Spatial Reasoning:
Activities and Questioning Strategies to Elicit Complex Reasoning in K-2

Lindsey Perry, PhD, @lindseyeperry

Eloise Aniag Kuehnert, PhD, @EAKuehnert

Leanne Ketterlin-Geller, PhD, @KetterlinGeller

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Measuring Early Mathematical Reasoning Skills (MMaRS) Project

Purpose: Develop formative assessment tools for teachers to use in K-2 to measure and monitor progress with numeric relational reasoning and spatial reasoning

Current activities:

- Developing learning progressions
- Conducting cognitive interviews with K-2 students

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Presentation Outline

1. Spatial reasoning
   - What is spatial reasoning and why is it important in K-2?
2. Questioning strategies
   - What are different question types and how can they be used in instruction to deepen children’s thinking?
3. Integrating spatial reasoning into K-2 instruction
   - What activities can I use to promote spatial reasoning?
   - How can I design questions within spatial reasoning activities to elicit student reasoning?

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Spatial Reasoning

What is spatial reasoning and why is it important in K-2?

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Spatial Reasoning

Spatial reasoning

- is consistently identified as one of the core components of early mathematics education (Clements, 2004 Engaging Young Children in Mathematics; NCTM, 2000; National Research Council, 2009).
- is the ability to interact with, navigate in, and understand one's environment (NRC, 2001, 2009)

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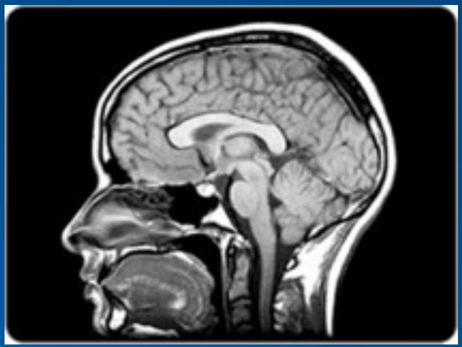
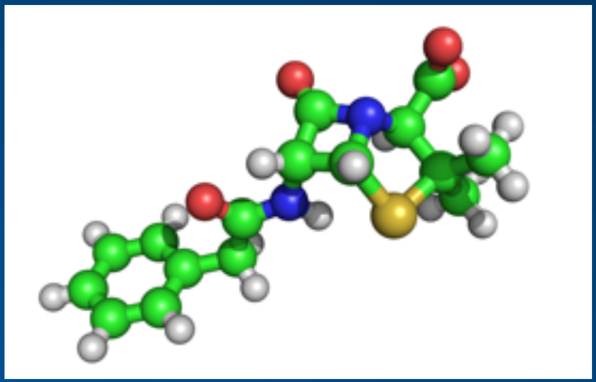
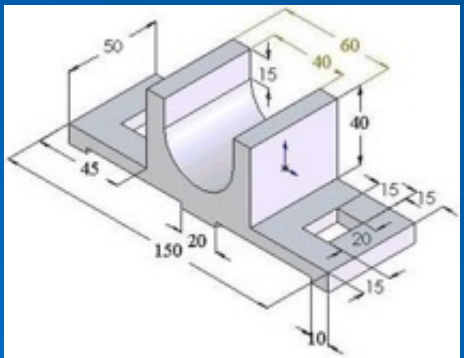
Why is spatial reasoning important?

Spatial reasoning is used everyday and in almost every profession.







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The links between spatial reasoning and mathematics are so strong that “spatial instruction will have a ‘two-for-one’ effect, yielding benefits in mathematics as well” (Verdine et al., 2014).

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Spatial Reasoning

Spatial Visualization

- Mentally create and transform two- and three-dimensional objects
- Develops between the ages of 3-7 and continues to improve with maturation

Spatial Orientation

- Views spaces or objects from different perspectives
- Develops between the ages of 3-6 and continues as students age

(Bishop, 1980; Blaut & Stea, 1974; Burnett & Lane, 1980; Clements & Battista, 1992; Connor & Serbin, 1980; Dalke, 1998; Eliot & Smith, 1983; Marmor, 1975; McGee, 1979; Michael et al., 1957; NRC, 2009; Pellegrino et al., 1984; Perham, 1978; Sarama & Clements, 2009; Tartre, 1990)

Spatial Reasoning in Grades K – 2

CCSS - Mathematics

K.G.6: Compose simple shapes to form larger shapes.

