Wrapping Around – Understanding Fractions and "Super" Units

NCTM April 2010

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DRL - 0732217

A Longitudinal Examination of Children's Developing Knowledge of Measurement: Mathematical and Scientific Concept and Strategy Growth from Pre-K through Grade 5

Principal Investigators

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  - Douglas H. Clements
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Outline of Presentation

- Background/Introduction
  - 4 year longitudinal NSF
  - Measurement through comparison
- Individual Interviews
- Follow-up in whole classrooms (2)
  - Unwrapping Demo
  - Measuring integer and non-integer lengths
  - Tool modifications
  - Extensions

Background/Introduction

- 4 year, longitudinal NSF-funded project
- Grades 2 – 5 at ISU, PreK – 2 at UB
- Eight students, grade 3 at the time of the individual interviews and whole class lesson
- The goal is validating and adding to Hypothetical Learning Trajectories for measurement with "predict and check" cycles

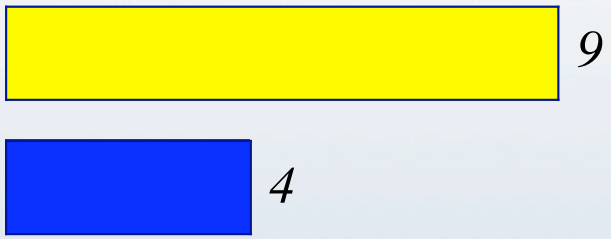
Measurement through Comparison

- First ask to compare two objects by some attribute.
  - Qualitative Comparison (this one is longer, taller,…)
  - Does the student identify the correct attribute?

- Now ask how much (longer, taller,…)
  - Quantitative Comparison
  - Select an appropriate unit to produce a difference or a ratio

Quantitative Comparisons and Units

- Compare length as a difference
  - The yellow strip is 5 units longer. (9-4=5)
  - The unit is a third object (cm in this case)



- Compare length as a ratio
  - The yellow strip is 2.25 times as long as the blue strip. (9/4=2.25)
  - One of the two objects serves as the unit (the smaller in this case)

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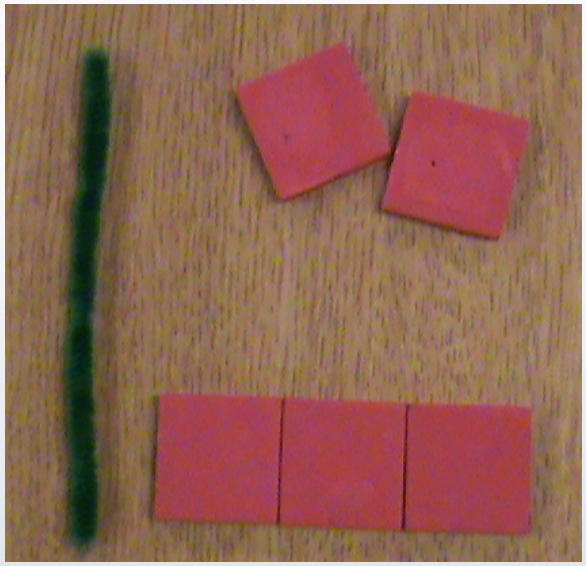
The Interview Task

Materials: four-inch pipe cleaner, one-inch square tiles, paper, pencil



Show the student that the length of the pipe cleaner is equal to the length of the path around one square tile.

The Interview Task

Questions/Purpose:

