Investigating How Setting Up Cognitively Demanding Tasks is Related to Opportunities to Learn in Middle-Grades Mathematics Classrooms

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Over the past several decades, mathematics education researchers have generally achieved consensus regarding a set of goals for students' mathematical learning, which are represented in documents like the National Council of Teachers of Mathematics' (NCTM; 2000) *Principles and Standards for School Mathematics* and the more recent *Curriculum Focal Points* (NCTM, 2006). These documents describe a relatively concrete set of learning goals that encompass both conceptual understanding and procedural fluency in a range of mathematical domains. They also describe "process" goals for students' learning, for example, the ability to participate in mathematical argumentation and to make connections between various mathematical ideas and representations.

At the same time, mathematics education researchers have worked to specify visions of instruction, or what should happen between teachers and students in classrooms, in order to accomplish these ambitious goals for students' learning (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). For example, it is generally well accepted that high-quality mathematics instruction, or what some call "ambitious mathematics teaching" (Lampert, et al., 2010), includes the use of challenging mathematical tasks in discourse-rich environments where students have frequent opportunities to articulate their mathematical reasoning and connect their ideas with those of their peers, and where teachers press and support students to make connections between various mathematical representations (e.g., tables, graphs, equations) (Franke, Kazemi, & Battey, 2007; Hiebert et al., 1997; Lappan, 1997; Stein, Smith, & Silver, 1999). Moreover, this form of teaching is ambitious because it necessarily requires that teachers teach in response to what students do as they engage in solving mathematical tasks (Kazemi, Franke, & Lampert, 2009).

Another central and challenging aspect of ambitious teaching is that it aims at supporting all students to substantially participate in rigorous mathematical activity (Lampert & Graziani, 2009). In our view, although the field of mathematics education has made considerable headway in detailing concrete forms of instructional practice that aim at ambitious learning goals for students, mathematics education research has made less progress in specifying concrete forms of practice that are associated with equitable learning opportunities (Boaler, 2002; Jackson & Cobb, 2010). From our perspective, a key characteristic of classrooms in which learning opportunities are equitable is that all students are supported to substantially participate in each phase of instruction (e.g., individual work, small group work, whole-class discussion), but not necessarily in the same ways. A central goal of our work is to contribute to literature that specifies concrete forms of instructional practices that are likely to support all students to participate substantially in instruction aimed at ambitious learning goals. In other words, we aim to contribute to the elaboration of the equitable dimension of ambitious teaching.

Towards this end, in this paper we focus on what happens when a task is first introduced to students as a crucial phase of instruction. We report on an empirical study of 132 middle-grades mathematics teachers' instruction—in particular, the nature of the ways in which they introduced tasks, and the relationship between how they introduced tasks and the nature of students' opportunities to learn mathematics in the concluding whole-class discussion. Based on our analysis, we specify aspects of introducing cognitively demanding tasks that are likely to support equitable opportunities to learn in classrooms aimed at ambitious learning goals.

The paper is organized as follows. We first present our conceptual framework. Second, we describe the methodology that guided our analysis of the classroom instruction of 132 middle-grades mathematics teachers. Third, we share our findings regarding the nature of how teachers introduced tasks to students, and how introducing tasks related to learning opportunities in concluding whole-class discussions. Finally, we describe the implications of this work for research and practice.

Conceptual Framework

Our research builds on the work of Stein and her colleagues' efforts to conceptualize key relationships between features of classroom instruction and students' opportunities to learn mathematics (e.g., Stein, Grover, & Henningsen, 1996a; Stein, Grover, & Henningsen, 1996b; Stein, Smith, Henningsen, & Silver, 2000). In what follows, we situate our research with respect to their work, specifically the Mathematical Tasks Framework. We clarify our rationale for focusing on the nature of how tasks are introduced, and we elaborate on what we have identified as key aspects of "setting up" complex tasks.

Mathematical Tasks Framework and Phases of Instruction

Lessons aimed at ambitious forms of teaching can take a variety of forms. In this paper we are concerned with one form of lesson structure—a three-phase lesson (task is introduced, students work on solving the task, teacher orchestrates a whole-class discussion). This lesson structure is common in *Standards*-based middle-grades curriculum, for example *Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997), and it is commensurate with what Stein and her colleagues suggested in the Mathematical Tasks Framework (Stein, et al., 2000). Figure 1 is an adaptation of the Mathematical Tasks Framework and, in particular, it illustrates how a three-phase lesson structure relates to the Framework. We explain our use of the Mathematical Tasks Framework and elaborate on the three-phase lesson structure below.

=======================[INSERT FIGURE 1 ABOUT HERE]=====================

The Mathematical Tasks Framework offers a characterization of central features of classroom instruction that Stein and colleagues found impacted the extent to which students were provided opportunities to develop significant mathematical understandings (Stein, et al., 2000). It is based on empirical analyses of middle-grades mathematics classroom instruction in urban settings in which teachers were supported to develop ambitious forms of teaching (Stein, et al., 1996a). The Framework reflects a basic assumption that students' opportunities to learn mathematics depend on the cognitive demand of the activity in which students participate. Cognitive demand refers to what students need to do to (e.g., the nature of reasoning) in order to solve a particular problem, or, at a broader level, participate in a given activity (Doyle, 1988). The Framework illustrates crucial points at which the "cognitive demand" of what students are engaged in might be altered.

A key feature of the cognitive demand of classroom activity (and thus the nature of students' learning opportunities) is the nature of the task that a teacher chooses to use in instruction, or the "task as it appears in instructional or curricular materials" (Stein & Lane, 1996). Stein, Grover, and Henningsen (1996a) systematically identified characteristics of mathematics tasks with low and high cognitive demand. Tasks with low cognitive demand require students to memorize or reproduce facts, or perform relatively routine procedures without making connections to the underlying mathematical ideas. Tasks with high cognitive demand tend to be somewhat ambiguous (i.e., a solution strategy is not immediately apparent), require students to make connections to the underlying mathematical ideas, and engage students in disciplinary activities of explanation, justification, and generalization. Based on analyses of middle-grades mathematics instruction aimed at ambitious learning goals, Stein and Lane (1996) found that that the use of tasks with high cognitive demand was related to greater student gains on an assessment requiring high levels of mathematical thinking and reasoning.

However, as Stein et al. (1996) illustrated, selecting a task with high cognitive demand does not ensure that students will be provided opportunities to engage in rigorous mathematical activity. Instead, tasks have to be understood as part of "classroom activity," and interactions between the teacher, students, and the task determine the extent to which the cognitive demand is maintained across the course of a lesson, and thus the nature of students' learning opportunities (Stein, et al., 2000, p. 25).

Stein et al. identified two phases of instruction that mattered regarding the extent to which the cognitive demand of a task as it appeared in materials was maintained—what they refer to as the *set-up* and *implementation* phases of instruction (*task as set-up* and *task as implemented* in Figure 1, respectively). In Stein et al.'s terms, the set-up phase of instruction refers to how a teacher introduces the task. It "includes the teacher's communication to students regarding what they are expected to do, how they are expected to do it, and with what resources" (Stein, et al., 2000, p. 25). It is quite possible that teachers can lower the cognitive demand of a task during this phase of instruction. For example, in introducing a given task, a teacher or students might suggest a solution path.

All activity that occurs after the task is set up and students begin to work to solve the task constitutes the implementation phase. Stein et al. (1996b) found that in classrooms where tasks with the potential for high levels of cognitive demand were assigned, teachers and/or students often decreased the cognitive demand during implementation of the task.

In this paper, we are especially concerned with how aspects of the set-up—in addition to the cognitive demand of the task as posed—are consequential for students' opportunities to participate in mathematical activity. Like Stein et al., we will refer to the set-up as the first phase of a lesson. However, we have separated what Stein et al. refer to as implementation into two phases of instruction.

For the purposes of our analyses, the second phase of instruction refers to when students work individually and/or in groups to solve the task. During this phase of instruction, ideally the teacher circulates among the students, paying close attention to what students are doing as they complete the task and deciding what mathematical ideas and whose solutions to make focal during the whole-class discussion that follows (Lampert, 2001; Stein, Engle, Smith, & Hughes, 2008).

The third phase of instruction refers to when the teacher “orchestrates” a concluding whole-class discussion aimed at developing students’ increasingly sophisticated understanding of the key mathematical goals (Stein, et al., 2008). Ideally, the teacher has identified and sequenced particular students’ solutions to ensure that the discussion advances the teacher’s instructional agenda (Stein, et al., 2008).

During the discussion, the teacher should press students to explain and justify their solutions, evaluate their peers’ solutions, and make connections between different solutions (Ball & Bass, 2000; Chapin, O'Connor, & Anderson, 2003; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Forman, McCormick, & Donato, 1998; Hiebert, et al., 1997; Stein, et al., 2000). Additionally, the teacher plays a crucial role in mediating the communication between students to help them understand each others’ explanations (McClain, 2002).

Orchestrating *productive* whole-class discussions (i.e., those that support the learning of all students) is challenging (Stein, et al., 2008). Research has documented that what often happens is a “show and tell” in which students take turns sharing solutions (Ball, 2001). In show and tell forms of discussions, teachers do not facilitate key connections between different solution strategies nor do they link student-generated solution methods to disciplinary methods and important mathematical ideas (Stein, et al., 2008). There is also little attention as to which solution strategies help to illustrate certain mathematical ideas or which strategies might be the most efficient or productive in a particular circumstance (Stein, et al., 2008).

A critical feature of productive whole-class discussions is the nature of the talk. Building from the work of A. Thompson and colleagues (A. G. Thompson, Philipp, Thompson, & Boyd, 1994), Cobb and colleagues (Cobb, Stephan, McClain, & Gravemeijer, 2001) described a distinction between two types of classroom discourse—*calculational* and *conceptual*. Calculational discourse refers to discussions that only emphasize *how* one arrived at a solution. In contrast, conceptual discourse refers to discussions that emphasize *why* one chose to solve a problem in a given way, in addition to the 'how'. The explainer provides an explicit rationale for using particular methods, and grounds any talk of mathematical quantities or relationships in the context of the given task. Conceptual explanations are therefore more likely to support all students' learning, particularly the listening students, because the explainer provides an "explicit account of the task interpretations that underpin particular solution strategies" (Jackson & Cobb, 2010, p. 24).

Why the Set-Up Phase of Instruction is Consequential for Equity in Students' Learning Opportunities

In this section, we clarify why we are focusing specifically on the set-up phase of instruction as crucial for equity in students' learning opportunities. Prior research has suggested that an effective set-up phase of instruction clarifies both the final work product and how students should work (i.e., individually, in groups) to solve the problem (Boaler & Staples, 2008; Smith, Bill, & Hughes, 2008). We focus on complementary aspects of the set-up—in particular students' understandings of the task statement.

A key reason why we view the set-up phase as crucial for supporting equitable learning opportunities in the classroom is because it provides teachers with an opportunity to a) gauge what their students understand about a given task statement, and b) take action to ensure that all students have requisite understandings such that they can engage productively in solving the task at hand (cf. Boaler, 2002). By engaging productively in solving a task, we mean in a manner that is likely to lead to the development of conceptual understanding of central mathematical ideas.