K.G.5: Model shapes in the real world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.1: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

1.G.2: Compose two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.

Texas Essential Knowledge and Skills - Social Studies

(K) 4B: locate places on the school campus and describe their relative locations; and

(1) 4A: locate places using the four cardinal directions; and

(1) 4B: describe the location of self and objects relative to other locations in the classroom and school.

(1) 5A: create and use simple maps such as maps of the home, classroom, school, and community;

(2) 5A: interpret information on maps and globes using basic map elements such as title, orientation (north, south, east, west), and legend/map keys;

(2) 5B: create maps to show places and routes within the home, school, and community.

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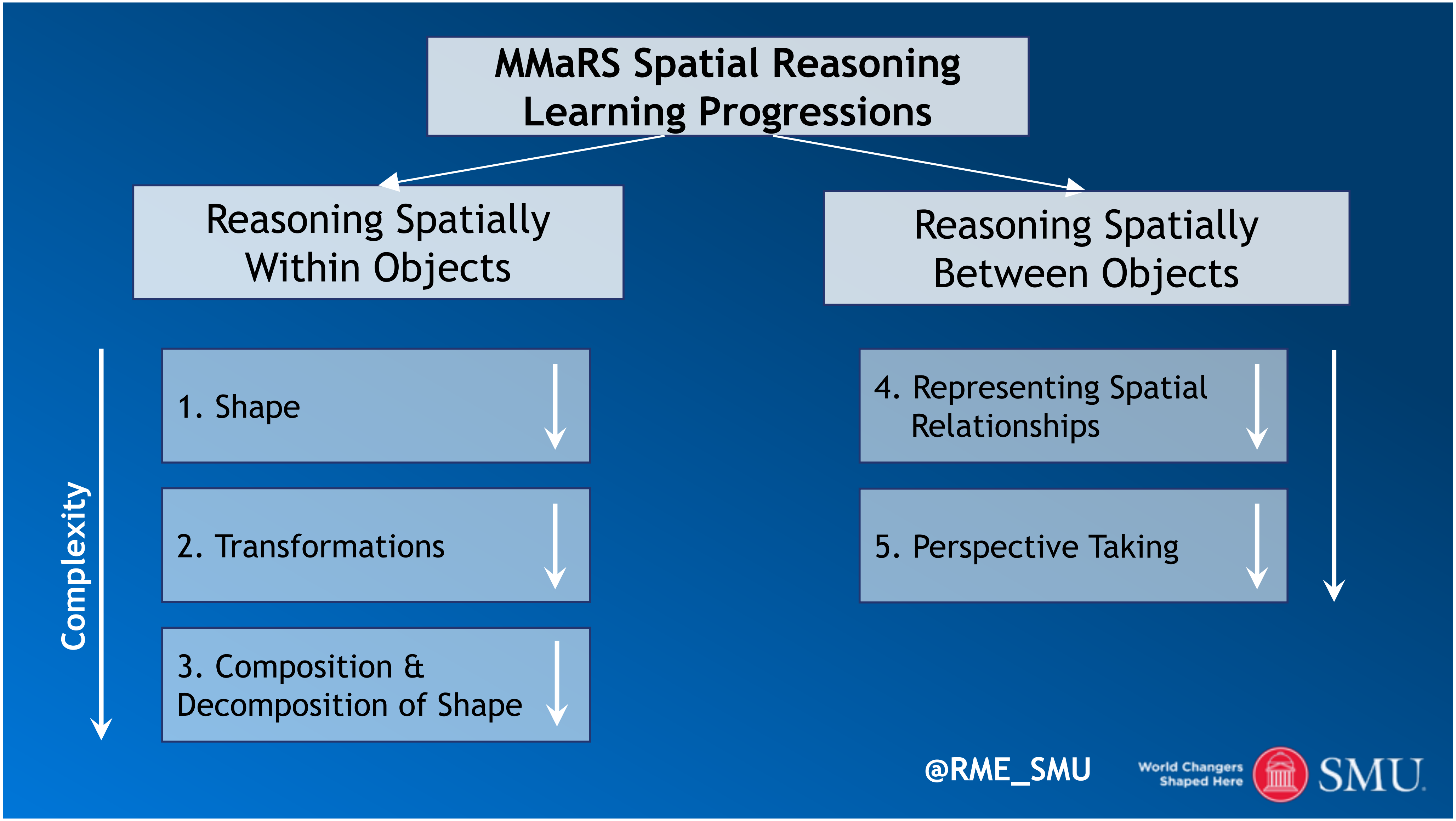
MMaRS Spatial Reasoning Learning Progressions
Reasoning Spatially Within Objects
Reasoning Spatially Between Objects
Complexity
1. Shape
2. Transformations
3. Composition & Decomposition of Shape
4. Representing Spatial Relationships
5. Perspective Taking
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Questioning Strategies

What are different question types and how can they be used in instruction to deepen children's thinking?