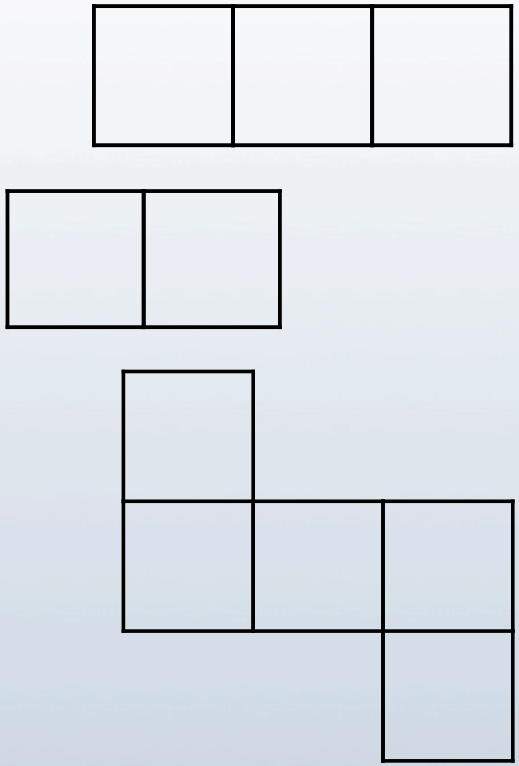
How does the distance around this set of square tiles compare to the length of the pipe cleaner?

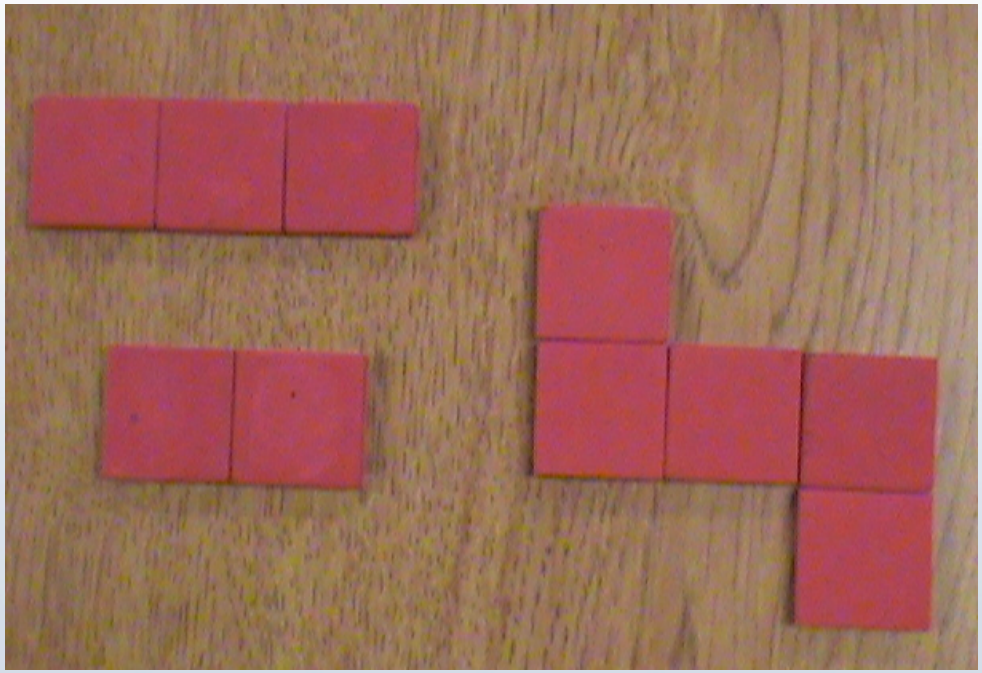
Purpose: Does the student identify and compare (qualitative) the correct attributes?

How much longer is the distance compared to the length of the pipe cleaner?

