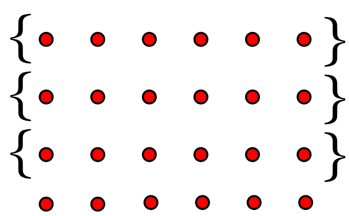
- **omposite unit:** a unit of units.
- **Inits coordination:** distribute the lements of one composite unit across the lements of another composite unit.
- hree stages of levels of units coordination:
  - Influences fraction knowledge, proportional reasoning, equation writing, combinatorial reasoning…
  - Transition between them can be protracted (Steffe & Cobb, 1998; Steffe & Olive, 2010)
- **roblem:** There are 18 flower pots. Each ne can be filled with 6 pounds of soil. How uch soil is needed to fill all of them?

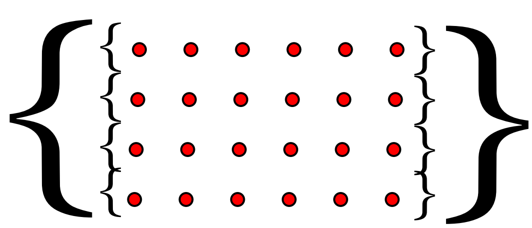
- **Stage 1:** 6 is a composite unit
  - But, no multiplicative relationship between 1s and the 6
  - Can track groups of 6s and 1s in activity, often counting on by 1s past known skip-counting patterns



Jnits Coordination

- **tage 2:** 6 is a composite unit with a ultiplicative relationship between 1s nd 6
  - Takes the coordination of 18 6s as given and can break apart 6s to reason with them
- **tage 3:** 6 is a composite unit and erations of 6, such as 24, are omposite units of composite units
  - Takes coordination of 18 6s as a three-levels-of-units structure
  - Can reason with 6s as if 1s but not lose them as 6s
  - Can switch to view the 24 as six 4s





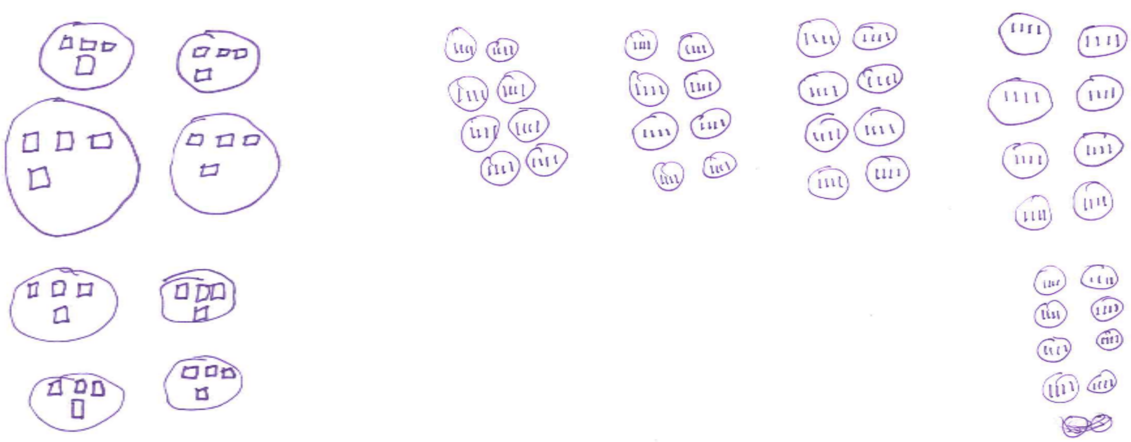
ate Problem: There are 4 cans of juice in a package and 8 packages in a box. :rate contains 6 boxes. How many cans of juice are in a crate, and can you ıw a picture to show how you know?

Units Coordination Framework		
Stage 1	Students can take one level of units as given, and may coordinate two in activity.	Often must "build up from ones" to nest quantities, and cannot keep multiple levels in mind when operating further.
Stage 2	Students can take two levels of units as given, and may coordinate three in activity.	They can iterate composite units, so a package can be both 1 package and 4 cans even as they're building up 8 of them into a box. Sometimes conflate boxes and packages when working with a crate.
Stage 3	Students take three levels of units as given, and can flexibly switch between three level structures.	They can usually move flexibly among packages, boxes and crate without conflation.

ere are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes. w many cans of juice are in a crate, and can you draw a picture to show how you know?

