# Conceptualizing Important Facets of Teacher Responses to Student Mathematical Thinking

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*International Journal of Mathematical Education in Science and Technology*
https://doi.org/10.1080/0020739X.2021.1895341

# Conceptualizing Important Facets of Teacher Responses to Student Mathematical Thinking

We argue that progress in the area of research on mathematics teacher responses to student thinking could be enhanced were the field to attend more explicitly to important facets of those responses, as well as to related units of analysis. We describe the Teacher Response Coding Scheme (TRC) to illustrate how such attention might play out, and then apply the TRC to an excerpt of classroom mathematics discourse to demonstrate the affordances of this approach. We conclude by making several further observations about the potential versatility and power in articulating units of analysis and developing and applying tools that attend to these facets when conducting research on teacher responses.

Keywords: teacher responses, responsive teaching, mathematics, teaching practice, mathematics teaching methods and classroom techniques

## Introduction

Several decades of research on classroom instruction that supports students' mathematical understanding and learning has highlighted the importance of using student mathematical thinking as part of whole-class instruction (e.g., Kilpatrick et al., 2003; National Council of Teacher of Mathematics [NCTM], 2014; Spangler & Wanko, 2017). An essential aspect of productive use of student mathematical thinking is the way teachers respond to that thinking (Bishop, 2020; Robertson, Scherr et al., 2016). To illustrate the critical nature of teacher responses, consider the following vignette:

> A class has been working on this task: *The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?* During whole-class discussion of the task, this student claim is displayed: "The price is the same because it was increased and decreased by the same amount." Here are two possible teacher responses to that student contribution:

Teacher Response 1: “This claim doesn’t seem to take into account what we’re taking 50% of.”
Teacher Response 2: “Class, what do you think about this claim?”

Although both teacher responses in this vignette are examples of teachers using student mathematical thinking as part of whole-class instruction, they use that thinking in very different ways. Whereas in Teacher Response 1 the teacher engages with the student contribution, in Teacher Response 2, the students are invited to engage with the contribution. The latter response aligns with NCTM’s (2014) vision of teachers facilitating meaningful mathematical interactions in which students share, analyze, and build upon each other’s ideas.

Although both of these teacher responses could be characterized as responsive teaching (e.g., Robertson, Atkins et al., 2016), the shift in who engages with the contribution illustrates how “subtle differences in teaching practices can affect students’ opportunities to engage in conceptual thinking” (Kazemi & Stipek, 2001, p. 60). Better understanding these subtle differences will support the development of high quality responsive teaching—teaching that engages students in actively making sense of mathematics (NCTM, 2014). Recognizing subtleties resulting from variations in teacher responses to student mathematical contributions that occur during classroom instruction is critical to developing this understanding. Our work on teacher responses to high-leverage student mathematical contributions (e.g. Stockero et al., 2019; Stockero et al., 2020) has suggested that certain facets of teacher responses may provide insight into these subtleties. In this paper we provide a means for characterizing and framing these important facets of teacher responses.

Teacher responses have been characterized in many different ways, including how teacher responses encourage student talk (e.g., Chapin et al., 2009), take up student thinking (e.g., Bishop et al., 2016; Lineback, 2015), and focus on the mathematics in students' ideas (e.g., Conner et al., 2014; Selling, 2016). This body of research has produced helpful insights about in-the-moment teacher responses to student contributions. Although it has produced coding schemes, frameworks, and collections of teacher moves for investigating teacher responses, we have found no existing characterizations that adequately capture the collection of important facets that are the focus of this paper. Instead, in extant characterizations of teacher responses, these facets are often entangled and related units of analysis are often left implicit. In this theoretical paper we (a) conceptualize important facets of teacher responses and associated units of analysis that should be made explicit in this work, (b) illustrate how we have made these facets and units explicit in our own theorizing about teacher responses, and (c) discuss how these facets and units provide insights into "subtle differences in teaching practices" (Kazemi & Stipek, 2001, p. 60).

