TEACHERS' SUPPORT OF STUDENTS' INSTRUMENTATION IN A COLLABORATIVE, DYNAMIC GEOMETRY ENVIRONMENT [1]

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We report on a case study that seeks to understand how teachers' pedagogical interventions influence students' instrumentation and mathematical reasoning in a collaborative, dynamic geometry environment. A high school teacher engaged a class of students in the Virtual Math Teams with GeoGebra environment (VMTwG) to solve geometrical tasks. The VMTwG allows users to share both GeoGebra and chat windows to engage in joint problem solving. Our analysis of the teacher's implementation and students' interactions in VMTwG shows that his instrumental orchestration (Trouche, 2004, 2005) supported students' instrumentation (Rabardel & Beguin, 2005) and shaped their movement between empirical explorations and deductive justifications. This study contributes to understanding the interplay between a teacher's instrumental orchestration and students' instrumentation and movement towards more deductive justifications.

Keywords: dynamic geometry, teacher practice, student reasoning, mathematical discourse, collaboration

INTRODUCTION

Dynamic geometry environments (DGEs) afford learners the ability to construct, visualize, and manipulate geometric objects, relations, and dependencies. These affordances support empirical explorations and theoretical justifications or proofs (Christou, Mousoulides, & Pittalis, 2004). In DEGs, empirical explorations are experienced immediately while the need to formulate proofs is latent and requires either a learner's disposition towards justification or pedagogical intervention. Pedagogically motivated transitions from empirical explorations to theoretical justifications depend on carefully designed tasks, teacher guidance, and classroom climates that support conjecturing and deductive justifications (Arzarello, Olivero, Paola, & Robutti, 2002; Hölzl, 2001; Öner, 2008).

Technologically-enabled collaboration, which for mathematics teaching and learning supports social conjecturing and justification, can occur in computer-supported, collaborative-learning (CSCL) environments (Öner, 2008; Silverman, 2011). In such CSCL environments, mathematics education researchers can investigate how pedagogical interventions support learners' appropriation of the environment and promote their movement between exploration and deductive justification. Knowing how to support this appropriation and promote this movement will enable mathematics education researchers and educators to realize the potential of DGEs to improve geometry learning and of CSCL environments to engage learners in developing mathematical ideas through online collaboration that parallel the real-world online, collaborative work of mathematicians, including Fields Medal recipients (Alagic & Alagic, 2013).

From a larger design-experiment project, this paper reports on a case study that aims to understand and describe a teacher's pedagogical interventions–using the framework of instrumental

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orchestration (Trouche, 2004, 2005)–that support learners' instrumentation (Rabardel & Beguin, 2005) and shape their movement between exploration and deductive justification as they work on geometric tasks in a CSCL environment. We understand pedagogical interventions to be instructional actions initiated by teachers that precede, invite, sustain, monitor, or reflect on students' activity. These actions are organized according to instrumental orchestration to understand the teacher's support of students' instrumentation of the Virtual Math Teams with GeoGebra environment (VMTwG). By instrumentation and "movement between exploration and deductive justification," we mean discursive, recursive actions in VMTwG through which learners are motivated to notice relations while manipulating objects to develop and communicate convincing arguments that satisfy their peers about the relations. A guiding research question frames our analyses: What pedagogical interventions promote learners' instrumentation and their movement between exploration and deductive justification? To understand students' instrumentation and their movement between exploration and deductive justification, we analyze a teacher's pedagogical interventions and his students' consequent actions.