Units Coordination - Alyssa

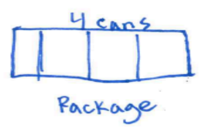
ıge 1	Students can take one level of units as given, and may coordinate two in activity.	Often must build up from ones to nest quantitie and cannot keep multiple levels in mind when operating further.



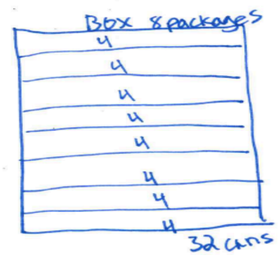
ere are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes.
w many cans of juice are in a crate, and can you draw a picture to show how you know?

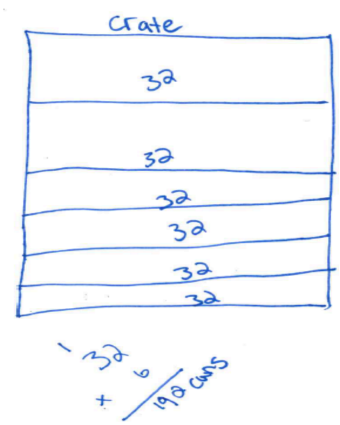
Units Coordination – Joanna

ge 2	Students can take two levels of units as given, and may coordinate three in activity	They can iterate composite units, so a package can be both package and 4 cans even as they're building up 8 of them a box. Sometimes they conflate boxes and packages when working with a crate.



ay. So are a box and a package the
Or are they different?
're the same.
e same. Okay.
, like kind of. In a box there's 8 cans,
tions of 4, versus 6 sections of 32.





pring 2017 design experiment

Participating classroom: 8^{th} grade pre-algebra, 20 students
Selected two other classrooms for comparison: 23 students
Gathered initial information:

- Initial written assessments on units coordination and fractions knowledge
- Individual interviews with 31 students

Results:

Units Coordination Level	Participating Class	Comparison Classes
Stage 1	5	3
Stage 2	13	15
Stage 3	2	5

Selected 6 participating focus students (two stage 1, two stage 2, two stage 3) and 6 comparison focus students matched on units coordination and aspects of fraction knowledge

ata Collection and Analysis

he Unit, Say It With Symbols: writing and combining equivalent expressions, solving equations, **writing expressions to represent linear relationships**

ata:

Daily: whole-class and small group video, copies of written student work

Middle of Unit – 6 focus students from participating class

End of the Unit – 12 focus students – participating and comparison

nalysis:

evelopment of second-order models of student thinking – nalysis of all data sources, discussions with research team

xample of tiering instruction

ction 4.1 of Say it With Symbols introduces the following uation:

Iagnolia Middle School needs to empty their pool for resealing. Ms heodora's math class decides to collect data on the amount of water in the ool and the time it takes to empty it.

he class writes the following equation to represent the amount of water *w* n gallons) in the pool after t hours.

$$w = -250(t - 5)$$

What information does the -250 represent?

What units should you use for -250?

What information does (t – 5) represent? What units should you use for (t – 5)?

What units should you use for -250(t – 5)? Explain.

Tiering the Pool Problem

Version A

2) Pauline comes in at noon to fill the pool. There are already 1080 gallons of water in it. She fills it at 12 gal/min:

$$w = 12t + 1080$$

Version B

3) Pauline comes in at noon to fill another pool, and the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. She fills it at 12 gal/min:

$$w = 12(t - 5)$$

g) An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?

Current Findings: Overview

Tiering instruction seemed effective because students' work revealed conceptual difficulties with the delayed start time.

Some students made progress cognitively.