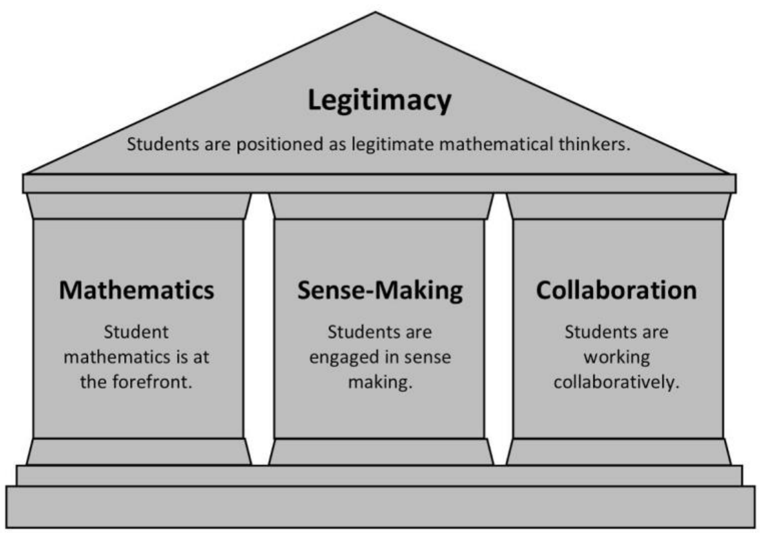
We begin by discussing the theoretical lens that undergirds our work and through which we view mathematics teacher responses. Next we draw on the teacher response literature to illustrate important facets of teacher responses and present our Teacher Response Coding Scheme (TRC) as an example of a tool that captures these facets. Then we identify important units of analysis related to the facets and demonstrate the added value of a tool such as the TRC by applying the TRC to a well-known transcript of a mathematics classroom discussion. We conclude by discussing ways the field can draw on these facets and units in order to productively work together to better understand responsive mathematics teaching.

**Principles Underlying Productive Use of Student Mathematical Thinking in Instruction**

We view teacher responses to student mathematical thinking through the lens of current ideas about effective teaching and learning of mathematics as captured in NCTM's *Principles to Actions* (2014). As discussed elsewhere (e.g., Stockero et al., 2020; Van Zoest, Peterson et al., 2016), we see embedded in that document four Core Principles underlying productive use of student mathematical thinking during instruction (see Figure 1).

**Figure 1**

*Core Principles Underlying Productive Use of Student Mathematical Thinking During Instruction*




The overarching Legitimacy Principle emphasizes the importance of positioning students as legitimate mathematical thinkers who are able to engage with mathematical ideas in a deep and meaningful way. Such positioning requires listening to students' contributions to discern what they are saying mathematically in order to make decisions about the pedagogical potential of incorporating such contributions into the lesson. For example, engaging the class with student mathematical contributions that contain ideas that are too easy or too hard for them would undermine their ability to consider those contributions in an authentic way. The other three principles undergird the Legitimacy Principle.

The Mathematics Principle establishes that productive use of student mathematical thinking requires foregrounding the mathematics of the student's thinking, and backgrounding the teacher's way of thinking about that mathematics. This foregrounding lays the foundation for instruction that uses student mathematical thinking as the driver for learning. The Sense-Making Principle makes it clear that students are to be engaged in making sense of mathematical ideas, rather than simply receiving information or practicing procedures. The Collaboration Principle addresses the social nature of learning by highlighting the importance of students working with each other and their teacher, rather than in isolation, as they engage collectively in learning mathematics.

It is by engaging in teaching practices that simultaneously embody these Core Principles that we see the potential for instruction that is responsive to student mathematical thinking in powerful ways. We view these Core Principles as the essence of *productive use of student mathematical thinking during instruction*. As such the Core Principles are the lens through which we view the productivity of teacher responses and the important facets of these responses that are the focus of this paper.

**Important Facets of Teacher Responses**

As we viewed the literature on teacher responses through the lens of our Core Principles and attempted to account for variations among the different approaches for describing teacher responses, three facets emerged that we found to have explanatory power for characterizing productive teacher responses to student contributions: the *who*, the *what*, and the *how* of teacher responses. As shown in the opening vignette, although two different teacher responses might both involve sense making, *who* is engaged in that sense-making matters. The action that a teacher response focuses on—the *what* of the response—is also important. Different situations call for different teacher responses; for example, correcting the student contribution in the opening vignette would provide a different learning opportunity than engaging in making sense of that thinking. Finally, a subtle difference in teacher responses that has emerged from our work is the extent to which students recognize that a teacher response is truly engaging with their mathematical thinking—the *how* of the response. This facet is underrepresented in the literature and will be elaborated on throughout this paper.

To illustrate these important facets of teacher responses to student contributions, we draw on scholarship in mathematics education that researched *teacher actions during classroom instruction in response to a student contribution*—referred to henceforth as a *teacher response*. Specifically, we draw on literature related to teachers' in-the-moment responses to student thinking that emerges during classroom instruction. The foci of this literature include teacher responses to specific types of student contributions (e.g., Drageset, 2015; Schleppenbach et al., 2007); variation among different teacher responses (e.g., Scherrer & Stein, 2013; Wood, 1999); teacher actions to support student talk (e.g., Chapin & O'Connor, 2007; Franke et al., 2009);

teachers' responsiveness to student thinking during classroom discourse (e.g., Bishop et al., 2016; Robertson, Atkins et al., 2016) and teacher actions to support specific mathematical activity (e.g., Conner et al., 2014; Ellis et al., 2019).