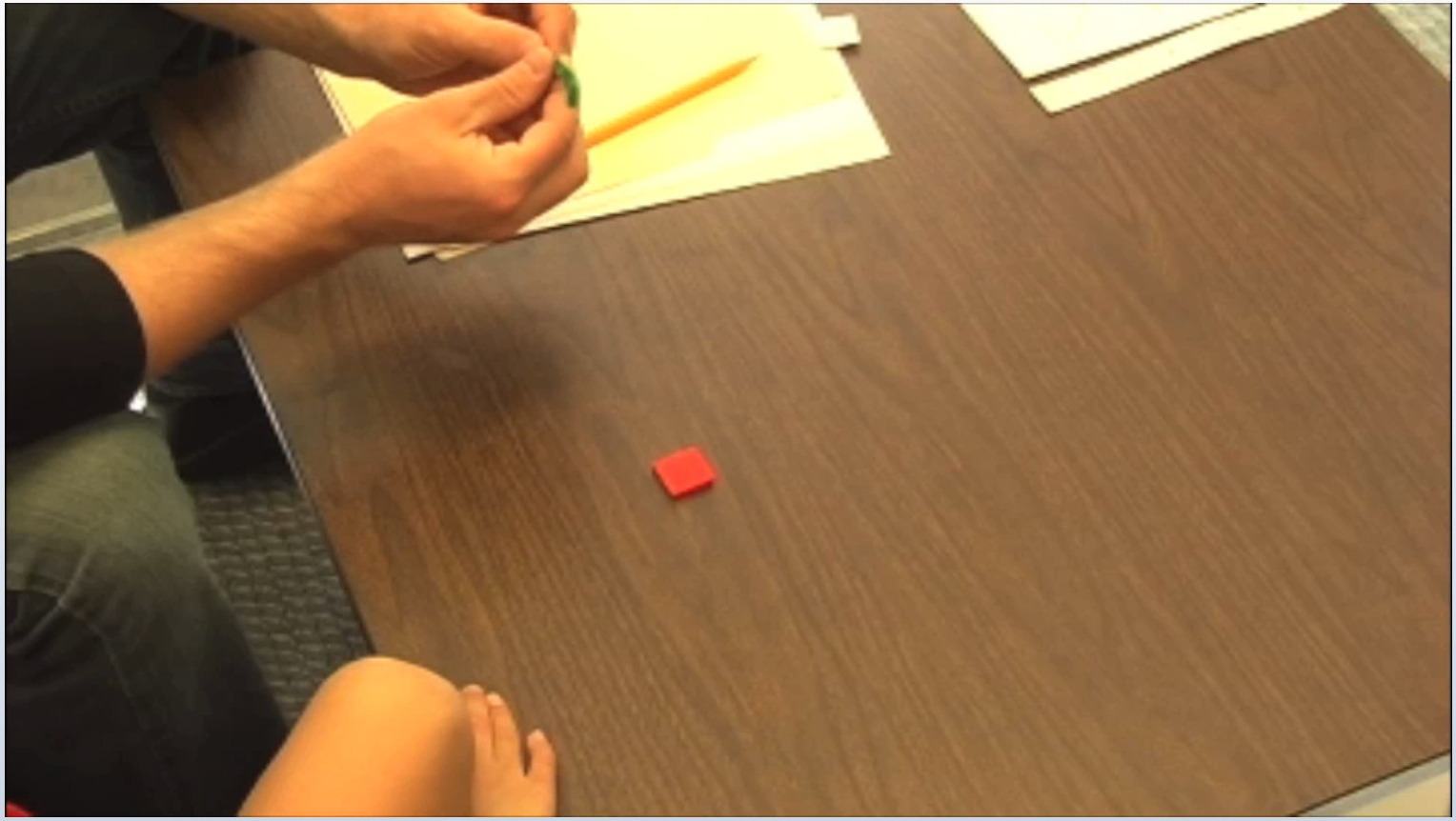
Purpose: Can the student select an appropriate unit and produce a quantitative comparison?