RELATED LITERATURE AND CONCEPTUAL FRAMEWORK

When integrating technology in the learning, the instrumental approach provides insight into understanding how learners interact with technological tools. Some studies investigate how learners appropriate technological tools while learning mathematics (Guin & Trouche, 1998; Kieran & Drijvers, 2006). To understand this appropriation, some researchers examine the role that mathematics teachers play when integrating technology (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Sutherland, Olivero, & Weeden, 2004). Using Trouche's (2004, 2005) notion of instrumental orchestration, investigators study how mathematics teachers support students' appropriation of technological tools. Sutherland et al. (2004) studied how a secondary school mathematics teacher taught proofs with a DGE. They found that the teacher's orchestration focused on relationships between geometric construction of objects and their properties, where students used the "drag test" to identify mathematical properties. Drijvers et al. (2010) investigated how three mathematics teachers orchestrated the teaching of functions using an applet that allows users to create functions and visualize their graphs. They identified six different orchestration types that the teachers used to support students' appropriation of the tool. In these two studies, the teachers were the focal point of classroom interaction as they orchestrated students' learning with technology, which corresponds to the theory of instrumental orchestration (Trouche & Drijvers, 2014). In contrast, research is needed to understand teachers' instrumental orchestration when they are not the focal point of classroom interaction, for example, when students work in a collaborative environment. In such environment, students are the focal point of classroom interactions and the teacher's role is significant but peripheral.

Instrumental orchestration aims explain teachers' role in supporting learners' appropriation of technological tools. To understand this appropriation, we draw on a Vygotskian perspective about goal-directed, instrument-mediated action and activity. Instrumental genesis (Lonchamp, 2012; Rabardel & Beguin, 2005) theorizes how learners interact with tools that mediate their activity on a task. To appropriate a tool, learners (teachers or students) develop their own knowledge of how to use it, which turns the tool into an instrument that mediates an activity between learners and a task. Learners engage in an activity in which actions are performed upon an object (matter, reality, object of work…) in order to achieve a goal using a tool (technical or material component). Rabardel and

Beguin (2005) emphasize that the instrument is not just the tool or the artifact, the material device or semiotic construct, it “is a composite entity made up of an artifact component and a scheme component.” (p. 442). Learners appropriate artifacts by developing their own utilization schemes. The transformation of an artifact into an instrument, or instrumental genesis, occurs through two important dialectical processes that account for potential changes in the instrument and in leaners, instrumentalization and instrumentation. Instrumentalization is “the process in which the learner enriches the artifact properties” (Rabardel & Beguin, 2005, p. 444). Instrumentation is about the development of the learner side of the instrument; the learner assimilates an artifact to a scheme or adapts utilization schemes. Instrumentation plays an important role in understanding the relationship between technology and mathematics but it can be a complex process (Artigue, 2002).

Part of the complexity of instrumentation lies in its multidimensionality. It has an individual as well as social dimensions (Trouche, 2005). Instrumental genesis mainly considers the individual aspect of instrumentation. To account for the social aspect, Trouche (2004, 2005) introduces instrumental orchestration to describe how teachers manage mathematics classroom when integrating technological tools. It is defined by the arrangements of artifacts in the environment, didactical configurations, and teacher and student moves within these configurations, exploitation modes (Trouche, 2004, 2005). Teachers use different combinations of didactical configurations and exploitation modes to support their students’ instrumentation. Their orchestration acts on three levels: artifacts, instruments, and students’ relationship with the instruments. In each level, teachers choose certain configurations and exploitation modes to support students’ instrumentation. Ruthven (2014) provides a summary of Trouche’s example of instrumental orchestration with using “Customised calculator” to teach about limits in which he describes the specific didactical configurations and exploitation modes. For example, the artifact level has two configuration modes: a) “classroom calculators are ‘fitted out’ with a guide affording three levels of study of the limit concept” and b) “these are designed to support the shift from kinetic concept of limit to an approximative concept”. The exploitation modes for this level are a) “guide can be available always or only during a specific teaching phase”, b) “students can use guide freely when available, or be constrained to follow the order of the levels”, c) “components can be fixed, or updated in response to classroom lessons”, and d) “recording of steps of instrumented work, can be required, or not” (p. 382). For Trouche (2005), instrumental orchestration tries to answer questions about what technological artifacts mathematics educators should introduce to learners and what guidance they should provide so learners can appropriate and use artifacts as instruments to mediate their activity.

METHODS

The research setting is a professional development project that involves middle and high school teachers in two semester-long, technology-focused online courses. The first course engages teachers in interactive, discursive learning of dynamic geometry through collaborating on construction and problem-solving tasks in VMTwG. The teachers reflect in writing on the mathematics, collaboration, and technological components of the course and collaboratively plan how to implement course modules with their students. In the second course, a reflective practicum, the teachers engage their students in at least 10 hours of class sessions to learn dynamic geometry through use of VMTwG to work on construction and problem-solving tasks.