Tiering had an affective effect on some.

ummary of Days 20-23

) Worker filling an empty pool starting at noon, using a hose that
owed at a rate of 12 gallons per minute

ay 20
- Task 1 introduced
- Whole class discussion then small group work

ay 21
- Each small group given one of two tiered tasks (#2 and 3)

ay 22
- Continued group work
- Whole class discussion of both tasks, including
- Each group given one of two tiered tasks for homework (#4 and 5)

ay 23
- Small group work
- Whole class discussion of 4 and 5, focused on graphs

arrin and Kathy

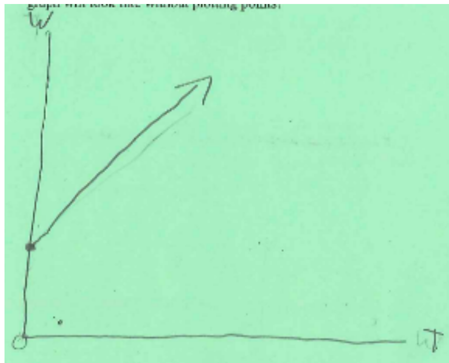
want to show you now two examples of students with very different
ys of thinking, both working at their edge.

The big idea: we wanted students to think about the coordination of both quantities (time, water in pool) as they were changing.

DARRIN: in interviews and school work, he struggled to coordinate multiple quantities but did well connecting context to representations and could simplify situations to help himself be successful

KATHY: Didn't see herself as good at math but was very engaged in class, thought deeply and asked great questions.

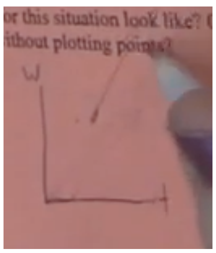
Darrin: Task 2, w = 12t + 1080

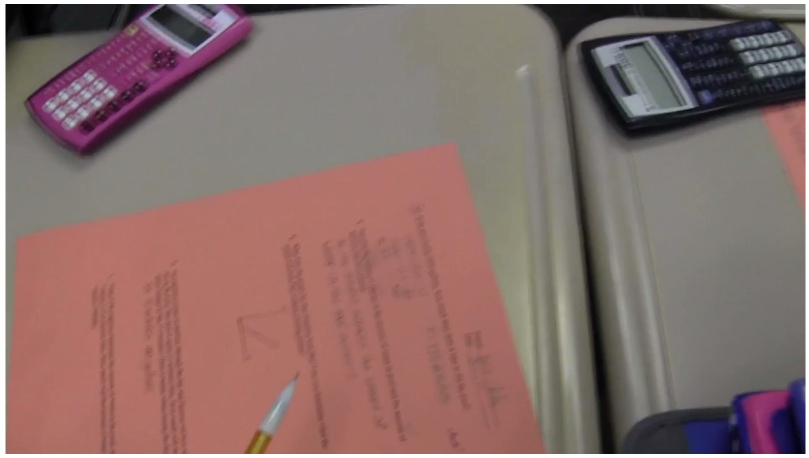


Darrin: Task 4, w = -12t

Kathy

Kathy’s initial graph for w = 12(t – 5)





Second attempt, ‘starting’ at 12 gallons, 6 minutes

Affective Influences

- CONCERNS
  - Several mentioned the possibility of students feeling ‘dumb’, but only one was a comment in the first person.
  - Lack of flexibility. Don’t hold them back if they’re ready to move on.
  - Group dynamics – one student didn’t want to feel more or less affinity for others based on how they were grouped.
- OPTIMISM
  - Student involvement might increase with greater match between level of understanding and level of assignments
  - Both struggling and advanced students could have the work they need.
  - Recognition that students are already aware of cognitive differences. This just addresses them.
  - Enthusiasm for learning from those who thought differently.

Thank you!

- With BIG thanks to other members of the **IDR²eAM** project team: Ayfer Eker, Mark Creager, Sharon Hoffman, Serife Sevis, Pai Suksak
- *What IDR²eAM stands for:* **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School
- **http://www.indiana.edu/~idream/** ← paper and references will be available here

Future Work

- Paper in preparation about our theory of differentiation based on Phase I & II
- Continued analysis of data from the two experiments in Phase III
- Paper in preparation based on the findings shared here today
- Proposal to present at 2019 NCTM meeting in collaboration with teachers from the two experiments in Phase III