Arrangements of Tiles



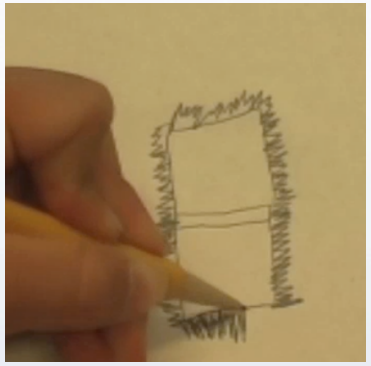


Introduction of the Task



Student Solutions

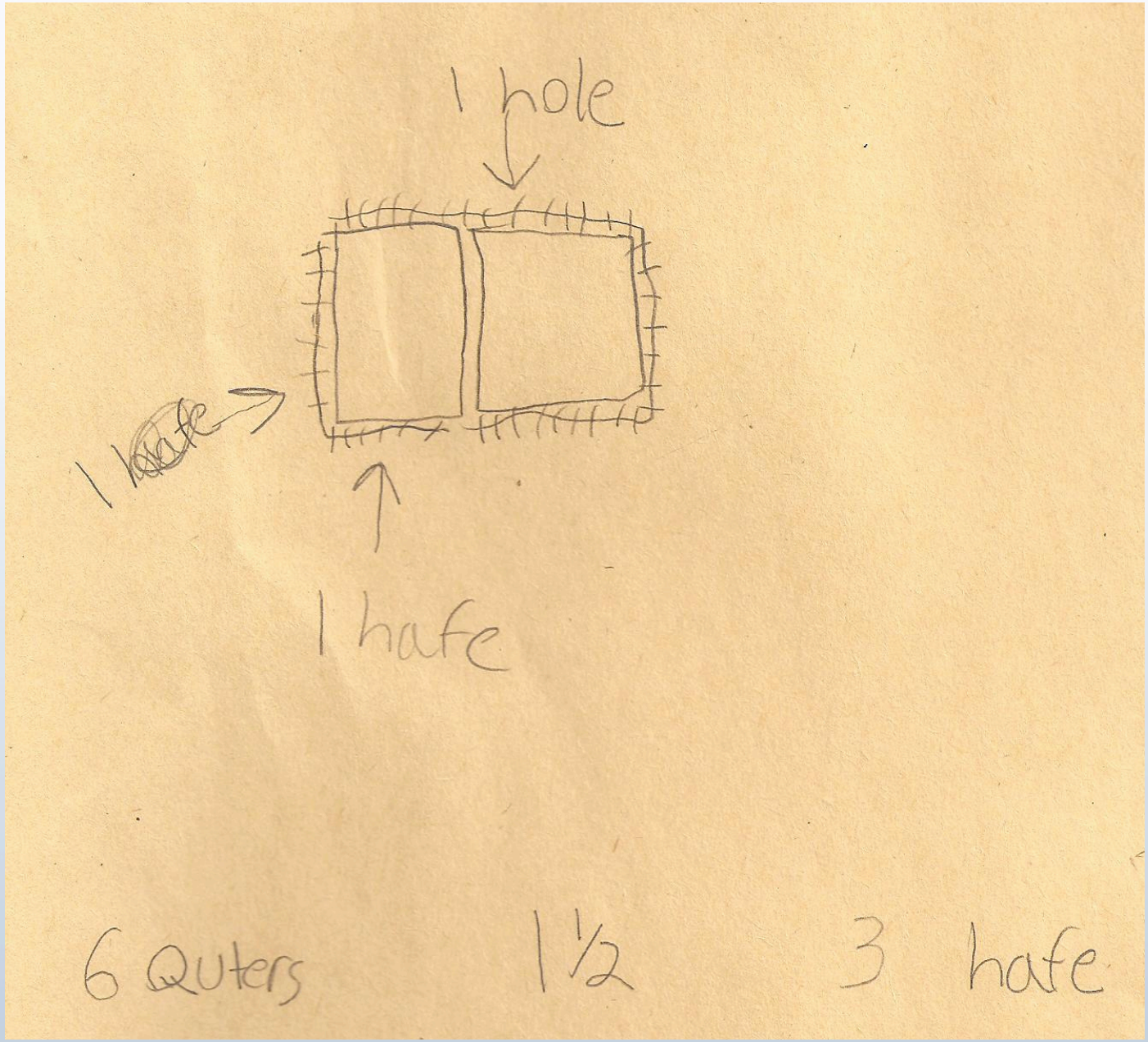
"I'm starting to think it's one quarter and one half… well actually, just one half"… "Because if I split it [string] in half and took that piece out and wrapped it around that [two edges] it would be one half." -Abby



- When comparing the string to the set of two tiles: "This one is eight and this one is four." -Owen

- When comparing the string to one tile: "You need one tile for this to wrap all the way around it." -Anselm