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Question Types

Question Type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	How many [trapezoids/rhombuses/triangles] are needed to make a hexagon?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	As you were finding ways to make a hexagon with pattern blocks, what did you think about?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	How are triangles related to [rhombuses/trapezoids/hexagons]?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that you have found all of the ways to make a hexagon?

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

Integrating spatial reasoning into PreK-2 instruction

- What activities can I use to promote spatial reasoning?
- How can I design questions within spatial reasoning activities to elicit student reasoning?

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Activity 1: I spy from this perspective!

Challenge:

- Make a prediction about where each photo was taken.
- Test your prediction by finding and standing where each photo was taken.

Activity 1: I spy from this perspective!

Question Type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	Which photo was taken from the farthest away?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	How did you decide this photo was taken or drawn from this perspective?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	How would the photo change if you moved closer to the location?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that the object looks smaller as you move away from the object?

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Activity 2: What would you see as you move?

Challenge:

- Determine the order of the photos based on the path taken.

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Activity 2: What would you see as you move?

Question Type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	Which photo was taken from the [front/side/back] view?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	Why is this photo after this photo?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	How is the front view different from the back view?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that the pictures are in the right order?

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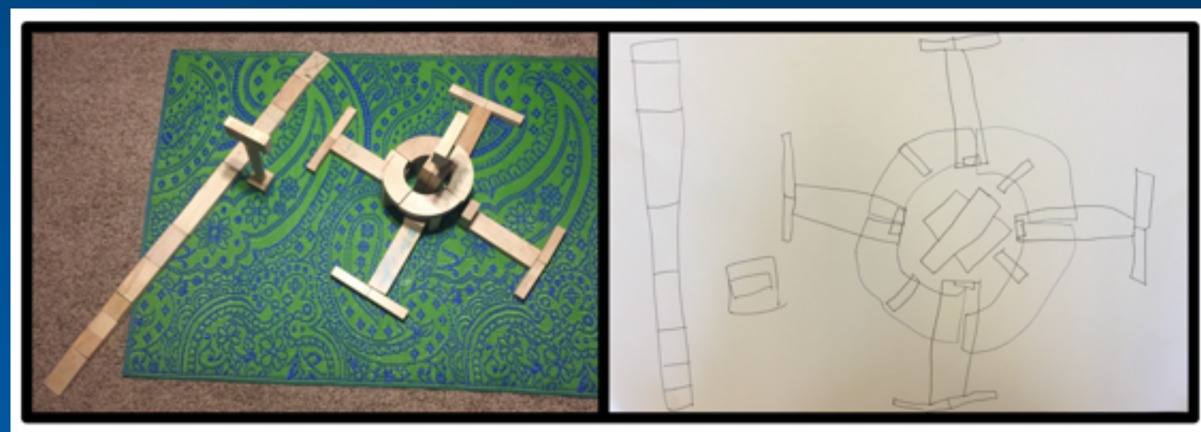
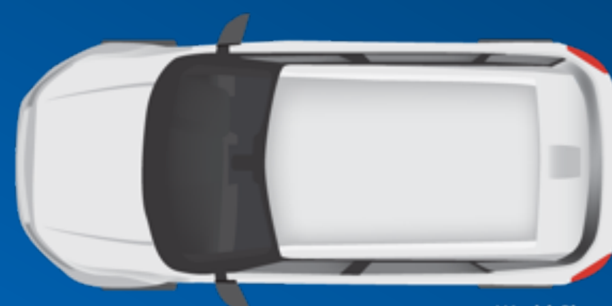
Activity 3: What do you see from a top view?



Challenges:

- What is this object/space?
- Draw the top view.