Across this literature, we found that researchers had developed a variety of frameworks, coding schemes, and collections of teacher moves—what we will collectively refer to as *tools*—that could be used to investigate teacher responses to student contributions. The three important facets that we identified as critical to characterizing productive teacher responses to student contributions—the *who*, the *what*, and the *how* of teacher responses—were often embedded in these tools. Although sometimes the facets were explicitly identified, often they were implicit in the tools' codes and code descriptions. In the following sections we describe these facets and use examples from these tools to argue for the importance of disentangling these facets from each other.

### ***The* Who *of a Teacher Response***

The who facet of a teacher response describes the member(s) of the classroom community publicly given the opportunity to consider the student contribution as a result of the teacher response to it.[1] Information about who is given this opportunity—the actor of the teacher response—is prevalent in tools used to investigate teacher responses to student contributions, although sometimes this information is implied rather than explicitly addressed. Even when the

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[1] When a student contribution emerges during classroom instruction, we acknowledge both that teachers initially make an internal decision about how to respond to a student contribution and that students could also be engaging internally with a student contribution. The focus here is on public engagement.

actor is explicitly stated, it is typically embedded in the codes and code descriptions of the tools. This embeddedness can obfuscate subtle differences in learning opportunities created by teacher responses with different actors—differences like those we saw in the opening vignette.

In some tools, a single code description might include more than one actor. Bishop and colleagues' (2018) Probing and Publicizing code, for example, applies when teachers revoice student ideas themselves as well as when they request that students "correct, evaluate, or indicate agreement with another student" (p. 7). In other cases, each code description is limited to a particular actor. For example, Selling's (2016) Reprising Moves have descriptions such as "the teacher names some aspect of student engagement in mathematical practices" (p. 550) or "the teacher makes explicit reference to [ideas related to the mathematical practices]," (p. 551) where the teacher is the actor doing the naming and referencing. Other descriptions focus on the student as actor. For example, the description of Chapin and O'Connor's (2007) Repeating Talk Move states that the teacher "asks students to repeat their classmates' contributions during a discussion" (p. 121). Although the teacher is the one who is doing the asking, the students in the class are the ones being invited to do the repeating. Thus, for this teacher response, students are the actors. Whether the actor is the student or the teacher, the description of who is being asked to engage with the student thinking is embedded in these code descriptions. This embeddedness makes it difficult to determine subtle, yet important, differences in teacher responses. For example, a "repeat" action with students as the actor might serve as a formative assessment, whereas a "repeat" action with the teacher as actor might serve to focus students on the student contribution.

### *The* **What** *of a Teacher Response*

The *what* facet of a teacher response describes *what the response gives the actor the opportunity to do with respect to the student contribution*—the action offered to the actor by the teacher response. For example, a teacher response may give the actor the opportunity to contribute mathematical reasoning related to a student contribution, as was illustrated in the opening vignette. Response 1 gave the teacher the opportunity to contribute that reasoning, while Response 2 gave the students in the class that opportunity. Regardless of the actor, in both of these responses the action is to contribute mathematical reasoning related to a student contribution. Another teacher response to the student contribution in the vignette might be, "Actually, the price wasn't increased and decreased by the same amount." This response has a quite different action, that of correcting the student, and hence provides a very different learning opportunity than the responses in the vignette. Thus the action of a teacher response matters.

The importance of the action of a teacher response—the *what* facet—is clearly recognized by the field. In fact, most tools are organized around actions. These actions, however, are often described from the perspective of what the teacher is doing (that is, the teacher *move*). The problem with this type of description is that the teacher is not always the actor despite the fact that the teacher is always doing *something* during a teacher response. The *what* facet focuses on what the response gives the actor, whoever that may be, the opportunity to do with respect to the student contribution.

To illustrate the distinction between the teacher move and the *what* of the teacher response, consider the description of Ozgur and colleagues' (2015) *Pressing for Justification* teacher move: "Teacher asks students to explain why something works or to justify (logically, conceptually) their idea, strategy, or solution" (p. 1067). The teacher move is "pressing;" the *what* or *action* of this pressing is "justify" (where the students are the actor). Thus although teacher moves are central to existing tools for studying teacher responses, looking at the *what* facet of teacher responses focuses our attention on the intellectual work that is initiated by those responses. Distinguishing between the intellectual work that is initiated and how the teacher initiates it is an important part of recognizing subtle, yet important, differences in teacher responses.

### *The* How *of a Teacher Response*

The *how* facet of a teacher response describes *how the teacher response relates to the student contribution to which it is responding*—arguably a critical aspect of any mathematics teaching that is characterized as *responsive*. The *how* facet will require some work to describe and explain, which we do later in the "Capturing *How*" section. Here our goal is to motivate, rather than to fully describe the facet. In the opening vignette, the student who made the contribution would likely recognize their contribution in both teacher responses. This student might have a quite different reaction to the following teacher response: "What is the first thing that we do when we take percents?" In this case, although the teacher may recognize how their question relates to the student's contribution, the student is not likely either to recognize that relationship or to feel that the teacher is incorporating their ideas into the response. These three teacher responses illustrate that *how* a teacher response relates to the student contribution to which it is responding sends messages to students, for good or ill, about how much the teacher values their mathematical contributions. Thus the extent to which students recognize that a teacher response is truly engaging with their mathematical thinking is a critical, yet subtle facet of teacher responses.

