CAREER: Proof in Secondary Classrooms: Decomposing a Central Mathematical Practice

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Abstract

This design and development research study focuses on secondary students' success with mathematical proof. The goal of this project is to develop a new and improved intervention to support the teaching and learning of proof. This study takes as its premise that if we introduce proof by first teaching students particular sub-goals of proof, then students will be more successful in constructing their own proofs.

Geometry Proof Scaffold

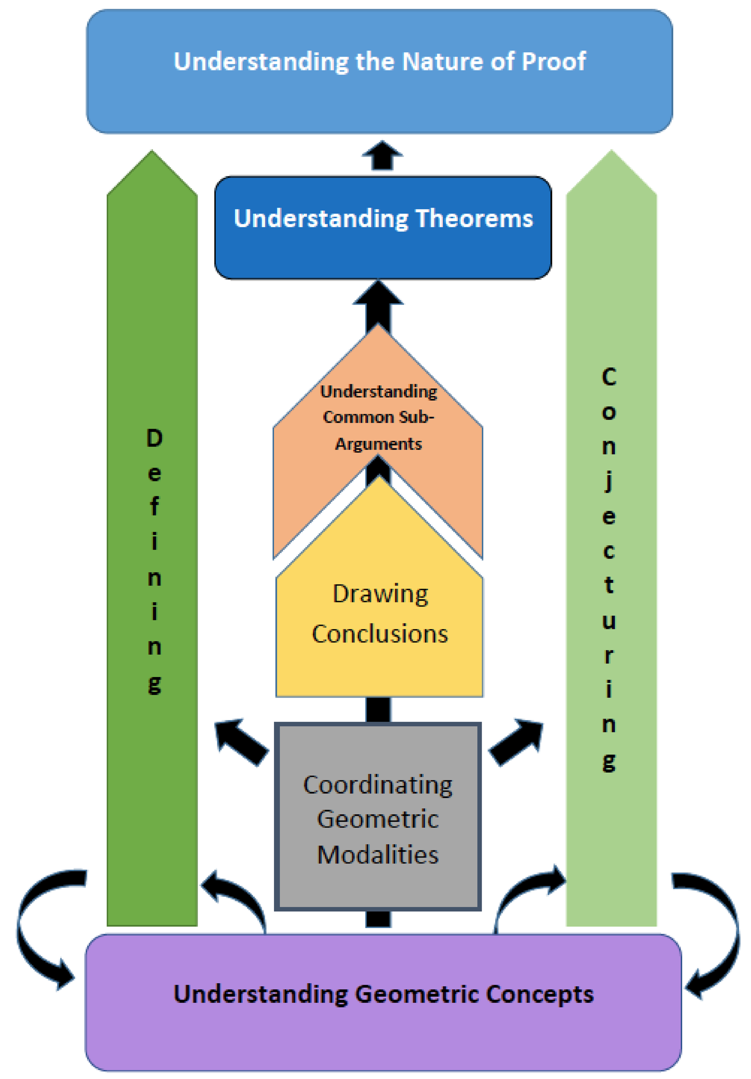
Understanding Geometric Concepts	This sub-goal highlights the importance of understanding the building blocks of geometry.
Coordinating Geometric Modalities	This sub-goal highlights the ways in which the mathematics register draws on a range of modalities.
Defining	This sub-goal highlights the nature of definitions, their logical structure, how they are written, and how they are used.
Conjecturing	This sub-goal recognizes that conjecturing is an important part of mathematics and proving.
Drawing Conclusions	This sub-goal presents the idea of an open-ended task that leads to conclusions that can be drawn from given statements and/or a diagram.
Understanding Common Sub-arguments	This sub-goal recognizes that there are common short sequences of statements and reasons that are used frequently in proofs and that these pieces may appear relatively unchanged from one proof to the next.
Understanding Theorems	This sub-goal highlights the nature of theorems, their logical structure, how they are written, and how they are used.
Understanding the Nature of Proof	This sub-goal highlights the nature of proof, proof structure, and how the laws of logic are applied.