At the start of a lesson, it is not reasonable to expect that all students will have requisite or similar understandings of a given task statement. Some tasks, particularly those associated with middle-grades *Standards*-based curricula, often embed problem-solving in scenarios. As other research has suggested, tasks that embed mathematics problem-solving in a scenario pose a challenge. It is not likely that all students will be equally familiar with the situation described in a task statement (Ball, Goffney, & Bass, 2005; Boaler, 2002; Lubienski, 2000; Silver, Smith, & Nelson, 1995; Tate, 1995). For example, consider the *Dollars for Dancing* task in Figure 2.

=======================[INSERT FIGURE 2 ABOUT HERE]=====================

In this task, students are asked to represent and interpret various ways of earning money in a dance marathon. Scenarios provide a context to support students' initial engagement in the problem and their mathematization (e.g., Gravemeijer & Doorman, 1999) of key mathematical ideas and relationships; in this case, the dance marathon provides a context about which students can reason to develop meaning for linear relationships.

If a student does not understand key aspects of the scenario, it is unlikely that s/he will engage in solving the task productively. And, without substantially engaging in solving the task, it is unlikely that a student will be able to participate in or benefit much from a concluding whole-class discussion. Colloquially speaking, then, the set-up is the phase of instruction in which a student is either supported to be "in the game" or not.

An additional reason to attend to the set-up phase of instruction is because how a task is introduced also affects the work of teachers. When students are not supported to understand key aspects of the task statement, teachers often spend the next phase of instruction re-introducing the task to individual or groups of students while others begin to solve the task. This is not necessarily bad—it may be that some students need additional or different information about the task to begin to solve it in productive ways. However, re-introducing tasks for an extended period of time is a loss of valuable time. As we noted above, ideally, once students begin to solve a problem, a teacher circulates to assess how students are approaching the problem and the nature of their current mathematical understandings, and to plan for the subsequent whole-class discussion (Stein, et al., 2008). However, if a teacher spends the second phase of instruction re-introducing the task, s/he is unlikely to be able to carefully plan a concluding whole-class discussion.

Key Aspects of High-Quality Set-Ups

In our work, we have identified four key aspects of setting up tasks in middle-grades mathematics lessons that appear to support students' subsequent engagement in solving complex mathematics tasks. We initially identified these four aspects in a qualitative analysis of 40 video-recordings of middle-grades *Standards*-based instruction. (That analysis took place prior to the study that we report on below.) We elaborate on these aspects for two reasons. First, our elaboration provides necessary background information on what we systematically examined in our analysis of 132 middle-grades teachers' classroom instructional practices on which we report in the latter half of the paper. Second, there has generally been minimal research about what setting up tasks aimed at supporting all students' participation in classroom activity might entail. Consequently, there are minimal images in the field regarding this form of practice in middle-grades mathematics.[1] We aim to offer an image of what high-quality set-ups might entail. Our empirical analysis described in the latter half of this paper provides evidence that these aspects are, indeed, related to the nature of students' learning opportunities in subsequent phases of instruction.

High-quality set-ups took the form of a whole-class discussion prior to students solving a task. The following four aspects characterized set-ups in which the majority of students were able to productively engage in solving the task at hand and contribute to

[1] Franke, Kazemi, and colleagues (Franke, 2006; Kazemi, et al., 2010) have identified the practice of "problem posing" as critical for supporting all students' learning in the context of elementary mathematics education. Their work provides useful images of what high-quality problem posing, or setting up tasks, looks like in the context of elementary mathematics.

whole-class discussions aimed at developing conceptual understandings of key mathematical ideas.

1. Key contextual features of the task scenario are explicitly discussed.
2. Key mathematical ideas and relationships as represented in the statement are explicitly discussed.
3. Student participation in the set-up is aimed at developing a taken-as-shared understanding (Cobb, Wood, Yackel, & McNeal, 1992) of contextual features and key mathematical relationships.
4. The cognitive demand of the task is maintained over the course of the set-up.

We discuss these aspects of setting up in relation to using tasks that include a problem-solving scenario. However, as we clarify in our Results section, we found that the second, third, and fourth aspects of high-quality set-ups were also relevant to instruction that used tasks that did *not* include problem-solving scenarios.

We ground our elaboration of these aspects in an actual set-up from classroom video collected in year one of a research project, which we describe in detail in the Methodology section.[2] This set-up took place in a seventh grade classroom.[3] The teacher, Mr. Lewis,[4] posed the *Dollars for Dancing* task, described above, about midway in the school year. In prior lessons, the students used tables, graphs, and equations to

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[2] We describe this set-up in more detail in Jackson, Shahan, Gibbons, & Cobb (submitted).

[3] The majority of the students in the class received free or reduced-price lunches. Most students identified as Latino/a, several of whom were identified as English language learners. The class also included African American and White students. For this class, the teacher's value-added score, based on students' performance on the state mathematics assessment, indicated better than expected growth.

[4] All names are pseudonyms.

solve problems involving linear relationships with y-intercepts of zero. This lesson was their first encounter with a linear relationship with a non-zero y-intercept. Mr. Lewis' goal was to leverage the Dance Marathon scenario to develop students' understandings of the y-intercept as an initial value and its relationship to slope. Mr. Lewis devoted eight minutes of an hour-long lesson to setting up this task.

Key contextual features. As suggested above, the extent to which a student is familiar with a task scenario will impact whether the student can productively engage in solving the task. Thus, one key aspect of an effective set-up is the explicit discussion of the key contextual features of the task scenario. Key contextual features are aspects of the scenario that students would not understand unless they had prior experience with it. For example, key contextual features of the *Dollars for Dancing* scenario include knowing what a dance marathon might involve and why people organize and participate in dance marathons to raise funds.

To develop his students' understandings of the key contextual features in *Dollars for Dancing,* Mr. Lewis focused first on eliciting students' prior understandings about dance marathons. He projected Internet pictures of dance marathons and asked students to discuss them:

Mr. L: I want you to take a look at the pictures…and see if you can tell anything that might be going on in today's math class that we're going to be talking about. OK. Eva.
Eva: Dance.
Mr. L: OK. Dance. Somebody add on to that please. Chris.
Chris: A dance marathon.
Mr. L: A dance marathon. Let's see if anybody can add on to what a dance marathon is or talk about it, Keith?
Keith: A group of people.

Mr. L: Okay. A dance marathon might be a group of people. Let's talk about what happens at a dance marathon or why people might do it. Okay Jamal?
Jamal: A certain amount of people dance for like a certain amount of time.
Mr. L: Okay. People dance for a certain amount of time. Can anybody add on to that? What else do you know about a dance marathon?

Mr. Lewis capitalized on students' contributions to develop a provisional description of dance marathons as "groups of people who dance for a certain amount of time." He then pressed students to explain why people might hold a dance marathon and built on several of their proposals to explain that this task involved holding a dance marathon to raise money. He connected this to their school's need to raise money to hire a DJ for the upcoming Valentine's Dance.

Key mathematical ideas and relationships. In addition to discussing the contextual features of a scenario, we identified a second, related critical aspect of high-quality set-ups—the explicit discussion of how the key mathematical ideas and relationships are represented in the task statement. Our attention to developing students' understandings of how key mathematical ideas and relationships are represented in the statement are based on the work of P. Thompson (1996) and McClain and Cobb (1998), who argued that students' initial understandings of the mathematical relationships described in the task statement provide a basis for any mathematizations they might make as they attempt to solve a task. For example, in *Dollars for Dancing*, the students were expected to use tables, graphs, and equations to represent the accumulation of money over time in three different plans. In order to do so, it was essential that the students understand that money accumulates as a participant continues to dance for a greater number of hours. Furthermore, there are different ways of accumulating money—starting with a fixed amount (a donation) and/or earning a fixed amount of money per every hour of dancing (a pledge). Absent an understanding of how these key mathematical relationships are represented in the statement, or what Thompson (1996) calls "situation-specific imagery," students' efforts to solve tasks typically become "decoupled" from their interpretations of problem situations (McClain & Cobb, 1998, p. 65). In the case of *Dollars for Dancing*, without these understandings, students are unlikely to make connections between accumulating quantities of money, slope, and y-intercept, as the task intends. It is also probable that some students will struggle to create appropriate tables, graphs, and equations themselves, or understand their peers' representations.

Mr. Lewis's set-up illustrates this second aspect of high-quality set-ups well. He did not end his set-up after explicitly discussing key contextual features of the scenario. Instead, Mr. Lewis went on to support his students' understandings of how a key mathematical relationship (the accumulation of money) was represented in the statement. He began with the difference between an up-front donation and an hourly pledge.

Mr. L: [T]here's two ways you can raise money in a dance marathon that we're going to talk about. One way is to dance for a long time. Chris and some others said a marathon takes a long time and people dance for a long time. So if you dance for a long time, and let's say I give you 50 cents every hour you're going to make a lot of money. But there's another way that you could raise money and that is to ask for a pledge. Not per hour, but just a donation. Okay we call that a donation. And you might go up to your teacher and say, "Can you give me $6 for being in the dance marathon?" Now that's different. Can anybody explain how that is different if I say, "Can you give me $6?" or instead "Can you give me 50 cents an hour?"

As the exchange continued, Mr. Lewis asked students to restate what he and others said about these mathematical relationships, and he took up students' ways of describing them:

Mr. L: What's the difference Cesar?
Cesar: Because you start with some money and then they add more money…
Mr. L: Jasmine, add on to that.
Jasmine: It's like, either they pay you up front or you continue so like they continue to pay you for however long you dance.
Mr. L: Great. So we have one where they pay you up front, one where they add on to it. How many people understand … what we're talking about here? [*waits to see students' hands*] Two kinds of fund raising. Pay you up front or pay you where you add on. Marisa, can you say it in your words? There's two ways that you could raise money, what are they?
Marisa: Well like one of them you already start with it and the other one you have to kind of work for it to get more.
Mr. L: Exactly. I like the way that's worded. One of them you start with it, you just have it. The other one you got to work for it to get the money.

Shortly after explicitly discussing the two ways to earn money in the scenario, Mr. Lewis handed out the tasks and briefly explained students' responsibilities in their groups. The students then began to solve the task.

It is worth clarifying two points. First, our division of "contextual features" from "key mathematical ideas and relationships" is artificial; the key mathematical ideas and relationships as represented in the task scenario are *also* contextual features of the scenario. We have chosen to separate key mathematical ideas and relationships from key contextual features because they capture two distinct foci of high-quality set-ups. A teacher can attend to what we term contextual features without attending to key mathematical ideas, and vice versa. As we elaborate on in our findings, we have found that attention to *both* aspects appear to be related to high-quality whole-class discussions.