The online environment, VMTwG, is an interactional, synchronous space. It contains support for chat rooms with collaborative tools for mathematical explorations, including a multi-user version of GeoGebra, where team members can construct dynamic objects and drag elements to visually explore relationships (see Figure 1). VMTwG records users' chat postings and GeoGebra actions, which participants can review and even replay at various speeds. The research team designed dynamic-geometry tasks to encourage participants to discuss and collaboratively manipulate and construct dynamic-geometry objects, notice relations and dependencies among the objects, make conjectures, and build justifications.

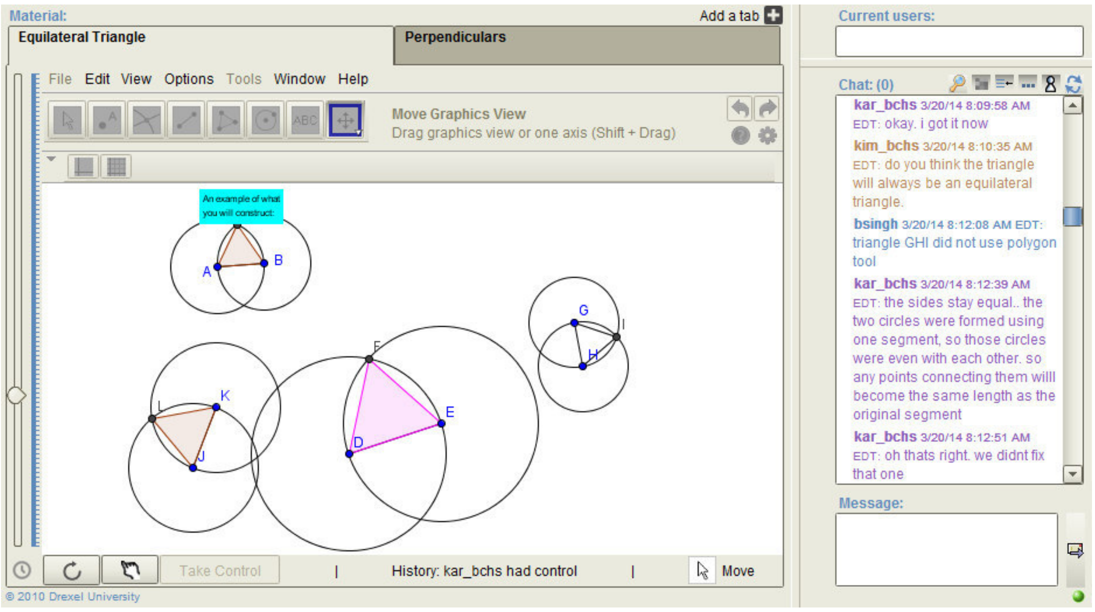

Figure 1: Screenshot of VMTwG environment with the work of Mr. S.'s students

The data for this case study come from the second course and concerns the work of a high school mathematics teacher, Mr. S. He engaged his class in VMTwG in small teams of three to four students each. The class worked in a computer lab, and Mr. S. encouraged students to communicate only through VMTwG. To understand his instrumental orchestration and how his pedagogical interventions promote students' movement between explorations and justifications, we analyze qualitatively four sources of data: (1) the tasks he used with his students, (2) the modifications he made to the tasks after reviewing teams' work, (3) the logged VMTwG interactions of two teams of his students on the tasks, and (4) his reflections on their work, which he wrote after each class session. We chose to analyze two teams, Team 1 and Team 6, since Mr. S. considered those teams to be most collaborative.

On each of the four data sources, we performed conventional and directed content analysis (Hsieh & Shannon, 2005). We were particularly interested in coding and categorizing both Mr. S.'s pedagogical interventions and the deductive justifications of two teams of his students. The data drives our analysis, and we interpret them using the theories of instrumental genesis and orchestration whenever there are links between the data and the theories.