PISC Lesson Topics

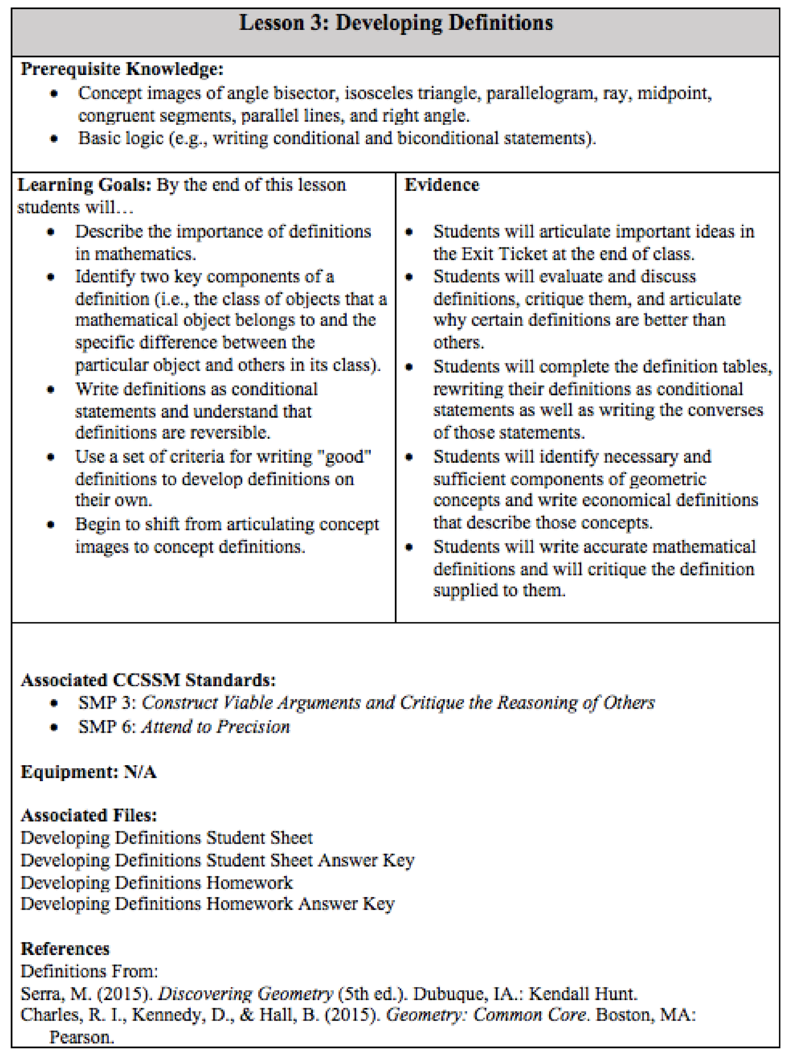
PISC Lessons 1-8	
Lesson 1: Getting Started in Euclidean Geometry	This lesson serves as an introduction to Euclid and the Euclidean system. Basic notation and geometry terms, undefined terms, and important assumptions for the course, including postulates and what can be assumed from diagrams, are covered.
Lesson 2: Investigating Geometric Concepts	Students use patty paper to explore important geometric concepts and begin to draft definitions and conjectures about these concepts. The geometric concepts selected appear frequently in geometry proof and they are concepts that students sometimes have trouble with.
Lesson 3: Developing Definitions	Students learn about features of a "good" mathematical definition and practice defining and critiquing others' definitions.
Lesson 4: Coordinating Geometric Modalities – Day 1	After being introduced to (or reviewing) basic geometric notation, students begin to coordinate geometric modalities, translating between notation, diagrams, and verbal or written descriptions.
Lesson 5: Coordinating Geometric Modalities – Day 2	Students sketch and label diagrams that require coordination of multiple "Given" features (e.g., segments intersect but are not perpendicular).
Lesson 6: Coordinating Geometric Modalities – Day 3	After selecting statements that correspond with markings on a complex diagram, students work with partners to "Sketch My Figure." That is, students take turns trying to describe diagrams that only they can see so that their partner draws the same figure.
Lesson 7: Drawing Conclusions – Day 1	Students are provided with a "Given" statement and they are asked to draw a valid conclusion from the information provided.
Lesson 8: Drawing Conclusions – Day 2	Students engage in two types of tasks: (1) Students draw multiple conclusions from one "Given" statements and (2) Students work backwards to determine what must have been "Given" to warrant the provided conclusion.