Second, throughout the paper, we use the language of “key” features and mathematical ideas and relationships. Scenarios associated with cognitively demanding tasks are often quite dense, both contextually and mathematically. The scenarios could lend themselves to extended talk about a number of contextual features (which may or may not be critical to solving the task), and a given task could be used to develop several key mathematical ideas and relationships. Clearly, time is of the essence in classroom instruction. Therefore, it is worth stressing that teachers need to make judgments regarding the *key* features and mathematical ideas and relationships represented in the scenario on which to focus in the set-up. These judgments will need to be made against a clear set of mathematical goals for instruction and knowledge of what is likely to be contextually unfamiliar to students.

Building taken-as-shared understandings. A third aspect of a high-quality set-up relates to the nature of student participation. In the effective set-ups that we identified, teachers did not simply talk *to* students about the key contextual features and mathematical relationships in the task scenario. Instead, they solicited input *from* multiple students and asked questions that required more than a yes or no response. Broad and active student participation helps the teacher assess students’ understandings of the key contextual features and mathematical relationships specific to the task scenario in order to determine the level of support that students may need (Boaler, 2002). Mr. Lewis’ set-up is illustrative in this regard, in that he elicited students’ understandings of the key contextual features and mathematical relationships in the scenario.

In addition to eliciting broad and active student participation, high-quality set-ups aimed at developing *taken-as-shared understandings* (Cobb, et al., 1992) of the key contextual features and mathematical ideas and relationships. Prior research has identified how developing taken-as-shared understandings among students serves as a basis for communication during instructional activity, for example when communicating and representing mathematical ideas in small groups (Cobb, et al., 1992; McClain & Cobb, 1998). Establishing taken-as-shared understandings during the set-up can also address the problem of how to support students with less developed understandings of the mathematical ideas at stake to participate in and benefit from a whole-class discussion. McClain and Cobb (1998) identified the teacher move of "folding back" as a useful tool for addressing this problem. Folding back refers to making explicit connections to situation-specific imagery that has been previously established as taken-as-shared. Consider the *Dollars for Dancing* lesson. Imagine that in the whole-class discussion, some students were struggling to make sense of the meaning of the y-intercept in a given set of equations, or on a graph. Mr. Lewis might have folded back to the idea of two ways to earn money in the context of the dance marathon—either you start with money or you have to earn it.

Mr. Lewis's set-up illustrates the establishment of taken-as-shared understandings of key contextual features and mathematical relationships. In particular, he used several "talk moves" (Chapin, et al., 2003) to support students to develop taken-as-shared understandings. For example, he revoiced or adopted students' language for describing aspects of the scenario, asked students to state or restate ideas in their own words the key features, asked students to “add on” to their peers’ ideas, and marked particular ideas as important. Asking students to revoice and add on to their peers’ ideas also provided Mr. Lewis with an informal assessment of the extent to which the students had developed both an understanding of the key contextual features and key mathematical relationships specific to the scenario.

Maintenance of the cognitive demand. The aspects of effective set-ups described above are aimed at ensuring that all students can engage productively in solving complex tasks *while maintaining the cognitive demand* of the task during the set-up. As we discussed above, Stein et al. (2000) found, however, that it is quite plausible that a teacher and/or students might lower the cognitive demand of a task in the set-up. For example, Mr. Lewis could have reduced the cognitive demand of the *Dollars for Dancing* task by suggesting to the students how to solve the problem (e.g., setting up tables or beginning graphs). Instead, he maintained the cognitive demand of the task by leaving solution pathways open for students to explore. He used the set-up to ensure that all students developed the requisite understandings to reason themselves about significant mathematical ideas. Providing students with access to key ideas of complex tasks while maintaining the cognitive demand is delicate work. We attend to this relationship in the empirical analysis we present below.

Methods

The intent of this study was to characterize middle-grades mathematics teachers’ attention to the aspects of setting up tasks specified above and to explore relationships between the set-up phase of instruction and the concluding whole-class discussion. The following research questions guided our analysis:

1) What is the nature of the set-up phase of instruction? In particular:
   a. To what extent do teachers attend to contextual features and/or key mathematical ideas of a task statement?
   b. To what extent do teachers maintain the cognitive demand during the set-up, especially when they attend to the contextual features and/or key mathematical ideas of the task?
2) How is the quality of the set-up related to the quality of the concluding whole-class discussion?

We asked these questions to understand how setting up cognitively demanding tasks might support equitable opportunities to learn in classrooms aimed at ambitious learning goals.

Research Context

To answer these questions, we used data that were collected in year three (2009-2010) of a four-year study designed to address the question of what it takes to improve the quality of middle-grades mathematics teaching, and thus student achievement, at the scale of large, US urban districts. Each year (2007-2011), several types of data were collected to test and refine a set of hypotheses and conjectures about district and school organizational arrangements, social relations, and material resources that might support mathematics teachers' development of high-quality instructional practices at scale (Cobb & Smith, 2008). In each of the four districts, approximately 30 teachers and their instructional leaders (principals, assistant principals, and coaches) participated in the study. Data collected include interviews with all participants, video-recordings of classroom instruction, assessments of teachers' mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004), video- and/or audio-recording of professional development sessions, and collection of student achievement data. For the purposes of this analysis, we focused on one form of data—video-recordings of classroom instruction—which we explain in further detail below.

Participating districts. Each of the four districts (all located in large US cities) was purposively invited to participate in the study for a few reasons. On the one hand, the districts were chosen because the challenges they faced were typical of most large, urban districts—limited resources, large numbers of traditionally low-performing students in mathematics, high teacher turnover, under-prepared teachers, and disparities between sub-populations of students' performance on state assessments (Darling-Hammond, 2000). On the other hand, the districts were chosen because their response to high-stakes accountability pressures to improve students' performance in middle-grades mathematics was atypical. Namely, each district was attempting to achieve an ambitious vision of instruction in middle-grades mathematics classrooms (Lampert, et al., 2010), or a vision that is broadly compatible with the National Council of Teachers of Mathematics' (2000) *Standards*. Each district adopted curricular materials that were, for the most part, aligned with an ambitious vision of instruction. Three of the four districts (which we will call Districts A, B, and D) adopted *Connected Mathematics Project 2* (CMP2), a *Standards*-based curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009). District C mathematics specialists created a curriculum, which was a blend of CMP2 and a more conventional mathematics text. Additionally, each district provided a number of supports aimed at enabling teachers to develop ambitious forms of instructional practices (e.g., curriculum frameworks, coaching, regularly scheduled time to collaborate with colleagues on issues of instruction, professional development for instructional leaders).

It is worth noting that Districts B, C, and D all adopted a new curriculum (described above) in year one of the study; prior to 2007-2008, mathematics teachers in those districts had used a conventional mathematics text. District A adopted CMP2 in year two of the study; however, District A teachers had been using the first edition of CMP for several years prior to the adoption of CMP2.

Participating teachers. Six to ten schools in each district were selected to participate in the larger study. Schools were purposively sampled to reflect variation in student performance and in capacity for improvement across schools. Our sample consists of 132 teachers across the four districts, located in the selected schools: 30 teachers from District A, 38 teachers from District B, 32 teachers from District C, and 32 teachers from District D. Teachers in our sample average 9.8 years of experience teaching mathematics (with a significantly more experienced group of teachers averaging 14.7 years of experience in District A).

Data Source: Video-recordings of Classroom Instruction

Videographers recorded two days of instruction (consecutively, when possible, to account for the fact that a lesson might extend over more than one day) in January, February, or March for each participating teacher. We asked that teachers include a problem-solving activity and a related whole-class discussion in their instruction, which is compatible with the lesson structure described above in our conceptual framework and with CMP2 lessons as well as the lesson structure suggested in the District C curriculum. To be clear, the goal of the video-recordings was *not* to capture the nature of teachers' everyday practice, but rather to assess the quality and extent to which a teacher might enact the particular kind of instruction articulated by district leaders as the goal of the instructional reform. Given our directions to include a problem-solving lesson and a whole-class discussion, it would be appropriate to think of what was video-recorded as teachers' "best shot" at enacting reform-oriented instructional practices. While the majority of the lessons were contained within the same class period, 22 of the lessons we video-recorded spanned two days. Therefore, we analyzed a total of 242 lessons for the 132 teachers.

Measuring Students' Opportunities to Learn Mathematics

In what follows, we describe how we measured the quality of various phases of instruction. The two video-recorded lessons for each teacher were coded using an expanded version of the *Instructional Quality Assessment* (IQA; Crosson, Junker, Matsumura, & Resnick, 2003). The traditional IQA, developed at the University of Pittsburgh, is based on the Mathematical Tasks Framework, and is consistent with the districts' ambitious instructional visions and professional development programs. The IQA is designed to measure the cognitive demand of the task as it appears in curricular materials, the cognitive demand of the task as implemented, and the quality of the whole-class discussion. The IQA rubrics have been tested for reliability and validity by the IQA team (Boston & Wolf, 2006; Matsumura, Garnier, Slater, & Boston, 2008). However, specific to the task-as-set-up phase of instruction, the IQA only measures the clarity of expectations regarding a final work product.[5] In the latter half of this section, we describe our development of a set of rubrics to measure the quality of the set-up phase of instruction. We refer to the use of the traditional IQA and the task-as-set-up rubrics as the *expanded IQA*. (See Figure 3 for a summary of the expanded IQA rubrics and the aspects of instruction upon which they focus.) The expanded IQA focuses on what teachers *and* students do in the classroom, however it does not directly measure what students actually learned via instruction. Therefore, we explicitly refer to students' *opportunities to learn*, with the assumption that the higher the scores, the more likely it is that students were given opportunities to learn significant mathematics.

=======================[INSERT FIGURE 3 ABOUT HERE]=====================

Traditional IQA Measures. In our analyses, we used the following IQA measures of opportunities to learn: Task Potential and Academic Rigor of the Discussion. In addition, we used two finer-grained measures of the quality of discussion: Student Linking and Student Providing.[6] They are explained in detail below, and complete rubrics are provided in the appendix.

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[5] At the start of the larger study, we chose not to use the rubrics related to Clear Expectations for two reasons: 1) pilot studies revealed that they were the most challenging rubrics in producing reliable estimates, and 2) many of the elements of the Clear Expectations rubrics were designed to be used with student work samples, which we did not collect from our participating teachers.

[6] The traditional IQA includes several other measures of the quality of discussion: teacher linking, teacher asking, and participation. For the purposes of this analysis, we

More generally, the task refers to the activity that the majority of the students participated in, for the majority of the time. *Task Potential* measures the cognitive demand of the task (usually written) that is posed. For task potential, scores of 1 or 2 indicate that a task has relatively low cognitive demand (e.g., students are asked at most to apply a standard procedure to solve a relatively routine problem), and scores of 3 or 4 indicate that a task has relatively high cognitive demand (e.g., students are asked to solve a relatively non-routine problem and to provide evidence of their mathematical reasoning) (Stein, et al., 2000). As an example, the *Dollars for Dancing* task (given in Figure 2) would be scored a 4 because there are multiple solution pathways, students are expected to make connections between representations, and students are expected to explain their reasoning.