Flexibility with Fractions



6 Quters 1½ 3 hafe

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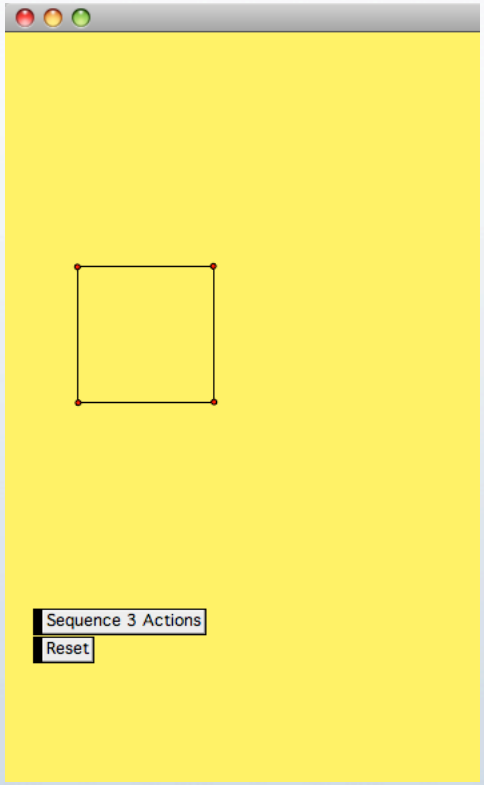
Whole Class Lesson Outline

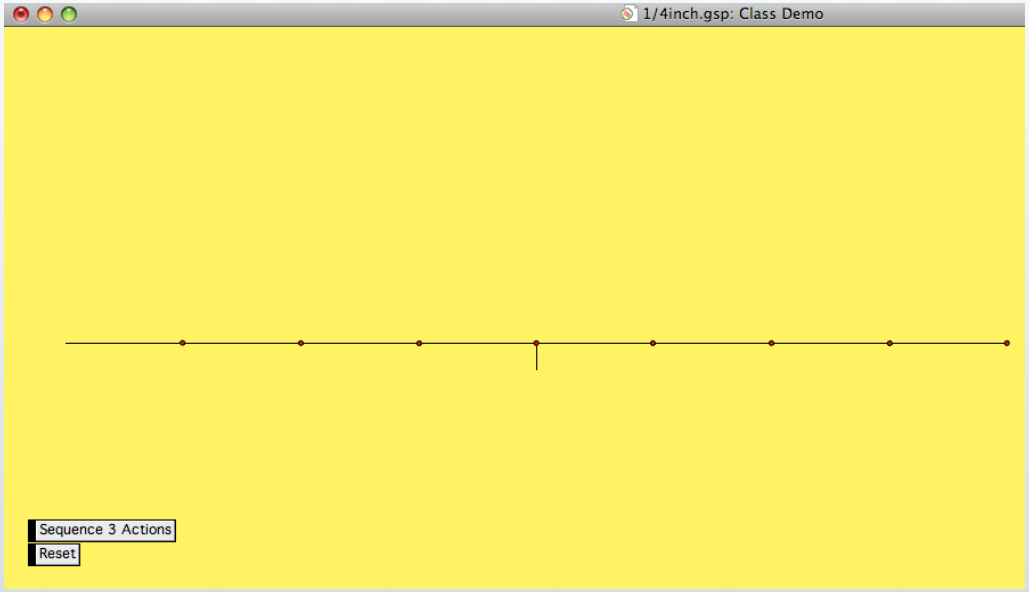
- Sheet 1: Integer Length Sides
  - **Ask the students to figure out how many wraps could be made from each of the wires shown.**
- Sheet 2: Conversions
  - **Ask students to convert from wraps to sides and sides to wraps.**
- Sheet 3: Non-Integer Lengths
  - **Ask students to figure out how many wraps could be made from each of the wires shown. If a student reports measures as 5 wraps and 2 sides ask them how many wraps that would be. (Encourage students to report answer as 5 ½ wraps)**

Whole Class Lesson Purpose

- Purpose:
  - We want students to see a quarter of a unit as a unit itself rather than a fraction of the whole.
  - We want students to coordinate quarter units and units and be able to measure seeing either or both as a unit.
  - We want students to be able to convert between different representations of numbers: proper fractions, improper fractions, and mixed numbers.