FINDINGS

Based on our analyses of Mr. S.'s implementation of the project design, his interventions were directed at supporting students' actions that can be grouped in three categories: collaboration, mathematical reasoning, and the use of technology. In addition, the analysis reveals that Mr. S. followed a trajectory of pedagogical interventions focused on his students' discursive interactions and their emerging knowledge of dynamic geometry. In his reflections on his students' work, Mr. S. expresses an overall goal that, within their teams, students manipulate and construct dynamic geometric objects and notice and discuss relations among them, particularly relations of dependency. To achieve this goal, Mr. S.'s pedagogical interventions focused on how the teams of students collaborate. Having given his students a task designed to promote collaboration, Mr. S. expressed concern in his weekly reflection that the teams did not collaborate successfully. He reported that to ensure successful collaboration sessions he subsequently discussed with his class features of successful collaborative sessions and presented examples of what he considered good collaboration moves. To underscore his advice, he distributed a list of behaviors that can help to ensure successful collaboration and called it "The Pledge". It contained behaviors such as "Include everyone's ideas" and "Ask what my team members think and what their reasons are".

These pedagogical interventions and ones that we will present below focused on collaboration reveal that Mr. S. is choosing exploitation modes (instructional decisions) that encourages students to be reflective of their work within their groups. His pedagogical interventions are mostly focused on the second and third level of his instrumental orchestration. Those levels are concerned, respectively, with the instrument and the students' relations with the instrument. Mr. S. used collaboration as a vehicle to orchestrate his students' appropriation of VMTwG artifacts and movement towards deductive justifications. In his weekly reflections, he assessed his students' reasoning by tracking their collaboration and their use of mathematical language.

Closely following Mr. S.'s interventions concerning collaboration, he focused on aspects of students' use of the technology. This focus is at the first level–artifact level–of his instrumental orchestration. In his weekly reflections, he reported that during his students' engagement in VMTwG, he "monitored progress and resolved some tech issues." He helped students gain insights into the use of particular GeoGebra commands by modifying tasks and by directing his students to view specific YouTube GeoGebra clips.

As Mr. S.'s teams of students increased their effective collaborative interactions, he shifted his pedagogical interventions more explicitly towards supporting their mathematical reasoning. He discussed with his class the concept of dependency in dynamic geometry to contrast it with dependency in other mathematical domains and modified the tasks to explicate particular mathematical ideas. He posed detailed questions to foreground mathematical discourse. For example, he found that the tasks' original questions were not specific enough to elicit mathematical reasoning, so he included the following questions in one of the tasks, "Constructing an Equilateral Triangle":

1. What kinds of triangles can you find here?
2. Drag the points. Do any of the triangles change kind? Discuss this in the chat.
3. Are there some kinds [of triangles] you are not sure about?
4. Why are you sure about some relationships?
5. Does everyone in the team agree?

These questions prompted students to attend to particular objects and relations in the construction and to discuss their behavior.

Mr. S.'s instrumental orchestration and his other pedagogical interventions contributed to his teams of students' instrumentation and movement towards greater collaboration and deductive justifications. For example, according to Mr. S.'s and our analyses, a team of three students (Team 6) improved their collaboration, explorations, and mathematical reasoning. In their third session, the task asked them to construct an equilateral triangle, find the relationships among objects in their construction, and justify their claims. The students first dragged a pre-constructed figure of an equilateral triangle (see triangle ABC in Figure 1) to explore elements of the construction and their behavior. Afterward, they each constructed a similar figure (see Figure 1) and dragged their construction vigorously to validate and justify their construction. Below, an excerpt of their discussion shows how they articulated a valid justification of why their constructions were of equilateral triangles.