Academic Rigor of the Discussion measures the quality and nature of the discourse used in the concluding whole-class discussion phase of the lesson. (We alternately refer to the Academic Rigor of the Discussion as "Discussion.") For Discussion, scores of 1 or 2 indicate, at best, that in a whole-class format, students describe their written work for solving the task but do not engage in a discussion of their strategies, procedures, or mathematical ideas (e.g., discussions take on a "show and tell" form). Scores of 3 or 4 indicate that in a whole-class format, students describe their written work for solving the task *and* engage in a discussion of their strategies, procedures, or mathematical ideas. Additionally, students provide thorough

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decided to use the measures related to the content of student talk because of our focus on how the set-up might subsequently support students' participation in the whole class discussion.

explanations of why particular strategies are valid and make connections between strategies and the underlying mathematical ideas.

The traditional IQA includes additional measures that provide a finer-grained analysis of the quality of whole-class discussion*. Student Linking* measures the extent to which students' contributions link to (i.e., are connected) and build upon each other. Examples of linking include revoicing ideas, marking ideas as important, and relating an idea to someone else's idea (e.g., a student says, "I agree with Maria because ….”). Scores of 1 or 2, respectively, indicate no effort or superficial efforts are made to connect ideas (e.g., there are indications of connections between ideas without explicit talk about *how* they are connected). Scores of 3 or 4 indicate that efforts are made to connect ideas to each other with some talk of *how* they are connected (with a score of 4 indicating that explicit connections are made consistently).

Student Providing measures the extent to which students support their contributions with evidence and/or reasoning. A score of 1 indicates that there are no efforts on the part of students to provide evidence for their contributions or to explain their thinking. A score of 2 indicates calculational explanations or insufficient evidence, and the difference between a 3 and a 4 is the extent to which students provide conceptual explanations.

Task-as-set-up Measures. A team of mathematics educators and doctoral students, including the authors of this paper, developed a set of rubrics to measure the quality of the task-as-set-up phase of instruction. As described above, we identified four key aspects of high-quality set-ups in a qualitative analysis of 40 video-recordings collected in year 1 of the larger study (2007-2008). We then developed a set of rubrics and measures based on those observed aspects that could be used to measure the quality of the set-up, and that could be used to investigate relationships between the set-up and other phases of instruction.

The development of the rubrics was an iterative process carried out over two years. It involved regular meetings (often weekly), in which we refined versions of the rubrics, including the levels and language. We used drafts of rubrics to code existing lessons from years 1 and 2 of the larger study to both refine them conceptually (e.g., deciding how taken-as-shared understandings of particular features of a task might be developed) and pragmatically (e.g., deciding how to code tasks without a problem solving scenario). We regularly consulted with an expert in measuring the quality of instruction in mathematics education throughout the process of rubric development. Because we know of no instrument that attends specifically to the set-up, we were unable to externally validate our measures with an existing instrument. However, one approach to testing construct validity is to look for relationships between the measures and existing evidence of opportunities to learn (Kane, 2006); in our case, we examined relationships between what happened in the task-as-set-up and the concluding whole-class discussion. As we share in the findings, we have found significant relationships between these two phases of instruction using our instrumentation.

We developed two rubrics—*Contextual Features* and *Mathematical Relationships* (explained in detail below). We included the third aspect of high-quality set-ups, developing a taken-as-shared-understanding, in both of the rubrics. The fourth aspect, maintaining the cognitive demand of the task during the set-up, was evaluated using a measure we developed loosely based on the IQA Task Potential and Implementation rubrics. The set-up rubrics and measures were designed to complement the traditional IQA rubrics, and as such, they employ compatible language and have a similar structure as the traditional IQA rubrics described above.

Contextual Features (CF) measures the extent to which students are supported to develop taken-as-shared understandings of the contextual features of the problem-solving scenario (PSS). The CF rubric is only used if the task has a problem-solving scenario (i.e., the mathematical task is presented in the context of a story or scenario). For example, a naked-number task or a problem-solving task that does not include a scenario (e.g., find the angle of rotation of a set of objects and then conjecture about the relationship between angles of rotation and rotational symmetry) would not be scored using the CF rubric. A score of 0 indicates that no attention is given to the contextual features of the PSS. A score of 1 or 2 indicates that the students are, at best, minimally involved (e.g., giving yes or no responses) in a discussion of the contextual features of the PSS. A score of 3 or 4 indicates that the teacher and students connect ideas together, with a 4 indicating that these connections are consistent. In other words, a 3 or 4 indicates there was evidence that a taken-as-shared understanding of the contextual features of the PSS was developed.

Mathematical Relationships (MR) measures the extent to which students are supported to develop taken-as-shared understandings of the key mathematical ideas and relationships, as they are represented in the task statement. A score of 0 indicates that no attention is given to key mathematical ideas or relationships in the task. A score of 1 or 2 indicates that the students are, at best, minimally involved (e.g., giving yes or no responses) in a discussion of the key mathematical ideas and relationships in the task statement. A score of 3 or 4 indicates that the teacher and students connect ideas together, with a 4 indicating that these connections are fully developed and conceptual in nature. In other words, a 3 or 4 indicates there was evidence that a taken-as-shared understanding of the key mathematical ideas and relationships as represented in the task statement was developed.

Thus far, we have framed most of our discussion in terms of teacher and student talk. For both task-as-set-up rubrics, we acknowledge that visual representations can serve as a powerful focus of student talk, particularly as a way to establish a taken-as-shared understanding of an important idea. However, in order for the use of a visual representation to contribute toward establishing a taken-as-shared understanding, we decided that more than one student had to talk about the representation. As Moschkovich (1999) clarifies, “Although using objects to clarify meanings is an important ESL instructional strategy, it is crucial to understand that these objects do not provide 'extra-linguistic clues.' The objects and their meanings are not separate from language, but rather acquire meaning through being talked about and these meanings are negotiated through talk” (p. 13).

As described above in the conceptual framework, a challenge in setting up tasks, particularly those that are complex, is maintaining the cognitive demand of the task. In order to account for this, we included an additional measure in the coding of the instructional videos. We asked coders to assess whether the teacher maintained, increased, or decreased the cognitive demand of the task during the task-as-set-up phase of instruction; we call this the *Set-Up Maintenance*. A lesson is scored as "maintain" if the set-up does not alter the cognitive demand of the initial task the students are to complete. A lesson is scored as decrease if the teacher reduces the cognitive demand of the initial task. For example, had Mr. Lewis suggested that students might want to start by creating tables of equations to solve the task or reviewed a sample problem advocating for a particular approach to the problem, a coder would have assigned a score of "decrease" as the Set-Up Maintenance.

Coding of Video-Recordings

Coders were trained to use the expanded IQA in a reliable manner. Before actual coding began, coders were required to achieve 80% reliability on a set of previously coded videos, chosen to represent the variety of anomalies that the coders would encounter. Each of the coders was randomly assigned a list of teachers to code using the expanded IQA. The set of two class days for each teacher was coded chronologically, given that it was possible that the lesson from the first day might continue into the second day (which would result in just one set of scores for the spanning lesson). It is important to note that given this coder assignment process, the same coders scored the set-up and whole-class discussion phases of instruction, often in the same sitting. However, scoring is generally completed over the course of viewing the lesson; in other words, coders generally assigned scores for the set-up phase of instruction at the end of the set-up, prior to viewing the subsequent phases of the lesson. We mention this to acknowledge that the fact that the same coder assessed the set-up and the whole-class discussion could be a source of some bias in the tested relationships between the two phases of instruction; however, we do not think that it outweighs the potential benefits of this analysis.

Over the course of the coding period, one set of teacher scores for each coder was randomly checked for reliability once every two weeks to account for rater drift (which resulted in double-coding of approximately 10% of the teachers). Any discrepancies were consensus coded to maximize the accuracy of the data and to allow for ongoing learning on the part of the coders. The overall percent agreement for the coders was 81.6% with an average kappa score of 0.66. The percent agreement range for the expanded IQA was from 66.7% to 100%, and the kappa scores ranged from 0.54 to 0.80. Because this is the first time the task-as-set-up rubrics were used on a large scale, we give reliability information for each rubric separately in Table 1. We also provide reliability information for the rubrics from the traditional IQA that we use in our analyses. In general, the reliability scores for the task-as-set-up rubrics do not differ significantly from the reliability scores for the IQA rubrics.

========================[INSERT TABLE 1 ABOUT HERE]=====================

Analyses and Results

In order to answer our research questions, we pursued several lines of analysis. Our first set of analyses describes the nature of the set-up across a large sample of middle school mathematics classrooms. Our second set of analyses examines relationships between the quality of the set-up and the quality of the whole-class discussion. For ease of explanation and interpretation of our results, the details of our methods of analysis are presented separately for each set of research questions, immediately prior to the presentation of the corresponding results.

Prior to reviewing our analyses and results for each research question, we describe our sample of lessons' scores on the traditional IQA rubrics that we use to answer our research questions. In this paper, we use the scores from Task Potential, Academic Rigor of the Discussion, Student Linking, and Student Providing rubrics from the traditional IQA. Each rubric's score range, mean, and standard deviation are given in Table 2. The Task Potential score possibilities range from a 1 to 4 and the other three rubrics range from 0 to 4. As is evident in Table 2, none of the lessons achieved the maximum score on Student Linking and scores rarely exceed a 2.

=====================[INSERT TABLE 2 ABOUT HERE]======================

The Nature of the Set-Up

Attention to the Contextual Features and Key Mathematical Relationships. Our coding of middle school mathematics teachers' lessons with the expanded IQA provides us with a sense of the nature of the set-up phase of instruction. Initially, we focus on the extent to which students are supported to develop taken-as-shared understandings of the contextual features of the problem-solving scenario (PSS) and the key mathematical ideas and relationships as they are represented in the task statement. For ease of reference, we shorten our use of these terms to the quality of the attention to the contextual features and the quality of the attention to the mathematical relationships, respectively.

It is possible that the nature of the set-up of tasks might differ dramatically depending on whether a task included a PSS. For this reason, we generally examined these two types of lessons (lessons with PSS tasks, lessons with non-PSS tasks) separately as well as together to be able to identify important differences. In our sample of 242 lessons, 138 involved tasks with PSSs while the other 104 lessons involved non-PSS tasks. Regardless of whether a task has a PSS, the lesson is scored using the MR rubric. But, recall that the CF rubric only applies to tasks with PSSs, so the sub-sample of lessons with scores for both MR and CF consists of the 138 lessons involving PSS tasks. Table 3 gives frequencies of CF and MR scores for lessons with PSS tasks and non-PSS tasks.

As represented in Table 3, in lessons with PSS tasks, the mean attention to mathematical relationships is significantly higher than the mean attention to the contextual features ($p<.001$). In other words, teachers appear to attend in higher quality ways to the key mathematical ideas than to the contextual features. Additionally, there does not appear to be any statistical difference between lessons with PSS and those with non-PSS tasks in the mean quality of attention to the mathematical relationships ($p=.312$).