Although the *who* and the *what* facets consistently have been embedded in tools for capturing teacher responses, the *how* facet rarely appears and seems connected to more recent notions of students as authentic mathematical learners. There is some evidence of the facet in some teacher response tools. For example, Selling (2016) describes a *Highlighting* code, which "involves teacher talk" that shifts "the conversation into a metalevel that reflects on what one or more students just did mathematically in the discussion with respect to mathematical practices" (p. 550). Because the "highlighting" that is done is focused on "what the students just did mathematically" this code has a clear emphasis on *how* the teacher response engages with the mathematics in the student contribution.

Although Selling's (2016) *Highlighting* code emphasizes the *how*, it entangles the *who* and the *what*. That is, one could imagine multiple actors and actions associated with a teacher response that gets this code. For example, a teacher might do the highlighting through connecting the student contribution to a particular mathematical practice, or a teacher might ask the class to make sense of how what the students did was an example of a particular mathematical practice. In the first case, the actor would be the teacher and the action would be to connect, because it was the teacher who had the opportunity to connect the student contribution to the mathematical practice. In the latter response, the actor would be the whole class and the action would be to justify because the whole class was given the opportunity to provide a justification for the relationship between the student contribution and the mathematical practice. In both of these cases, the teacher response was closely related mathematically to the student contribution to which it was responding.

Although having a code that acknowledges a particular way that a teacher response can connect to the mathematical thinking of a student contribution is useful, we argue that the *how* facet should be assessed for each teacher response as there is much to be learned about nuances in the ways teacher responses relate to the student contributions to which they are responding. Few existing tools capture this facet, and those that do, such as Selling's, tend to limit their capture to particularly salient times when it occurs. Further conceptualizing the *how* facet and fully capturing this facet in teacher response tools is critical to analyzing subtle, yet important differences in the responsiveness of teacher responses to student mathematical contributions.

## An Example of Capturing the Important Facets: The Teacher Response Coding Scheme

We present the Teacher Response Coding Scheme[2] (TRC) as an example of a tool that captures and disentangles the facets just described: (a) *who* interacts with the contribution; (b) *what* they are asked to do in those interactions; and (c) *how* the teacher response relates mathematically to the student contribution. For each facet, we introduce the TRC coding categories designed to

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[2] We developed the TRC in tandem with our use of the literature to identify the facets as we analyzed data for our larger project (see LeveragingMOSTs.org). Over the past ten years we have analyzed the following data from grades 6-12 mathematics teachers: videotaped mathematics lessons chosen to reflect the diversity of teachers, students, mathematics, and curricula present in US schools (see Van Zoest et al., 2017 for more details), interview data of teachers responding to a common set of student mathematical contributions with varying potential (see Stockero et al., 2020), and videotapes of teacher-researchers' classroom use of MOST-eliciting prompts (see Leatham et al., 2020).

capture that facet, along with the codes within each category. In doing so, we use a set of four different teacher responses to an illustrative student mathematical contribution (SMC) to elaborate the TRC categories and codes and to highlight how they illuminate subtle, yet important differences in teacher responses.

### ***Capturing* Who*: Actor***

To capture *who* is likely to be engaged in considering the SMC as a result of the teacher response, the Actor category answers the question, "Who is publicly given the opportunity to consider the SMC?" with one of four codes (see Figure 2). The Actor determines the level of collaboration related to the SMC (Collaboration Principle), and sends strong messages about who is capable of considering it (Legitimacy Principle). As mentioned previously (see Footnote 1), in order to respond to student mathematical thinking that emerges in their classroom, the teacher is always likely to first privately consider the student mathematical thinking on some level. Additionally, it is possible for anyone who hears the thinking to be considering that thinking privately. However, neither of these considerations are how we determine the Actor. Instead, we determine the actor by examining the teacher response to identify *who* is publicly given the opportunity to consider the thinking.

**Figure 2**
*Definitions of Codes for the Actor Category*

| Code | Definition |
|---|---|
| | The teacher response … |
| *Teacher* | does not publicly invite or allow anyone other than the teacher to consider the SMC. |
| *Same Student(s)* | invites or allows the contributor(s) to consider the SMC. |
| *Other Student(s)* | invites or allows a subset of students other than the contributor(s) to consider the SMC. |
| *Whole Class* | invites or allows the whole class to consider the SMC. |

To illustrate distinctions among these codes, we consider four sample teacher responses to the illustrative SMC in Figure 3. For a teacher response to be coded *Teacher*, it is only the *Teacher* who publicly has the opportunity to engage in considering the SMC, as is the case in Teacher Response 1 (TR1). By directing their response to a specific student, TR2 and TR3 publicly provide the opportunity for the student who contributed the thinking (*Same Student*) and another student in the class (*Other Student*), respectively, to consider the SMC. In contrast to the other responses, TR4 publicly invites the *Whole Class* to consider the SMC.