18	kar_bchs:	looks like we both got it [both successfully construct and drag the figures vigorously]
19	kim_bchs:	yay, it seem like for a second one of the circles appeared much larger. but that was my imagination.
20	kar_bchs:	oh. lol. why is the third point dependent on the distance between the first two points? (number 7)
21	kar_bchs:	it just connects the points and the circles. making them all one piece
22	kim_bchs:	as the segments change sides so does the radius of the circle. However, the triangle remains an equilateral traiangle
23	bsingh:	[the teacher] be sure to read directions, ALL, and make the pledge
24	kim_bchs:	triangle
25	kar_bchs:	yea. even though the sizes of the sides change, the fact that it is an equilateral triangle doesn't
26	kar_bchs:	each side has the same distance in between it. even when you move the points
27	kim_bchs:	i notice that point d and e are on the circumference of one circle. while point f is an intercetion of both circle. making it dependent on both points.
28	kar_bchs:	if you try and move the intersected point (F and I), it wont move. but yea you're right, the intersecting point depends on the segment that was made
29	kim_bchs:	*point f is an intersect of both circle
30	bsingh:	[the teacher] there is something missing, are you reading the directions
31	bsingh:	[the teacher] we are only doing tab 1 today
32	kar_bchs:	i didnt use the polygon tool.. thats missing in mine
33	kim_bchs:	i just notice that.
34	kar_bchs:	can i try?
35	kar_bchs:	okay. i got it now
36	kim_bchs:	do you think the triangle will always be an equilateral triangle.

37	kar_bchs:	the sides stay equal.. the two circles were formed using one segment, so those circles were even with each other. so any points connecting them willl become the same length as the original segment
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50	kim_bchs:	the radius of a circle is the same distance. segment AB is Sure. the radii of both circles and Segment AC and BC are also radii of both cicles. hense, the triangle should be euilateral.
51	kar_bchs:	the circles are equal. Making the circumference of each, equal to one another

The students noticed that the equilateral triangle depended on the relationship between the two circles that they created. They discussed their constructions and the relationships they noticed (lines: 18 - 29). Both students noticed that the construction maintains equilateral triangle with dragging (lines 22 and 25). They tried to explain how the intersection points of the circles are dependent on the centers of the circles (lines 27 - 29). In line 36, kim_bchs asks whether the triangle is always equilateral. In response, kar_bchs states that the sides of the triangle are equal and mentions that the two circles are "even" or congruent. In line 50, it seems that kim_bchs builds on kar_bchs's observation and notes that the radii of both circles are equal and that implies that the triangle is equilateral and, in line 51, that the circumferences of the two circles are equal. The students successfully build on each other's ideas and justify why their constructions yield equilateral triangles and justify other equivalences that they notice. They also note that the congruence of their circles depends on the segment that they share (line 37: "the two circles were formed using one segment, so those circles were even with each other") and that two sides of the given triangle are dependent on segment AB (line 50: "the radius of a circle is the same distance. Segment AB is Sure. The radii of both circles and Segment AC and BC are also radii of both cycles. Hence, the triangle should be equilateral."). This provides further evidence that these students are justifying mathematical relations, which moves them in the direction of deductive justification. This also indicates that the students transformed artifacts of the environment into instruments such as chat, dragging, and tools involved in constructing equilateral triangles.

CONCLUSIONS AND IMPLICATIONS

While integrating technology in mathematics, examining a teacher's pedagogical interventions provides insight into that part concerned with his instrumental orchestration and with fostering deductive reasoning. To promote learners' movement between exploration and deductive justification, our study indicates that the teacher's pedagogical interventions addressed different aspects of his geometry lessons—collaboration, mathematical content and practices, task design and instructions, and tool use—and were coded to be acting in the three different levels of instrumental orchestration. His orchestration followed a trajectory of pedagogical interventions that began with a focus on supporting teams of his students with different didactical configurations and exploitation modes to have effective collaborative interactions. Once he was satisfied those students within teams were listening to each other and building on each other's ideas, he shifted to focus his instructional interventions on mathematical reasoning and justifications. Our analysis of his weekly reflections and of his students' work show that, in parallel with his trajectory, his students progressed toward more pointed justifications of geometric relations they noticed, including relations of dependencies. Such relations are mathematically significant (Stahl, 2013; Talmon & Yerushalmy, 2004).

Finally, further research is needed to determine in general whether students' instrumentation and movement between exploration and deductive justification in a CSCL environment can be promoted effectively by a trajectory of pedagogical interventions that first focuses on their discursive interactions in collaboration and then attends to mathematical reasoning and justifications. Research would also need to account for effects stemming from the task design and the VMTwG environment. Such research would inform mathematics teacher educators about the instrumental orchestrations and pedagogical interventions teachers could employ to support students' collaborative instrumentation and learning of deductive justifications in dynamic geometry.

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