======================[INSERT TABLE 3 ABOUT HERE]======================

Given that the quality of the attention to mathematical relationships tends to be higher than the quality of the attention to the contextual features, one might wonder if high-quality attention to mathematical relationships occurs at the expense of quality attention to contextual features in lessons with PSS tasks. The cross-tabulation of scores for CF and MR given in Table 4 shows that this does not appear to be the case. In fact, scores for CF and MR are significantly positively correlated ($r = .439$, $p < .001$), meaning that in lessons with PSS tasks, scores for one measure generally tend in the direction of scores for the other. Despite that trend, as displayed in Table 4, only 9 of the 138 lessons involve attention to the mathematical relationships and contextual features in taken-as-shared ways (i.e., at a level 3 or 4 on both rubrics).

======================[INSERT TABLE 4 ABOUT HERE]=======================

Maintenance of the Cognitive Demand in the Set-Up. As mentioned above, we asked coders to assess the maintenance of the cognitive demand during the set-up. Table 5 provides an overview of the Set-Up Maintenance. First, in over half of the lessons (62.8%), the cognitive demand of the task was decreased. Such a decrease in cognitive demand can manifest itself subtly if a teacher suggests a particular approach to the problem, yet still requires students to carry out the task as described in the instructional materials. More dramatically, a teacher may decrease the cognitive demand during the set-up by modifying the task as described in the instructional materials (e.g., the teacher eliminates task sections that required students to explain their mathematical reasoning). In addition, we find that lessons with Task Potential of 2 or 3 are significantly more likely than lessons with Task Potential of 1 or 4 to decrease in cognitive demand during the set up ($p<.001$).

=====================[INSERT TABLE 5 ABOUT HERE]========================

Additionally, we examined the extent to which teachers were able to maintain the cognitive demand while providing students with access to key ideas of the task. We investigated all possible CF/MR score pairs (e.g., 1 for CF and 3 for MR, 2 for CF and 3 for MR, etc.) in relation to the Set-Up Maintenance. Table 6 shows the number of lessons with each possible CF/MR score pair. Along the bottom row and far right column, we provide the overall percent of lessons within each MR and CF score category in which the lesson received a Set-Up Maintenance score of decrease. Approximately 48.8% of lessons with an MR score of 1, approximately 76.6% of lessons with an MR score of 2, approximately 55.5% of lessons with an MR score of 3, and only 18.2% of lessons with an MR score of 4 decreased in cognitive demand during the set-up phase of instruction. The fact that only 18.2% of the lessons with MR scores of 4 decreased in cognitive demand is promising with regard to being able to attend to key ideas of complex tasks without decreasing the cognitive demand. Also, it is interesting that the highest percentage of lessons with decreases in cognitive demand occur in lessons in which the attention to MR is at a level 2. Lastly, we did not find the same trends in the percentages of lessons that decreased in cognitive demand with regard to the score categories for CF: the percentages do not differ dramatically by score category and range from 49.8% to 67.7%. We describe possible reasons for several of these findings in the discussion below. These findings suggest that while setting up a task so that all students have access and maintaining the cognitive demand is challenging, it is indeed possible to do so.

=====================[INSERT TABLE 6 ABOUT HERE]======================

Relationships between the Set-Up and Whole-Class Discussion

While descriptive information about the set-up is useful for describing the nature of that phase of instruction for a large sample of teachers, ultimately we are interested in how what is accomplished in the set-up might impact opportunities to learn in subsequent phases of instruction. That said, given the nature of our data, we are only able to descriptively report relationships rather than make causal claims. Several sets of hierarchical linear models (HLMs) allowed us to explore and describe the relationships between aspects of the set-up and students' opportunities to learn within the whole-class discussion. We used HLMs to account for clustering within schools and within teachers (due to our treatment of the two lessons for most teachers as independent observations).

For ease of interpretation of results, we describe the series of 8 models that were estimated for each of the three outcomes of interest (Academic Rigor of the Discussion, Student Linking, and Student Providing). It is possible that the relationship between the set-up and whole-class phases of discussion may vary depending on whether a task had a PSS. For this reason, we maintained our approach of examining the two types of lessons (those with PSS tasks and those with non-PSS tasks) separately as well as together. The relatively small sample sizes of the two types of lessons required that we make careful decisions about which variables to include at which times. For this reason, we used a set of HLMs for each outcome of interest, rather than one all-inclusive HLM, to look for trends in relationships and to highlight particular features. Although the outcome variables change for each set of HLMs, the independent variables included within each set of models is the same. In each model we include dummy variables to account for differences by district and scores for Task Potential, Maintenance, and Mathematical Relationships.

Models 1 and 2 were baseline MR models that regressed only Task Potential, Maintenance, District controls, and MR on the outcome of interest. Model 1 was limited to lessons with PSS tasks while Model 2 was limited to lessons with non-PSS tasks. Models 3 and 4 explored the differential relationships of each MR score level on the two different task types (i.e., PSS or non-PSS) by replacing the MR score in the models with dummy variables for each non-zero MR score category (i.e., MR-1, MR-2, MR-3, and MR-4), while keeping the rest of the model the same. Model 5 included the same covariates as Models 3 and 4, but it was estimated on the entire sample of lessons, so it also included a dummy variable to account for whether each lesson used a non-PSS task. Models 6, 7, and 8 were limited to lessons with PSS tasks because the focus was the addition of CF (which only applies when tasks involve a PSS). Model 6 included the same covariates as Model 1 with the addition of CF. Model 7 included the same covariates as Model 3, investigating the different MR score levels, with the addition of CF. Model 8 paralleled our approach of exploring differential relationships by score levels in Models 3-5, but this time, CF score categories (CF-1, CF-2, and CF-3 or 4) were included as dummy variables. (We combined 3s and 4s on CF because 3s or 4s only occurred in 12 lessons, with only 2 lessons scored at a 4.) As in the other CF-focused models, we included MR as a covariate in Model 8 as well. Lastly, we note that we report three different levels of significance in the tables of results for the HLMs, but in the narrative descriptions of our results, we use $p < 0.1$ as our cut-off for statistical significance due to our small sample size.

===================[INSERT TABLE 7 ABOUT HERE]==========================

We first describe the results from the models with Academic Rigor of the Discussion as the outcome. The full set of results is listed in Table 7, but there are several findings worth highlighting. First, Maintenance was nearly consistently (with the exception of only Model 4), significantly, positively related to Academic Rigor of the Discussion while controlling for the Task Potential, MR, and CF (where applicable).

Second, scores on Mathematical Relationships were nearly consistently (with the exception of two models: Model 3 and Model 7) significant and positively related to the Academic Rigor of the Discussion. Further exploration of the differential relationships of particular scores revealed that the scores of a 3 and 4 were the scores driving the positive relationship between MR and Academic Rigor of the Discussion (see Models 4 and 5 in Table 7). These models used an MR score of 0 as the comparison group, and indicate that scores of 1 and 2 on MR are not significantly different from an MR score of 0, while scores of 3 and 4 are significantly different from an MR score of 0. Even though the coefficients for 3 and 4 are not statistically different from one another, the increasing magnitude over the score categories is interesting as well. These results suggest that the quality of the attention to key mathematical relationships and ideas is crucial—the greater the attention to establishing a taken-as-shared understanding of MR, the stronger the relationship with the quality of the discussion. These findings resulted from using models with lessons with non-PSS tasks and a model that combined all lessons.

However, these findings did not hold true when we examined lessons that only involved tasks with PSS (both when controlling for CF, in Model 7, and when CF was omitted in Model 4). Also, none of these models that included CF resulted in a statistically significant relationship between CF and Academic Rigor of the Discussion when controlling for Task Potential, Maintenance, and MR. In lessons with PSS tasks, the positive relationship between MR and Academic Rigor of the Discussion held when MR was included as one variable; however, the high scores did not appear to drive the relationship in the same way as in the other models. While the coefficients generally behaved in a manner consistent with the other models, the standard errors increased at the high MR score levels. This signals that there is more variation in the relationship between high MR scores and Academic Rigor of the Discussion in lessons with PSS tasks than compared with lessons with non-PSS tasks. This is a result that warrants more attention in the future.

Third, this set of models generally suggests a significant difference between Districts A and C (represented by a significant coefficient on the dummy variable for District C) with regard to Academic Rigor of the Discussion that is not fully explained by differences in Task Potential or our measured aspects of the set-up. It appears to hold most strongly for models with lessons with PSS tasks and is less significant in models that control for attention to the contextual features. Although district differences are not our primary focus, we do comment on this result in the discussion, below.

=======================[INSERT Table 8 ABOUT HERE]======================

The second set of models explored relationships between aspects of the set-up and Student Linking (see the full set of results listed in Table 8). Recall that Student Linking measures the extent to which students' contributions link to and build on each other.

First, we see that Maintenance was somewhat consistently (with a few exceptions, Models 3, 4, and 5), significantly, positively related to Student Linking in the discussion while controlling for the Task Potential, MR, and CF (where applicable).

Second, scores on MR were consistently significantly and positively related to Student Linking. However, unlike the models with Academic Rigor of the Discussion as the outcome, further exploration of the differential relationships of particular scores for these models revealed that there was no specific score driving the positive relationship between MR and Student Linking (see Models 3, 4, 5 and 7 in Table 8). These findings resulted from using models with lessons that involved tasks without PSSs, a model that combined all lessons, as well as a model restricted to lessons that involved tasks with PSSs when controlling for CF.

Third, though one of the three models that included CF did not result in a statistically significant relationship between CF and Student Linking (see Model 7 in Table 8), we do see an indication of a relationship (see Model 6) between CF and Student Linking. In fact, in the model that investigates the differential impact of CF scores, we specifically see that "high" CF (i.e. CF scores of 3 or 4) is significantly, positively related to Student Linking when controlling for Task Potential, Maintenance, and MR (see Model 8).

====================[INSERT TABLE 9 ABOUTHERE]========================

The third set of models explored relationships between aspects of the set-up and Student Providing (see the full set of results listed in Table 9). Recall that Student Providing measures the extent to which students support their contributions with evidence and/or reasoning.

First, we see that Set-Up Maintenance was nearly consistently (with only one exception: Model 4), significantly, positively related to Student Providing in the discussion while controlling for the Task Potential, MR, and CF (where applicable).

Second, scores on Mathematical Relationships were consistently significant and positively related to Student Providing. Further exploration of the differential relationships of particular scores demonstrated that a score of a 4 was driving the positive relationship between MR and Student Providing (see Models 3, 4, 5 and 7 in Table 9). In other words, when we examined the MR levels separately, only the 4s were consistently significantly different from an MR score of 0, our reference category in those models. This again suggests that the *quality* of attention to key mathematical relationships is important.

Third, similar to our findings with Student Linking as the outcome, two of the three models that included CF did not result in a statistically significant relationship between CF and Student Providing (see Models 6 and 7 in table 9). However, in Model 8 we again see that “high” CF (i.e., CF scores of 3 or 4) is significantly and positively related to Student Providing when controlling for Task Potential, Maintenance, and MR (see Model 8 in Table 9).

Fourth, this set of models suggests a significant difference between Districts A and C (again represented by a significant coefficient on the dummy variable for District C) with regard to Student Providing that is not fully explained by differences in Task Potential or our measured aspects of the set-up. It appears to hold most strongly for models with lessons with PSS tasks. We comment on this result in the discussion, below.