**Figure 3**

*Actor Coding for Teacher Responses to an Illustrative Student Mathematical Contribution*

| | | |
|---|---|---|
| **Context:** Whole-class discussion about the graph of a situation relating the amount of money accumulated by saving both a one-time gift and babysitting money that was earned weekly.<br>**SMC: "**I put the money on the bottom and weeks on the side." | | |
| | **Teacher Response** | **Actor** |
| 1 | "Remember, we always put the independent variable on the x-axis." | *Teacher* |
| 2 | [To same student] "Why is the amount of weeks dependent on the amount of money which you put on the bottom?" | *Same Student* |
| 3 | [To another student] "And what do I like to do first when I make a graph?" | *Other Student* |
| 4 | "Did anyone label the axes a different way?" | *Whole Class* |

### *Capturing* What*: Action*

The Action category captures *what* the response gives the actor the opportunity to do with respect to the SMC. In capturing this action we do not try to infer teacher intent or what the actor might actually do in response to the teacher response, but rather, focus on how the teacher's response is likely to be interpreted by their students. With respect to the Core Principles, the action on the SMC determines the extent to which students are positioned to make sense of it (Sense-Making Principle). In addition, some actions position students as legitimate mathematical thinkers more than other actions (Legitimacy Principle).

We identified 14 mutually exclusive actions (see Figure 4) that captured all of the actions in the teacher responses we have analyzed (see Footnote 3). Many of these actions are recognizable from, and were informed by, other studies of teacher responses. For example, the TRC *Dismiss* code is similar to Scherrer and colleague's (2013) *Terminal* move, defined as "ignoring or discontinuing a response" (p. 121). Another TRC code, *Justify,* is fairly common in the literature, but named in different ways, including *Elaborating* (Even & Gottlib, 2011), *Pressing for Justification* (Ellis et al., 2019), and *Justification* (Drageset, 2014; Lineback, 2015).

**Figure 4**
*Definitions of Codes for the Action Category*

| Code | Definition |
|---|---|
| | The teacher response … |
| *Adjourn* | indicates (either explicitly or implicitly) that the SMC will not be considered at that time, but suggests the SMC may be considered later. |
| *Allow* | creates an open space for interaction with the SMC. |
| *Check-in* | gives an opportunity for the actor to self-assess their reaction to or understanding of the SMC. |
| *Clarify* | gives the actor an opportunity to make the SMC more precise. |
| *Collect* | gives the actor an opportunity to contribute additional ideas, methods, or solutions. |
| *Connect* | gives the actor an opportunity to make a connection between the SMC and other ideas, representations, methods/strategies, or solutions. |
| *Correct* | gives the actor an opportunity to rectify the student mathematics of the SMC. |
| *Develop* | gives the actor an opportunity to expand the SMC beyond a simple clarification, but does not request justification. |
| *Dismiss* | indicates (either explicitly or implicitly) that the SMC will not be considered. |
| *Evaluate* | gives the actor an opportunity to determine the correctness of the mathematics of the SMC. |
| *Justify* | gives the actor an opportunity to contribute mathematical reasoning related to the SMC. |
| *Literal* | gives the actor an opportunity to provide brief factual information related to the SMC. |
| *Repeat* | gives the actor an opportunity to repeat (verbally or in writing) the SMC (including minor rephrasing that does not change the meaning). |
| *Validate* | gives the actor the opportunity to affirm the general value of the SMC and/or encourage student participation (e.g., says "thank you," gives a thumbs up signal, asks for applause). |

Two important differences between the TRC Action codes and some of the similar codes in the literature are that in the TRC: (a) each discrete type of Action has its own code, and (b) the code descriptions do not include (i.e., are disentangled from) the facets of a teacher response captured in our other coding categories—namely, the *who* and *how*. Related to the first difference, consider Brodie's (2011) *press* subcategory of the *Follow-up* move, defined as, "The teacher pushes or probes the learner for more on their idea, to clarify, justify or explain more clearly (p. 180)." Rather than grouping the three embedded purposes of the *Press* move together—clarify, justify, and develop—the TRC defines these as three different Actions. Related to the second difference, consider Conner and colleagues' (2014) two actions *Evaluating* and *Requesting Evaluation.* The primary difference between these codes is who is engaged in evaluating—either the teacher or students, respectively. The TRC includes only one Action related to evaluate, because the *who* facet of the response is captured independently with the Actor codes.