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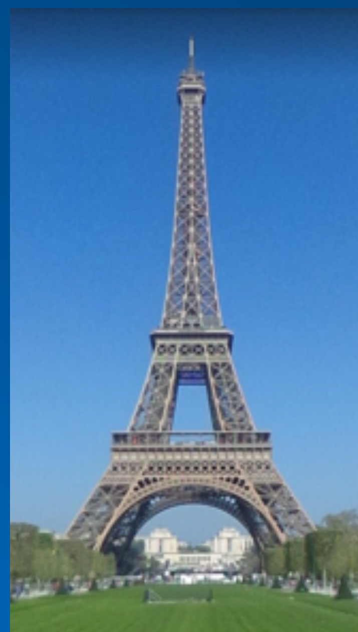
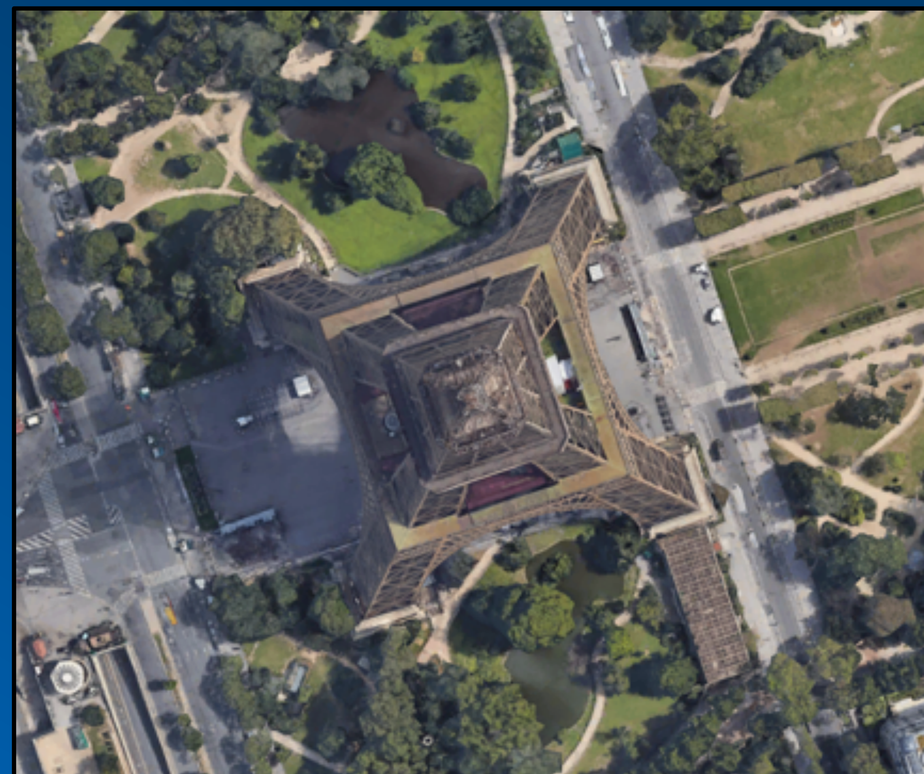
Activity 3: What do you see from a top view?

Pretend you are a bird. What would you see if you were flying directly above the Eiffel Tower?





Tweet your drawings using #MMaRS!

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Activity 3: What do you see from a top view?

Question Type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	What do you see from the top view?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	How did [what child saw] help you draw your photo?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	How are the shapes that you see different in the top view than in a front view?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How can you tell that this picture is of a [ball/cup] from the top view?

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Activity 4: Where should I stand to create an optical illusion?

Challenge:

- Create fun images that utilize distance and perspective to create optical illusions.

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Activity 4: Where should I stand to create an optical illusion?

- Focus on *predictions* and *reasoning*
- Emphasize the use of spatial language

"If you're far away from something, it'll shrink."

"Minis are far away big ones."

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Activity 4: Where should I stand to create an optical illusion?

Question Type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	Which bus is larger?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	Can you explain more about how you created the optical illusion?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	What happens in your photo when a bus moves closer to the camera?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How can you tell that the person becomes larger in the photo when the person moves closer to the tree?

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Conclusions

- Spatial reasoning is an important construct for students in Grades K-2.
  - Get your students engaged in spatial reasoning within their environments, such as in their classrooms or at home.
  - Incorporate student choice and a variety of representations and contexts within spatial activities.

- Use a mix of the 4 question types (NCTM, 2014) to elicit students' spatial reasoning.

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