More generally, our analyses indicate that the maintenance of the cognitive demand, attendance to the mathematical relationships of the task, *and* attendance to the contextual features of the task (particularly in shared ways) are all positively related to student linking and providing in the concluding whole-class discussion. Examining Student Linking and Student Providing as outcomes reveals that attendance to the contextual features during the set-up may positively relate to the way students build on each other's contributions and provide conceptual explanations during the discussion.

Limitations of the Study

Before turning to a discussion of our findings, we briefly state several limitations of this study. First, as we clarified above, our findings are descriptive in nature. Given our data and methods of analysis, we are unable to assert causality. Although we detected a relationship between the nature of activity in the set-up and in the concluding whole-class discussion, we cannot claim that what happened in the set-up necessarily impacted what happened in the whole-class discussion. However, we were able to identify characteristics of the set-up that appear to be significantly related to characteristics of the whole-class discussion, controlling for other characteristics of instruction. Second, as we stated above, the questions we asked of our data, and therefore our findings, generally assumed a three-phase lesson structure. That said, although investigating variability within lesson structures is beyond the scope of this study, it is important to note that even though teachers were asked to include a whole-class discussion, not all did. Relatedly, teachers spent varied amounts of time on the set-up versus other phases of instruction. Although we collected data on time spent on various phases of instruction, we did not analyze it for the purpose of this study. Third, we only investigated the relationship between the set-up and the concluding whole-class discussion; we did not investigate the relationship between the set-up and the second phase of instruction (when students work on solving the task). In fact, the evidence of whether a set-up is productive in terms of supporting students' participation is probably best determined by analyzing what happens when students start to solve the task. However, we were unable to systematically examine students' activity in phase 2 of the lessons because of the nature of the video data. At the start of the study, it was assumed that the video-recordings would be coded by the traditional IQA, which attends to phase 2 in a global manner (e.g., whether the teacher and students reduce the cognitive demand of the task as students work to solve it). The teacher wore a lapel microphone, and the videographers were instructed to place a paddle microphone by 1-2 groups of students to capture the nature of their talk during phase 2. In order to reliably compare activity in the set-up to student talk and activity during phase 2, we would have needed access to all of the students' talk and activity during that phase of instruction.

Discussion

We undertook this study in an effort to begin to specify what appears as important to attend to in the set-up phase of instruction, justified in terms of equity in students' opportunities to learn. In this section, we summarize and elaborate on our key findings as they relate to the larger goal of this study. We also raise questions about our findings, and suggest areas for future research specific to the set-up phase of instruction.

Perhaps most importantly, based on our findings, we have reason to believe that, indeed, the set-up phase of instruction matters in terms of students' opportunities to learn. Namely, it appears that the quality of the set-up is related to the quality of the concluding whole-class discussion. We identified several ways in which the nature of the set-up relates to the nature of the whole-class discussion.

First, we found that the extent to which the cognitive demand of the task was maintained in the set-up was almost always significantly, positively related to the quality of the whole-class discussion. In other words, in cases in which teachers and students maintained the cognitive demand of the task, with the potential of the task held constant, the concluding whole-class discussion was of higher quality. There were a few exceptions. Generally, when looking across the discussion-quality outcome measures, the models in which Set-Up Maintenance was not significantly related to the discussion outcome were models that explored the significance of different levels of MR. We hypothesize that this might be related to the fact that at different MR score levels, the rate of occurrence of decreases in the cognitive demand varied (as shown in Table 6). We plan to explore this more fully in the future.

Second, we found that the quality of the attention to mathematical relationships in the set-up was generally positively related to the quality of the whole-class discussion. In particular, engaging in activity aimed at establishing taken-as-shared understandings of mathematical relationships (i.e., a 3 or 4 on MR) in the set-up were more strongly (and significantly) related to the quality of the discussion than low-quality attention to the mathematical relationships (i.e., a 1 or 2 on MR) in the set-up. Furthermore, establishing taken-as-shared understandings of mathematical relationships in a consistent manner (i.e., a 4 on MR) was more frequently related to the quality of the discussion than doing so less consistently (i.e., a 3 or MR). This suggests the importance of not only attending to the mathematical relationships in the set-up, but doing so in ways that support students' to build upon one another's ideas. In other words, it is not enough to have students share their isolated ideas about key mathematical ideas in the set-up (this would receive a 2 on MR). It appears that some "orchestration" of discussion (Stein, et al., 2008) of those key mathematical ideas is also desirable in the set-up.

Third, we found positive relationships between the quality of the set-up and the quality of the whole-class discussion regardless of whether the task included a problem-solving scenario or not, although the nature of the relationships differed slightly for lessons with different task types. In other words, even in cases where the task did not include a scenario, we still detected positive associations between attending to the key mathematical ideas or relationships of the task in taken-as-shared ways in the set-up and the quality of the whole-class discussion.

Some of our initial interest in investigating the set-up was prompted by prior research that has suggested equity concerns in terms of the cultural suppositions of situations used in problem-solving scenarios (Ball, et al., 2005; Boaler, 2002; Silver, et al., 1995; Tate, 1995) as well as our own viewing of instruction in which it was clear that not all students were equally familiar with the contextual features of the scenario. A key question is, then, when a lesson includes a task with a PSS, how is attention to the contextual features (i.e., the non-mathematical features of a scenario) in the set-up related to the whole-class discussion? We did not detect a statistically significant relationship between CF and the Academic Rigor of the Discussion. However, we did detect statistically significant relationships between high-quality attention to CF and the nature of student contributions (i.e., student linking, student providing) in the whole-class discussion, when controlling for MR, Task Potential, and Set-Up Maintenance. In other words, our findings suggest that establishing taken-as-shared understandings of the key contextual features of the PSS in the set-up adds value to the nature of student contributions in the whole-class discussion. This is the fourth way in which we found the set-up relates to the quality of the whole-class discussion. Students were more likely to make connections to one another's ideas and to provide conceptual evidence for their reasoning in the whole-class discussion when taken-as-shared understandings of the contextual features of the PSS were established in the set-up.

An important, related finding of our study was that in lessons with PSS tasks, teachers tended to attend to the mathematical relationships much more than the contextual features, and in higher-quality ways. There was no attention to the contextual features in 49% of lessons with PSS tasks, while there was no attention to key mathematical ideas in only 6% of those lessons. Additionally, we found statistically significant differences in the quality of attention to the two features, with a significantly higher mean for MR. It is important to note that even with these differences, high-quality attention to either aspect was relatively rare, with scores of 3 or 4 occurring for MR in only 16% of all lessons and with scores of 3 or 4 occurring for CF in only 9% of lessons with PSS tasks. Given that we found that high-quality attention to the contextual features is positively related to students' contributions in the whole-class discussion, it seems important to research this phenomenon further. For example, how do teachers conceive of the contextual features of a problem-solving scenario? Do they see the aspects of the PSS as potentially alienating or unfamiliar to students? How does their thinking about the contextual features relate to their mathematical goals for the lesson?

As noted above, the positive relationships between the measured aspects of the set-up and the whole-class discussion (using both global and more specific, student-contribution measures) generally held. This implies that, assuming a fixed task potential, when teachers and students attended to contextual features (in the cases of lessons with PSS tasks) and mathematical relationships in taken-as-shared ways *and* maintained the cognitive demand of the task, discussions were generally of higher quality. A key question to ask is how likely is it that set-ups meet these conditions? As we illustrated in our Results, the percentage of lessons in which these conditions were met was relatively low (6.5 % of lessons with PSS tasks). Only 13 teachers attended to MR and CF (if applicable) at a level 3 or higher *and* maintained the cognitive demand of the task. That said, in our view, although it was rare, a central finding of our study is that teachers can develop taken-as-shared understandings of key contextual features (if using a task with a PSS) and key mathematical relationships in the set-up *and* maintain the cognitive demand of an activity. We view it is as valuable to have found evidence that it is *possible* to do both.

A related, important finding is that, unfortunately, in our sample of 132 teachers, it was very common for teachers and students to lower the cognitive demand of a task during the set-up phase of instruction. Given that the cognitive demand of an activity is a significant predictor of students' opportunities to learn, this finding suggests that future research is needed to detail how cognitive demand is lowered during this phase of instruction. Anecdotally, we note that it is quite common for teachers to decrease the cognitive demand of activity during the set-up by demonstrating how to do an example problem that is similar to the task asked of students or by telling students to use a particular procedure to solve a problem; however, systematic research is needed to detail the typical ways in which cognitive demand is decreased during this phase of instruction and *why*. For example, is it related to teachers' mathematical goals for the lesson (or sequence of lessons)? Is it related to teachers' views of students mathematics capabilities (Jackson, Gibbons, Garrison, & Munter, in progress)? Similarly, future research is needed to qualitatively describe teaching in which the cognitive demand is *maintained* decreased during the set-up. What does that teaching look like? How do teachers that maintain the cognitive demand conceive of their mathematical and academic goals in the set-up?

We should note, however, that although decreasing the cognitive demand of the task during the set-up was quite common, it was most common when the task chosen was procedural in nature (i.e., scored as a 2 on the Potential of the Task rubric). It was actually less common for the cognitive demand of a task to be decreased during the set-up if a teacher chose a task of high cognitive demand (i.e., a 3 or 4 on the Potential of the Task rubric) from instructional materials. One explanation for this is that procedural tasks lend themselves more easily to a decrease in the cognitive demand over the course of the set-up. Procedural tasks often include many problems of the same kind (i.e., sets of naked number problems). Given that there are multiple problems that require the application of very similar procedures, it is quite likely that teachers might do a few examples with the students beforehand; this would be a decrease in cognitive demand, assuming that a strategy was suggested. An alternative, and perhaps complementary, explanation is that teachers who choose tasks of higher cognitive demand might have more rigorous goals for their students' learning, and are therefore more concerned with maintaining the cognitive demand of activity during the set-up.

Last, we return to an issue we raised in the Results regarding what appeared to be district-level differences in the relationships between the set-ups and the whole-class discussions. Although investigating the reasons behind district-level differences were beyond the scope of this study, based on our research team's involvement in the four districts for several years, we are in a position to offer some conjectures regarding why there were differences. First, to clarify, the differences were statistically significant when comparing District C lessons to those in District A. As we noted in the Methods, District A teachers had significantly more experience than teachers in Districts B, C, and D. In addition, not only had they been teaching longer, most teachers had several years of experience using the *Connected Mathematics Project* curriculum. That said, we did not detect a significant difference for the set-up/whole-class discussion relationships between Districts B or D and District A. Why, then, were only District C lessons significantly different from District A lessons? One explanation is the nature of the curriculum that District C teachers used. Although it was designed to support a three-phase lesson structure (i.e., include a set-up and concluding whole-class discussion), it did not include as much teacher direction (or support) for a set-up as the CMP2 teacher's guide. Additionally, District C teachers were less likely to hold whole-class discussions than teachers in the other districts, even given our explicit request to do so. We suspect this is because they often taught quite conventionally (introduction of a procedure, students solve several problems that require the application of the procedure); if a concluding whole-class discussion was held, it typically consisted of the teacher and students discussing the answers to the problems in a calculational manner, at best. On the other hand, CMP2 (the curriculum used in Districts A, B, and D) consistently suggested the use of a task of high-cognitive demand, and the CMP2 teacher's guide provided explicit support on what a concluding whole-class discussion should entail.