We provide examples of four of the fourteen Action codes in the TRC in the rightmost column of Figure 5. In TR1, the teacher *Corrects* the student's labeling by reminding the class of labeling conventions. In TR2, the student who generated the SMC is asked to *Justify* their choice of money as the independent variable and weeks as the dependent variable. TR3 asks a *Literal* question to engage a different student in providing factual information about the teacher's preferred process for creating a graph. In TR4, the teacher is *Collecting* additional methods for labeling the axes from the class.

**Figure 5**

*Action Coding for Teacher Responses to an Illustrative Example of Student Mathematical Contribution*

| | | | |
|---|---|---|---|
| **Context:** Whole-class discussion about the graph of a situation relating the amount of money accumulated by saving both a one-time gift and babysitting money that was earned weekly. **SMC: "**I put the money on the bottom and weeks on the side." | | | |
| | **Teacher Response** | **Actor** | **Action** |
| 1 | "Remember, we always put the independent variable on the x-axis." | *Teacher* | *Correct* |
| 2 | [To same student] "Why is the amount of weeks dependent on the amount of money which you put on the bottom?" | *Same Student* | *Justify* |
| 3 | [To another student] "And what do I like to do first when I make a graph?" | *Other Student* | *Literal* |
| 4 | "Did anyone label the axes a different way?" | *Whole Class* | *Collect* |

### ***Capturing* How**

To capture *how* the teacher response relates mathematically to the student contribution to which it is responding, we draw on Robertson and colleagues' (2016) definition of responsive teaching. They define responsive teaching as "foregrounding the substance of students' ideas" (p. 1), "recognizing the disciplinary connections within students' ideas," and "taking up and pursuing the substance of student thinking" (p. 2). We see in this definition two important aspects of the *how*, which the TRC captures with two distinct but related categories: Student Recognition and Mathematical Alignment. The Student Recognition category captures the extent to which the student who contributed the SMC is likely to recognize their contribution in the teacher response, and thus the extent to which students might be positioned as legitimate mathematical thinkers (Legitimacy Principle). The Mathematical Alignment category focuses on the *how* from the perspective of alignment to important mathematical ideas in the SMC, and thus relates to the Mathematics Principle. Specifically, Mathematical Alignment captures the relationship between the mathematics of the teacher response and the mathematics of the SMC. We discuss these categories in greater depth below.

*Student Recognition*

The Student Recognition category captures the extent to which the student who provided the SMC would likely recognize their contribution in the teacher's response. Through our iterative work in analyzing classroom discourse we noticed two distinct ways this likely recognition might be apparent in a teacher response: through attention to Student Actions and attention to Student Ideas (see Figure 6).

**Figure 6**
*Definitions of Codes for the Subcategories of the Student Recognition Category*

| Sub-Category | Code | Definition |
|---|---|---|
| | | The teacher response... |
| Student(s) Actions | *Explicit* | uses the student's specific actions (typically words). |
| | *Implicit* | uses indexical language (e.g., pronouns, synonyms) or stops short of using the student's specific actions because of conversational conventions. |
| | *Not* | does not explicitly or implicitly use the student's specific actions. |
| Student(s) Ideas | *Core* | focuses on a main idea of the SMC in a way that the student who contributed the SMC would likely recognize the idea as their own. |
| | *Peripheral* | focuses on an idea that is related to, but not the main idea of the SMC, or focuses on a main idea, but recognizing that would require a leap of logic that students are not likely to make. |
| | *Other* | focuses on an idea that does not seem to be related in any way to a main idea of the SMC. |
| | *Indeterminate* | is worded such that it is not possible to infer its focus or the SMC cannot be inferred. |

The subcategory Student Actions encompasses unique words or specific phrasings a student has used (verbal), as well as any gestures or written work provided by the student (non-verbal). The codes (*Explicit*, *Implicit*, or *Not*) for Student Actions capture the extent to which the teacher response uses the actual student actions. We again return to the illustrative example to elaborate these codes (see Figure 7). To explore the subtle distinction between a response coded as *Implicit* and one coded as *Explicit* for Student Actions, consider TR2 and TR4. In TR2, the teacher uses language unique to the SMC ("on the bottom"). In contrast, in TR4 the teacher does not use this unique language, replacing "put" with the verb "label," and replacing "on the bottom" and "on the side" with the term "axes." Hence, TR2 *Explicitly* uses the student's actions while TR4 *Implicitly* uses the student actions. Responses that do not use the student's identifiable actions or clear replacements (such as TR1 and TR3) are coded as *Not*.