GSP Demonstration





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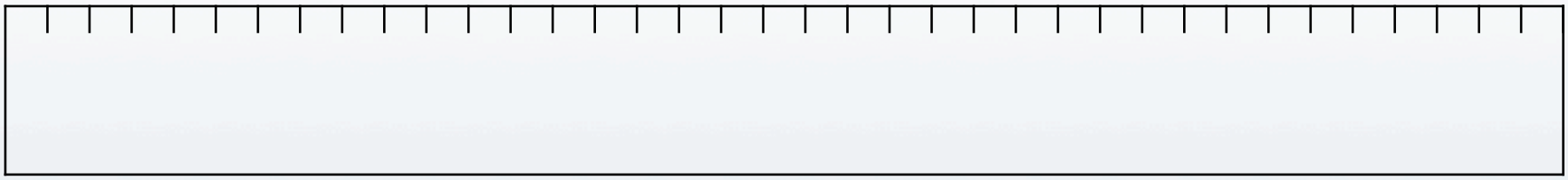
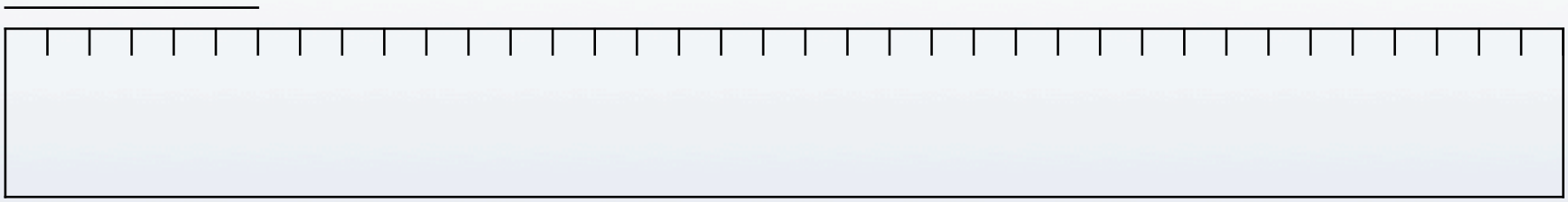
Sheet 1: Integer Lengths



Sheet 2: Conversions

- Fill in each of the blanks below.
- 1. 6 sides is the same as _____ wraps.
- 2. 8 wraps is the same as _____ sides.
- 3. 40 sides is the same as _____ wraps.
- 4. 6 wraps is the same as _____ sides.
- 5. 5 ½ wraps is the same as _____ sides.

Sheet 3: Non-Integer Lengths



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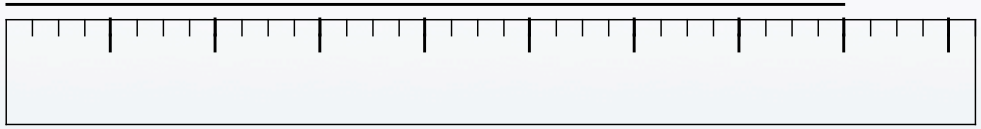
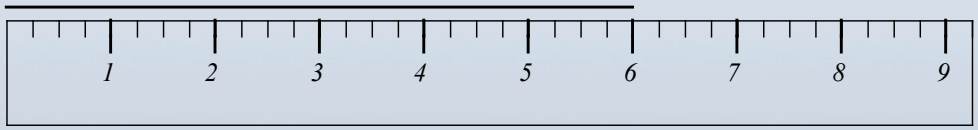
Tool Modifications