Conclusion: Setting-Up Tasks as a "High-Leverage Practice"

A major motivation for this study was to contribute to the elaboration of what might be called an equitable dimension of ambitious teaching. In other words, what are forms of practice that teachers can develop that will, in turn, support more of their students to substantially participate in mathematical activity aimed at ambitious learning goals? Based on our findings from this study, we are suggesting that setting up, or launching, tasks is a practice of this kind, or what Ball, Sleep, Boerst, and Bass (2009) call a "high-leverage practice."

Ball et al. (2009) advocate for the identification and elaboration of "high-leverage practices" in the context of describing a professional curriculum for mathematics teacher education. As they argue, rightly in our view, teacher education cannot include all that one needs to know and do to be an effective teacher. Instead, choices need to be made about what is "high-leverage," "or teaching practices in which the proficient enactment by a teacher is likely to lead to comparatively advances in student learning" (p. 461). Similarly, Lampert, Kazemi, Franke and colleagues (Kazemi, et al., 2009; Lampert, et al., 2010) advocate for the identification of what they call "instructional activities," or "tasks enacted in classrooms that structure the relationship between the teacher and students around content in ways that consistently maintain high expectations of student learning while adapting to the contingencies of particular instructional interactions" (Kazemi, et al., 2009, p. 3). In both cases, these groups of mathematics educators are arguing for identifying forms of practice that are 1) somewhat stable, or consistent, in ambitious teaching (i.e., teaching that leads to all students' development of increasingly sophisticated understandings of mathematics); and 2) able to be taught to pre- and in-service teachers.

Ball et al. (2009) suggest a set of criteria for determining candidates for high-leverage practices. The first four criteria were based on their knowledge of mathematics teaching, and the last four were "necessitated by [their] teacher education context":

1. Supports work that is central to mathematics.
2. Helps to improve the learning and achievement of all students.
3. Is done frequently when teaching mathematics.
4. Applies across different approaches to teaching mathematics.
5. Can be articulated and taught.
6. Is accessible to learners of teaching.
7. Can be revisited in increasingly sophisticated and integrated acts of teaching.
8. Is able to be practiced by beginners in their field-based settings. (p. 461)

In the remaining paragraphs, we first defend our claim that setting up tasks is a high-leverage practice (i.e., we reflect on our findings in light of the first four criteria). Our reflection on the first four criteria also highlights areas for future research. Then, using criteria 5-8 as a guide, we consider what further research would be required such that pre- and in-service teachers could be supported to develop competencies in setting up cognitively demanding tasks.

We provided evidence of a positive relationship between the quality of set-ups and the quality of whole-class discussion. Since participating in a conceptually-oriented discussion regarding mathematical ideas involves, for example, articulating and defending one's reasoning, and making connections between solution strategies and underlying mathematical ideas, we view this as evidence that set-ups support work central to mathematics (criterion 1).

Regarding the second criterion, we did not directly measure students' learning. However, we used measures of students' opportunities to learn that (particularly in the case of the cognitive demand of activity and quality of whole-class discussion) are grounded in the mathematics education research literature. Therefore, we feel fairly confident that our findings suggest it is likely that high-quality attention to the contextual features and mathematical relationships of a given task in the set-up improves the *learning opportunities* of all students. However, it would be very worthwhile for future research to investigate learning and achievement outcomes in relation to a careful analysis of the set-up, what happens while students work on solving the problems, and the whole-class discussion.

We consider the third and fourth criteria together—the extent to which set-ups are frequent and whether what we identified as key aspects of a set-up applies across teaching approaches. As we stated earlier, some middle-grades mathematics curricula specify the use of a "launch," and therefore, setting up tasks is arguably something that happens frequently when teaching mathematics in those contexts that use such curricula. Clearly, our findings were specific to middle-grades teaching that included the use of a curriculum that specified a launch. Even in the case of District C, which did not use CMP2, teachers were generally expected to include a set-up phase of instruction. However, organizing instruction in three phases is not the only way to organize a middle-grades mathematics lesson. Future research may want to investigate the effects of attending to similar aspects as we did (e.g., attention to contextual features and/or mathematical relationships in relation to task statements) in lessons that do not follow a three-phase lesson structure (e.g., where students and teachers explore connected mathematical ideas in several stages over the course of a lesson). It is also worth reminding the reader that our study was restricted to districts in which teachers were being explicitly supported to develop ambitious forms of teaching aimed at rigorous learning goals for students. We are unable to say whether attending to the key aspects of a set-up is critical in teaching that is not aimed at similar learning goals for students.

Assuming that we have provided evidence that setting up tasks is at least worthy of consideration as a high-leverage practice, a key question is what would it take (research- and practice-wise) to support teachers (both pre- and in-service) to develop high-quality forms of the practice? Another way to think about this is: what evidence is needed to evaluate whether or not setting-up tasks meet Ball et al.'s (2009) criteria 5-8, listed above?

In order to claim that setting-up tasks "can be articulated and taught" (criterion 5), it would first be necessary to flesh out in sufficient detail what setting-up tasks in high-quality ways includes. Specifically, it requires, in Grossman and colleagues' (Grossman et al., 2009) terms, the decomposition of the practice of setting up tasks. Decomposition involves making "visible the grammar of practice to novices and may require a specific technical language for describing the implicit grammar and for naming the parts" (p. 2069). We view our work on delineating four key aspects of setting-up tasks as making significant headway toward the decomposition of the practice. Specifically, we named what are potential "parts" of setting-up tasks in high-quality ways—namely supporting the development of taken-as-shared understandings of key contextual features and mathematical relationships while maintaining the cognitive demand of the task. However, we anticipate that more research is needed to flesh out the "grammar of practice" in detail sufficient to support novices to both analyze and begin to enact the forms of practice. Part of this work involves providing the field with images, or representations, of the practice of setting-up tasks. We provided one such image in our description of Mr. Lewis' set-up. Based on our viewing of numerous set-ups, we are fairly confident that there are multiple ways in which one might set-up a task that reflect the key aspects we identified in our study. However, research would be necessary to more systematically develop a set of images that together could serve as the foundation for "describing the implicit grammar" of setting-up tasks.

Evaluating evidence for criteria 6-8 will depend on research focused on supporting both pre- and in-service teachers to learn to set up tasks in teacher education contexts. To this end, we know of work in elementary and secondary mathematics teacher education that is promising in this regard. Kazemi, Franke, Lampert, and colleagues (Kazemi, et al., 2009) are involved in teacher education work that focuses on setting up tasks in the context of elementary mathematics. Shahan and Gibbons (2011) have been investigating how to support both elementary and secondary mathematics education teacher candidates to learn to set-up tasks. Both groups of mathematics teacher educators have designed pedagogies of investigation and enactment (Grossman & McDonald, 2008) aimed at supporting teacher candidates to learn to effectively set-up tasks. Namely, they are deliberately designing activities that support novices to analyze set-ups and to engage in increasingly sophisticated acts of setting up tasks (thus potentially providing evidence for criterion 7 and 8). More generally, it would be useful to have research documenting the extent to which teacher education designs meet criteria 5-8 regarding setting-up tasks (and other high-leverage practices), with suggestions as to how we can improve our teacher education designs.

Both of the projects mentioned above are focused on pre-service teachers. It will also be important to research effective designs for supporting in-service teachers to develop the complex practice of setting-up tasks in high-quality ways. Presumably, in-service teachers represent a different population than pre-service teachers in some regards. For example, it is likely easier to convince in-service teachers of the consequences of a poor set-up (given that they have likely experienced this in their teaching) than pre-service teachers.

In closing, we view what happens in the set-up phase of instruction as consequential for students' opportunities to engage in rigorous mathematical activity. Our data suggest that this is a phase of instruction that is not often carefully attended to in middle-grades mathematics teaching, yet appears to be related to the extent to which students are able to participate in concluding whole-class discussions in high-quality ways. This suggests evidence that what happens in the set-up phase of instruction has implications for equity in students' learning opportunities. More generally, we view this sort of analysis—as one that is generated from observations of teachers' practice and aimed at making visible forms of practice that teachers can develop to support all students' participation—as necessary to elaborate the equitable dimension of ambitious teaching.

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Appendix: Select Rubrics from the Expanded IQA

Task as Set Up

I. How effectively did the task-as-set-up phase of instruction* establish a shared understanding of the contextual features of the problem-solving scenario and what is to be mathematized such that the students are able to begin solving the task?
*****Includes prior to introduction of the task and/or in the context of introducing the task

RUBRIC 1: Contextual Features
To what extent were students supported to develop a shared understanding of the contextual features of the problem-solving scenario?

Rubric 1: Contextual Features	
4	Teacher elicits what students know about the problem-solving scenario. The teacher and/or students **consistently** make connections between ideas that are likely to support the establishment of a shared understanding of the contextual features of the problem-solving scenario (e.g., through accountable talk moves like marking, revoicing, linking, pressing; connecting to shared experiences; establishing a representation that students refer to). More than one student actively participates in this segment of instruction. Students respond in ways that demonstrate knowledge or understanding of the contextual features of the problem-solving scenario. **The teacher and/or students must consistently make connections between ideas (that could include the establishment of a representation) that support and/or build toward a shared understanding of the contextual features of the scenario.** *The difference between a 3 and a 4 is that the teacher and/or students* ***consistently connect*** *ideas that build toward a shared understanding of the contextual features of the problem-solving scenario.*
3	Teacher elicits what students know about the problem-solving scenario. The teacher and/or students **inconsistently** make connections between ideas that are likely to support the establishment of a shared understanding of the contextual features of the problem-solving scenario (e.g., through accountable talk moves like marking, revoicing, linking, pressing; connecting to shared experiences; establishing a representation that students refer to). More than one student actively participates in this segment of instruction. Students respond in ways that demonstrate knowledge or understanding of the contextual features of the problem-solving scenario. **The teacher and/or students inconsistently make connections between ideas (that could include the establishment of a representation) that support and/or build toward a shared understanding of the contextual features of the scenario.** *The difference between a 2 and a 3 is that the teacher and/or students connect ideas together (albeit inconsistently) to build toward a shared understanding of the contextual features of the problem-solving scenario.*
2	Teacher elicits what students know about the problem-solving scenario *but* the teacher and/or students **do not connect ideas** together in a way that would support students in establishing a shared understanding of the contextual features of the problem-solving scenario. (I.e., the