**Figure 7**
*Student Recognition Coding for Teacher Responses to an Illustrative Example of Student Mathematical Thinking*

**Context:** Whole-class discussion about the graph of a situation relating the amount of money accumulated by saving both a one-time gift and babysitting money that was earned weekly.
**SMC: "**I put the money on the bottom and weeks on the side."

| | Teacher Response | Actor | Action | Recognition | |
|---|---|---|---|---|---|
| | | | | **Student Actions** | **Student Ideas** |
| 1 | "Remember, we always put the independent variable on the x-axis." | *Teacher* | *Correct* | *Not* | *Peripheral* |
| 2 | [To same student] "Why is the amount of weeks dependent on the amount of money which you put on the bottom?" | *Same student* | *Justify* | *Explicit* | *Peripheral* |
| 3 | [To another student] "And what do I like to do first when I make a graph?" | *Other student* | *Literal* | *Not* | *Other* |
| 4 | "Did anyone label the axes a different way?" | *Whole class* | *Collect* | *Implicit* | *Core* |

The subcategory Student Ideas focuses on the relationship of the teacher response to the main idea(s) in the SMC. The codes for Student Ideas (*Core*, *Peripheral*, *Other*, and *Indeterminate*; see Figure 6) capture the extent to which the student is likely to recognize their idea in the teacher response. Thus, the coding is determined from the perspective of the student providing the SMC, not from the perspective of the teacher or of others who have a more advanced understanding of mathematics. TR4 focuses on the labeling of the axes, which the student is likely to recognize as their main idea (*Core*) since they had described which variable they had put on which axis. In contrast, although it would be clear to the teacher, it would not likely be clear to this student that their thinking relates to an important connection between the labels of a graph and the independent-dependent relationship between the variables. Instead, the student (and others in the classroom) are likely to interpret both TR1 and TR2 as picking up what the student had said, but taking it in a slightly different direction. Thus Student Ideas for both of these responses are coded *Peripheral*. TR3, on the other hand, uses the SMC as an opportunity to prompt the students to remember what the teacher "likes to do first" when they make a graph. This response, although prompted by the SMC, does not take up the student's idea about what the axes represent, asking instead about what the teacher likes to do first when graphing. Thus, TR3 is coded *Other* for Student Ideas.

*Mathematical Alignment*

Another element of *how* the teacher response relates to the SMC is the degree to which the mathematics in the teacher response—the mathematics the teacher seems to be moving toward—

aligns with the mathematical idea most closely related to the SMC. The Mathematical Alignment category documents this alignment (see Figure 8).

**Figure 8**
*Definitions of Codes for the Mathematical Alignment Category*

| Code | Definition |
|---|---|
| | The mathematics of the teacher response ... |
| *Core* | is closely related to the mathematical idea of the SMC. |
| *Peripheral* | is tangential to the mathematical idea of the SMC. |
| *Other* | does not seem to be related in any way to the mathematical idea of the SMC. |
| *Indeterminate* | cannot be compared to the mathematical idea of the SMC because either the mathematics of the teacher response or the mathematical idea of the SMC cannot be determined. |

We return to the four teacher responses to the illustrative SMC to discuss the Mathematical Alignment codes of the TRC (see Figure 9). The mathematical idea of the SMC, "I put money on the bottom and weeks on the side," is "The placement of the variables on the axes of a graph is determined by what makes the most sense in the problem, given the established convention of the x-axis representing the independent variable." In TR2 the mathematical idea in the teacher response seems to be the same as the mathematical idea of the SMC, hence the Mathematical Alignment of the teacher response is coded as *Core.* TR1 focuses on the mathematical conventions of labeling axes, thus the mathematical idea of the teacher response may be articulated as, "By convention, the x-axis represents the independent variable and the y-axis represents the dependent variable." Though this mathematical idea is contained in the mathematical idea of the SMC, it focuses on following the convention rather than on deciding which variable is independent and which is dependent. Thus, the mathematics of the teacher response is *Peripheral* to the mathematical idea of the SMC. When the mathematics in the teacher response is not even peripherally related to the mathematical idea of the SMC, it is coded as *Other*. For example, if a teacher responded, “That reminds me, I wanted to talk about [something different],” the response would be coded *Other*. TR3 and TR4 illustrate responses that are coded as *Indeterminate*; in both of these responses, it is not yet evident what mathematical idea the teacher is pursuing.