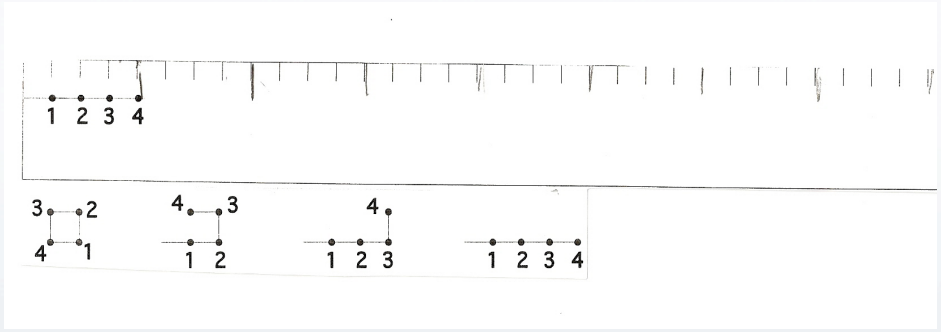
21 students

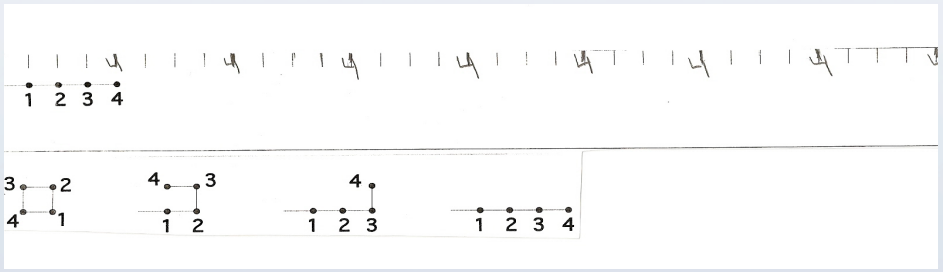
5 students

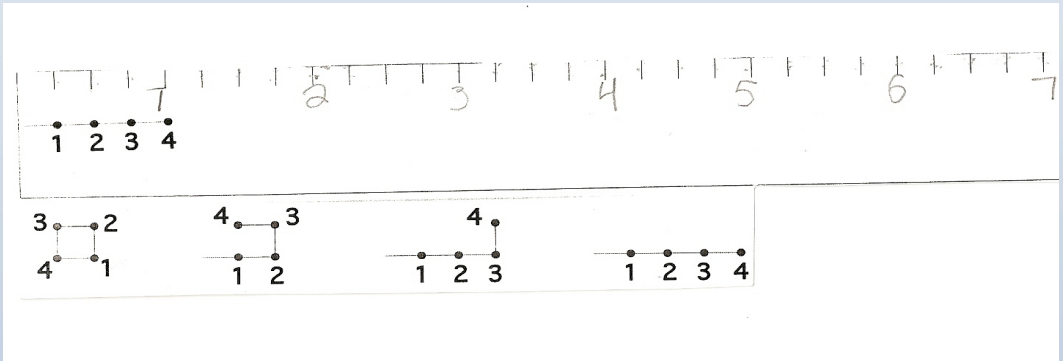


12 students

Records of Tool Modifications







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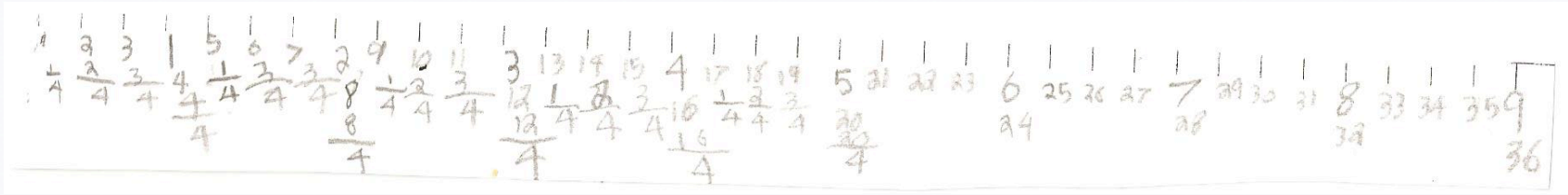
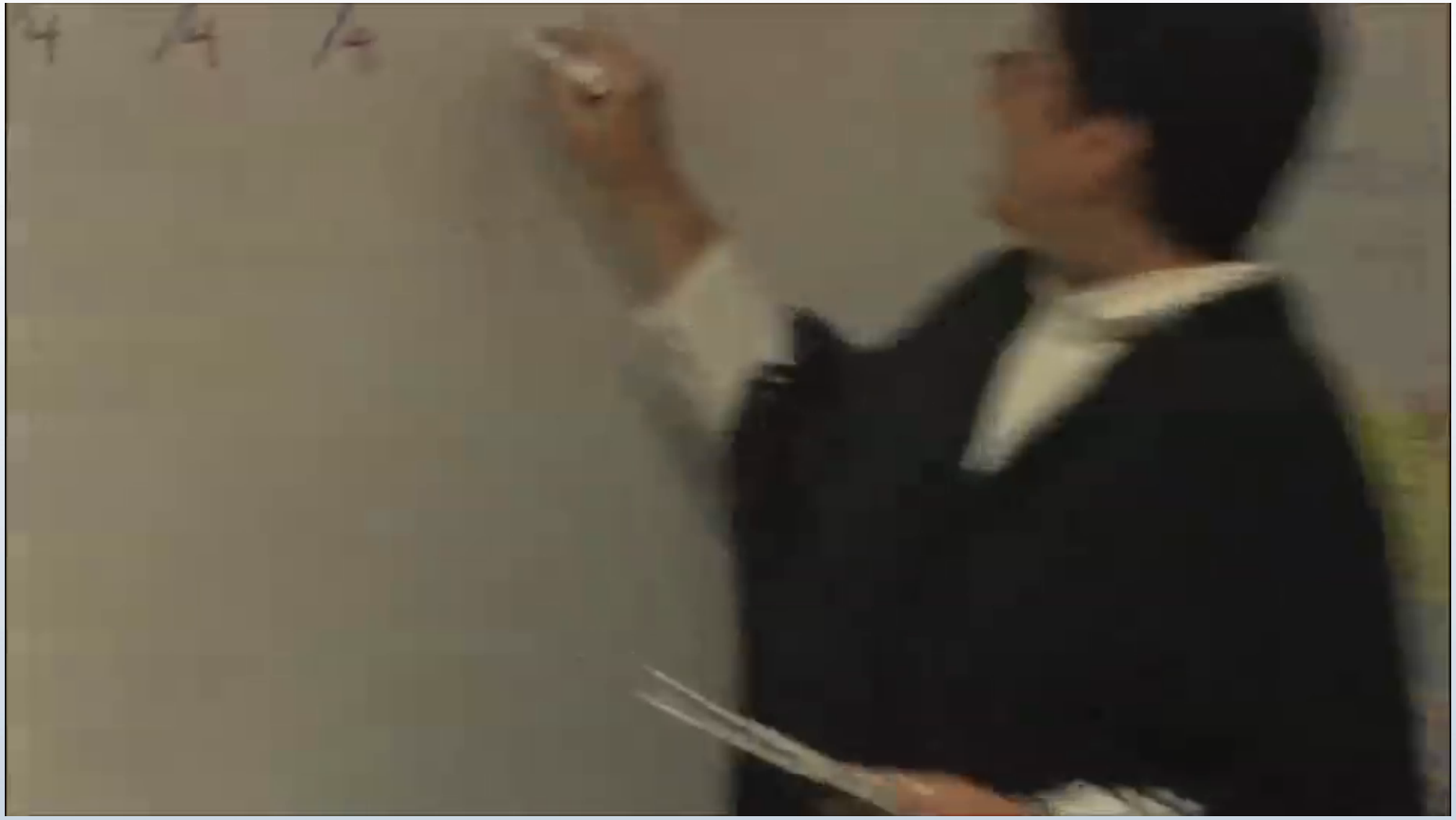
Miss Wilder's Extensions

- Additional tool modifications
- Look at equivalent fractions for the whole numbers
- Tie the activity to quarter inches and inches
- Comparing mixed numbers using < and >





Discuss

- What are your impressions of the activity?
- What would students get out of it?
- What extensions can you think of?