PISC Lessons 9-16	
Lesson 9: Deductive Structure	Students complete a reading about deductive structure which describes mathematical proof, theorems, and inductive versus deductive reasoning. Next, they prove simple theorems about linear pairs, right angles, and vertical angles.
Lesson 10: Proving Simple Theorems	Students prove simple theorems about parallel lines cut by a transversal, including the triangle angle sum theorem.
Lesson 11: Common Sub – Arguments	Students learn five common sub-arguments that they will frequently use in their proofs (i.e., perpendicular lines, vertical angles, linear pairs, line segment bisectors, and alternative interior angles).
Lesson 12: Hidden Triangles – Day 1	Students begin to explore "hidden triangles" whereby one student "director" tries to get their group-mates, the "drawers," to make a congruent copy of a triangle that only they can see.
Lesson 13: Hidden Triangles – Day 2	After establishing that only three parts of the triangle are needed to draw congruent triangles, students explore each potential case in a systematic way, using the hidden triangles and changing up roles as directors and drawers.
Lesson 14: First Triangle Proofs	After reviewing the triangle congruence criteria and reading about different proof formats, students experiment with the different proof formats and begin their work on proofs, making use of their Drawing Conclusions skills and the common sub-arguments.
Lesson 15: Conjecturing about Parallelograms – Day 1	Students use a Geogebra applet to conjecture about the sides and angles of parallelograms. When possible, they prove their conjectures.
Lesson 16: Conjecturing about Parallelograms – Day 2	Students use a Geogebra applet to conjecture about the diagonals of a parallelogram and a rectangle. When possible, they prove their conjectures.

GPS Visual



Sample Lesson Plan



Lesson 3: Developing Definitions

Prerequisite Knowledge:
- Concept images of angle bisector, isosceles triangle, parallelogram, ray, midpoint, congruent segments, parallel lines, and right angle.
- Basic logic (e.g., writing conditional and biconditional statements).