	teacher surfaces some initial ideas about the contextual features of the problem-solving scenario, but the ideas do not build to a greater or shared understanding of the contextual features. For example, the teacher may ask the same question to a number of students to gather information, "What is your favorite_____?" but does not support a shared understanding of the particular idea.) *Examples of establishing a shared understanding of the contextual features of the problem-solving scenario; examples apply to 2-4; score will increase depending on the extent* ***to which the teacher and/or students connect ideas together to build toward a shared understanding of the contextual features of the task****:* • Teacher elicits students' understandings of the contextual features of the problem-solving scenario (e.g., Teacher asks students, *Can anyone tell me what a dance marathon is?*). • Teacher asks that students imagine that they are participating in the problem scenario, and then elicits information from the students about the scenario. • Teacher elicits information from students about teacher-student shared experiences that are central to the contextual features of the problem-solving scenario (e.g., *Do you remember when we went on a bike ride together? Who can tell me how long it took us to ride around the lake?*) • Teacher adopts a student's way of naming or describing contextual features of the problem-solving scenario. • Teacher asks a student to restate or state in his/her own words something about the contextual features of the problem-solving scenario (e.g., *How are pledges and donations different?*). • Teacher marks a student's idea (e.g., *Melissa said something interesting about race-walking*). • Teacher makes a connection between the problem-solving scenario in the text and a cultural or social context that may be familiar to the students. • Teacher makes a connection between the problem-solving scenario and a person, place, or thing (e.g., a historical figure or event) that is likely to be familiar or of interest to the students in the class. • Teacher asks that a student or students act out something that is relevant to the contextual features of the problem-solving scenario (e.g., show what speed-walking is). *The difference between a 1 and a 2 is that students must actively participate in the discussion to warrant a 2.*
1	Teacher makes at least a brief mention of the problem-solving scenario that is central to completion of the task. The teacher is the *only* person providing information about the contextual features of the problem-solving scenario. The teacher may ask questions that require brief or one-word answers. Students do *not* actively participate in this chunk of instruction. At most, students provide yes/no responses or nod heads. *Examples:* • Superficial attempt (*Does anyone have a cell phone? Show a picture of a cell phone*.)
0	There is no attempt to discuss the contextual features of the problem-solving scenario.
N/A	The task as provided does not have a problem-solving scenario (e.g., it is a set of naked number problems).
NS	(No score). There is <u>no whole class discussion of the task prior to students starting the task</u> (e.g., teacher hands out the task and tells the students to start the task; teacher hands out the task and has students discuss it in groups prior to working on the task, but there is no whole class discussion prior to students starting the task). **Only assign NS for Contextual Features if the task*

	has a problem-solving scenario and there is no whole class discussion of the task prior to students starting the task.

RUBRIC 2: Mathematical Relationships

To what extent have students been given opportunities to develop understandings of key mathematical relationships (e.g., key mathematical ideas, relationships and/or quantities) as represented in the task (i.e., what is to be mathematized in the task)?

Rubric 2: Mathematical Relationships	
4	Teacher elicits the ideas students have developed *and* the teacher supports students in establishing a shared understanding of key mathematical ideas, relationships, and/or quantities (e.g., through marking, revoicing, linking, pressing) as represented in the task. Students actively participate in this segment of instruction. Students respond in ways that demonstrate understanding of how key mathematical ideas, relationships, and/or quantities are represented in the task. The students' responses connect to and build on each other. **In addition to what qualifies as a 3, the teacher and/or students must make at least one strong accountable talk move (e.g., teacher and/or student identifies connections between ideas and *how* the ideas are related, teacher presses on student's understandings of the mathematical ideas).** *The difference between a 3 and a 4 is the presence of **one strong accountable talk move** on the part of the teacher or students.*
3	Teacher elicits information about the mathematical ideas students have developed. The teacher and/or students **consistently use accountable talk moves (other than, or in addition to, repeating students' contributions)** to support students in establishing a shared understanding of how key mathematical ideas, relationships, and/or quantities are represented in the task. **Alternatively, the students (with or without the involvement of the teacher) jointly establish a representation that supports a shared understanding of relevant mathematical relationships**. Students actively participate in this segment of instruction. Students respond in ways that demonstrate understanding of how key mathematical ideas, relationships, and/or quantities are represented in the task. **The teacher and or/students consistently use accountable talk moves (other than, or in addition to, repeating students' contributions).** **OR** **More than one student is involved in the establishment of a representation that supports a shared understanding of mathematical relationships relevant to what is to be mathematized. Student talk must be aimed at developing a conceptual understanding of relevant mathematical ideas. (*More than one student must be involved in talk related to the representation.)** *The difference between a 2 and a 3 is that the teacher and/or students consistently use accountable talk moves (other than, or in addition to, repeating students' contributions) **or** students are involved in the **joint establishment** of a representation to develop a shared understanding of the mathematical relationships.*
2	Teacher elicits information about the ideas students have developed *but* the students (with or without the involvement of the teacher) do not **jointly establish** a representation nor are there consistent accountable talk moves that would support students in establishing a shared understanding of how key mathematical ideas, relationships, and/or quantities are represented in the task. **At best, there is inconsistent use of accountable talk moves (or consistent repeating of students'**

	contributions). *Examples of establishing how key mathematical ideas, relationships, and/or quantities are represented in the task; examples apply to 2-4; score will increase depending on the type of accountable talk moves made:* • Teacher adopts a student's way of describing or naming mathematical ideas, relationships, and/or quantities. • Students restate or state in their own words the key mathematical ideas, relationships, and/or quantities in the task. • Teacher and/or students develop a representation of the relationship between quantities (e.g., double number line) to support students' development of imagery. • Teacher and/or students act out something that supports students' development of imagery related to key mathematical ideas, relationships, and/or quantities (e.g., act out a box-and-whiskers plot given their heights, act out an exchange of money to illustrate the relationship between quantities). • Teacher and/or students refer to relevant mathematical representations that were established in the past. *The difference between a 1 and a 2 is that students must actively participate in the discussion to warrant a 2.*
1	Teacher makes at least a brief attempt to provide or suggest how key mathematical ideas, relationships, and/or quantities are represented in the task. The teacher is the *only* person suggesting or providing ideas regarding how mathematical ideas, relationships, and/or quantities are represented in the task. The teacher may ask questions that require brief or one-word answers. Students do *not* actively participate in this chunk of instruction. At most, students provide yes/no responses or nod heads.
0	There is no attempt to discuss key mathematical ideas, relationships, and/or quantities.
N/A	The task is not mathematical in nature.
NS	(No score). There is <u>no whole class discussion of the task prior to students starting the task</u> (e.g., teacher hands out the task and tells the students to start the task; teacher hands out the task and has students discuss it in groups prior to working on the task, but there is no whole class discussion prior to students starting the task).

Traditional IQA: Academic Rigor[7]

RUBRIC 1: Potential of the Task

Did the task have potential to engage students in rigorous thinking about challenging content?

<table>
<tr><td>4</td><td>The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:
• Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR
• Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.

The task must explicitly prompt for evidence of students’ reasoning and understanding.
For example, the task MAY require students to:
• solve a genuine, challenging problem for which students’ reasoning is evident in their work on the task;
• develop an explanation for why formulas or procedures work;
• identify patterns and form generalizations based on these patterns;
• make conjectures and support conclusions with mathematical evidence;
• make explicit connections between representations, strategies, or mathematical concepts and procedures.
• follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship.</td></tr>
<tr><td>3</td><td>The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a “4” because:
• the task does not explicitly prompt for evidence of students’ reasoning and understanding.
• students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy <u>or</u> too hard to promote engagement with high-level cognitive demands);
• students may need to identify patterns but are not pressed for generalizations;
• students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;
• students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions</td></tr>
<tr><td>2</td><td>The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it. The task does not require students to make connections to the concepts or meaning underlying the procedure being used. Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).

OR The task does not require students to engage in cognitively challenging work; the task is easy to solve.</td></tr>
<tr><td>1</td><td>The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced.</td></tr>
</table>

[7] Only a subset of the Traditional IQA rubrics are given in what follows. For information about the full set of rubrics see (Matsumura, Garnier, Slater, & Boston, 2008).

0	Students did not engage in a mathematical activity.

RUBRIC 3: Academic Rigor of the Discussion
To what extent did students show their work and explain their thinking about the important mathematical content?

4	Students show/describe written work for solving a task and/or engage in a discussion of the important mathematical ideas in the task. During the discussion, students provide complete and thorough explanations of why their strategy, idea, or procedure is valid; students explain why their strategy works and/or is appropriate for the problem; students make connections to the underlying mathematical ideas (e.g., “I divided because we needed equal groups”). OR Students show/discuss more than one strategy or representation for solving the task, and provide explanations of why the different strategies/representations were used to solve the task.
3	Students show/describe written work for solving a task and/or engage in a discussion of the important mathematical ideas in the task. During the discussion, students provide explanations of why their strategy, idea, or procedure is valid and/or students begin to make connections BUT the explanations and connections are not complete and thorough (e.g., student responses often require extended press from the teacher, are incomplete, lack precision, or fall short making explicit connections). OR Students show/discuss more than one strategy or representation for solving the task, and provide explanations of how the different strategies/representations were used to solve the task but do not explain why they were used.
2	Students show/describe written work for solving the task (e.g., the steps for a multiplication problem, finding an average, or solving an equation; what they did first, second, etc) but do not engage in a discussion of their strategies, procedures, or mathematical ideas. [There are presentations of students' work but no discussion.] OR Students show/discuss only one strategy or representation for solving the task.
1	Students provide brief or one-word answers (e.g., fill in blanks);
0	Class discussion was not related to mathematics or there was no discussion

Traditional IQA: Accountable Talk

Consider talk from the whole-group discussion only.

I. How effectively did the lesson-talk build Accountability to the Learning Community?

Students' Linking Contributions

Do student's contributions link to and build on each other?

Rubric 3: Students' Linking	
4	The students consistently connect their contributions to each other and show how ideas/positions shared during the discussion relate to each other. (e.g. I agree with Jay because…")
3	At least twice during the lesson the students to connect their contributions to each other and show how ideas/positions shared during the discussion relate to each other. (e.g. I agree with Jay because…")
2	At one or more points during the discussion, the students link students' contributions to each other, but do not show how ideas/positions relate to each other. (e.g., "I disagree with Ana.") OR only one strong effort is made to connect their contributions with each other.
1	Students do not make any effort to link or revoice students' contributions.
0	Class discussion was not related to mathematics.
N/A	No class discussion

II. How effectively did the lesson-talk build Accountability to Knowledge and Rigorous Thinking?

Student Providing

Did students support their contributions with evidence and/or reasoning?

Rubric 5: Student Providing	
4	Students consistently provide evidence for their claims, OR students explain their thinking using reasoning in ways appropriate to the discipline (i.e. conceptual explanations).
3	Once or twice during the lesson students provide evidence for their claims, OR students explain their thinking, using reasoning in ways appropriate to the discipline (i.e. conceptual explanations).
2	Students provide explanations that are computational, procedural or memorized knowledge, OR What little evidence or reasoning students provide is offered to back up claims is inaccurate, incomplete, or vague.
1	Speakers do not back up their claims, OR do not explain the reasoning behind their claims.
0	Class discussion was not related to mathematics.
N/A	No class discussion