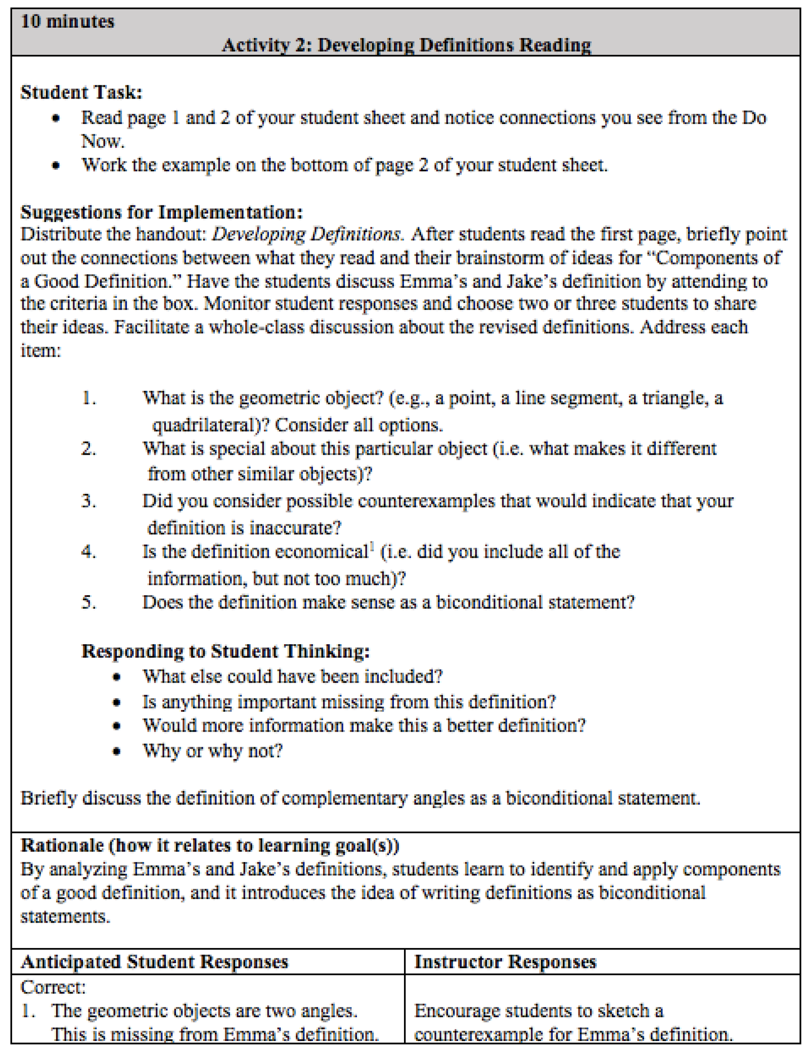
Learning Goals: By the end of this lesson students will…	**Evidence**
• Describe the importance of definitions in mathematics. • Identify two key components of a definition (i.e., the class of objects that a mathematical object belongs to and the specific difference between the particular object and others in its class). • Write definitions as conditional statements and understand that definitions are reversible. • Use a set of criteria for writing "good" definitions to develop definitions on their own. • Begin to shift from articulating concept images to concept definitions.	• Students will articulate important ideas in the Exit Ticket at the end of class. • Students will evaluate and discuss definitions, critique them, and articulate why certain definitions are better than others. • Students will complete the definition tables, rewriting their definitions as conditional statements as well as writing the converses of those statements. • Students will identify necessary and sufficient components of geometric concepts and write economical definitions that describe those concepts. • Students will write accurate mathematical definitions and will critique the definition supplied to them.

Associated CCSSM Standards:
- SMP 3: *Construct Viable Arguments and Critique the Reasoning of Others*
- SMP 6: *Attend to Precision*

Equipment: N/A

Associated Files:
Developing Definitions Student Sheet
Developing Definitions Student Sheet Answer Key
Developing Definitions Homework
Developing Definitions Homework Answer Key

References
Definitions From:
Serra, M. (2015). *Discovering Geometry* (5th ed.). Dubuque, IA.: Kendall Hunt.
Charles, R. I., Kennedy, D., & Hall, B. (2015). *Geometry: Common Core*. Boston, MA: Pearson.



10 minutes
Activity 2: Developing Definitions Reading

Student Task:
- Read page 1 and 2 of your student sheet and notice connections you see from the Do Now.
- Work the example on the bottom of page 2 of your student sheet.

Suggestions for Implementation:
Distribute the handout: *Developing Definitions.* After students read the first page, briefly point out the connections between what they read and their brainstorm of ideas for "Components of a Good Definition." Have the students discuss Emma's and Jake's definition by attending to the criteria in the box. Monitor student responses and choose two or three students to share their ideas. Facilitate a whole-class discussion about the revised definitions. Address each item:

1. What is the geometric object? (e.g., a point, a line segment, a triangle, a quadrilateral)? Consider all options.
2. What is special about this particular object (i.e. what makes it different from other similar objects)?
3. Did you consider possible counterexamples that would indicate that your definition is inaccurate?
4. Is the definition economical[1] (i.e. did you include all of the information, but not too much)?
5. Does the definition make sense as a biconditional statement?

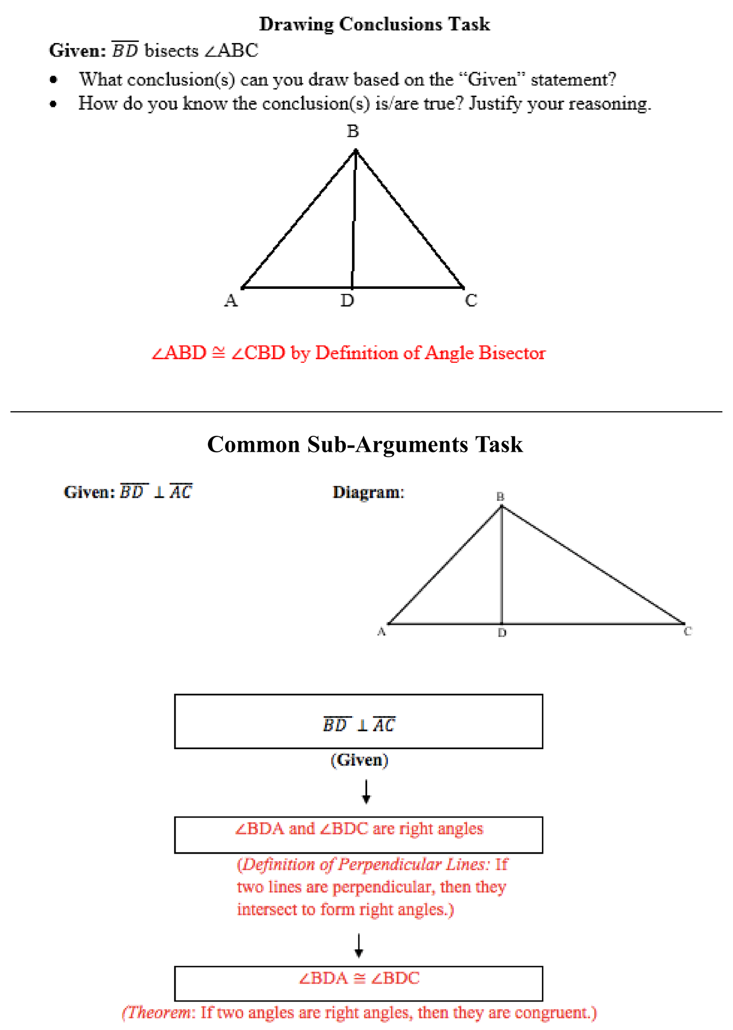
Responding to Student Thinking:
- What else could have been included?
- Is anything important missing from this definition?
- Would more information make this a better definition?
- Why or why not?

Briefly discuss the definition of complementary angles as a biconditional statement.

Rationale (how it relates to learning goal(s))
By analyzing Emma's and Jake's definitions, students learn to identify and apply components of a good definition, and it introduces the idea of writing definitions as biconditional statements.

Anticipated Student Responses	**Instructor Responses**
Correct: 1. The geometric objects are two angles. This is missing from Emma's definition.	Encourage students to sketch a counterexample for Emma's definition.

Sample Tasks



Research Questions

1. How do teachers *introduce* proof in geometry?
2. When engaging in lesson study based on introducing proof by first teaching particular sub-goals of proof, how do teachers respond to and execute the lesson plans?
3. How do students respond to these lessons?
4. How do students in the control and experimental groups think about proof and perform on a set of proof tasks?

Project Timeline

Planning Year	*Baseline Data Collection & Lesson Piloting*	*Professional Development & Summer Lesson Study*	*Pilot Lessons (Core Teachers)*	*Pilot Lessons(Again) (Core Teachers)*	*Publication & Dissemination*
Phase I	**Phase II**	**Phase III**	**Phase IV**	**Phase V**	**Phase VI**
2015-2016	2016-2017	Spring & Summer, 2017	2017-2018	2018-2019	2019-2020

Acknowledgments

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