**Figure 9**
*Mathematical Alignment Coding for Teacher Responses to an Illustrative Student Mathematical Contribution*

**Context:** Whole-class discussion about the graph of a situation relating the amount of money accumulated by saving both a one-time gift and babysitting money that was earned weekly.
**SMC: “**I put the money on the bottom and weeks on the side.”

| | Teacher Response | Actor | Action | Recognition: Student Actions | Recognition: Student Ideas | Mathematical Alignment |
|---|---|---|---|---|---|---|
| 1 | “Remember, we always put the independent variable on the x-axis.” | *Teacher* | *Correct* | *Not* | *Peripheral* | *Peripheral* |
| 2 | [To same student] “Why is the amount of weeks dependent on the amount of money which you put on the bottom?” | *Same Student* | *Justify* | *Explicit* | *Peripheral* | *Core* |
| 3 | [To another student] “And what do I like to do first when I make a graph?” | *Other Student* | *Literal* | *Not* | *Other* | *Indeterminate* |
| 4 | “Did anyone label the axes a different way?” | *Whole Class* | *Collect* | *Implicit* | *Core* | *Indeterminate* |

## Units of Analysis Related to Studying Teacher Responses

> It is the analysis you do in your study that determines what the unit is. (Trochim et al., 2015, p. 22)

Before illustrating an application of the TRC, we pause to discuss the importance of articulating units of analysis in work on teacher responses. We have had to make decisions about a number of units of analysis in order to effectively use the TRC in our work—decisions that all researchers must make as they operationalize their tools. As we looked at the work of others, we realized that despite having clearly made such decisions, researchers were seldom explicit about their units of analysis. This lack of explicitness is problematic for moving the field forward because it makes it difficult to accurately compare and accumulate knowledge across studies. Similar to type I and type II statistical errors, we run the dual risks of concluding that studies align or support each other when they do not and that they do not when they actually do.

We argue that making units of analysis explicit will support efforts to reconcile disparate research results for the purpose of developing a coherent understanding of what is known about teacher responses to student contributions. Here we outline general descriptions of the units of analysis we identified as important to teacher response research. When we proceed to apply the TRC in the next section, we will be explicit about these units of analysis in our work.

We have identified four important units of analysis related to research on teacher responses: *Student Contribution*, *Teacher Response*, *Teacher Response Referent,* and *Context*. The first two units of analysis articulate the grain size of the student contribution and of the teacher response, respectively, while the third unit describes the relationship between them. The fourth unit of analysis articulates the set of circumstances and information that one uses to make sense of the other three units. Figure 10 provides brief descriptions and examples of variations in these units of analysis.

**Figure 10**
*Units of Analysis for Studying Teacher Responses to Student Contributions*

| Unit | Description |
|---|---|
| Student Contribution | The grain size of the student contribution to which a teacher response is attached.<br>***Examples***<br>• conversational turn (e.g., Conner et al., 2014; Selling, 2016)<br>• portion of a turn (e.g., Brodie, 2011)<br>• related sequence of turns (e.g., Bishop et al., 2018) |
| Teacher Response | The grain size of the teacher actions that are considered in response to a student contribution.<br>***Examples***<br>• conversational turn (e.g., Conner et al., 2014; Selling, 2016)<br>• portion of a turn (e.g., Brodie, 2011)<br>• related sequence of turns (e.g., Bishop et al., 2018) |
| Teacher Response Referent | The student contribution with respect to which the teacher response is being analyzed.<br>***Examples***<br>• the immediately preceding student contribution (e.g., Brodie, 2011; Selling, 2016)<br>• a particular student contribution (e.g., Van Zoest, Peterson et al., 2016)<br>• a collection of student contributions (e.g., Connor et al., 2014) |
| Context | The set of circumstances and information that one uses to make sense of the other units of analysis.<br>***Examples***<br>• immediately preceding dialogue (e.g., Selling, 2016)<br>• all relevant preceding dialogue (e.g., Connor et al., 2014)<br>• preceding and subsequent dialogue (e.g., Drageset, 2014, 2015) |

## Illustrating the Value of a Tool that Attends to the Important Facets: Applying the TRC

In this section, we highlight the affordances of a coding scheme like the TRC that *distinctly* captures the who, what, and how of teacher responses by illustrating how it can be used to analyze classroom discourse. First, we operationalize our use of the TRC. Next, we introduce an excerpt of classroom discourse that we have analyzed using the TRC. Finally, we explore what the analysis of each TRC category, as well as combinations of the categories, reveals about classroom discussions.

### *Operationalizing our Use of the TRC*

In keeping with our argument about explicitness with regard to units of analysis, we articulate those choices for our illustrative application of the TRC (see Figure 11).

**Figure 11**
*Methodological Choices for Our Illustrative Use of the TRC*

| **Unit of Analysis** | **Our Methodological Choice** |
|---|---|
| Student Mathematical Contribution | "An observable student action or small collection of connected actions (such as a verbal expression combined with a gesture)" that is mathematical (Leatham et al., 2015, p. 92). |
| Teacher Response | The collection of observable teacher actions that begins as a given SMC ends[a] and concludes when the teacher turn ends or there is a clear shift to a different activity. |
| Teacher Response Referent | The SMC that immediately precedes the teacher response. |
| Context | The circumstances and information preceding the teacher response that are needed to make sense of the teacher response, including the SMC to which the teacher is responding, any ongoing classroom discussion, particular language commonly used in the class, and common experiences of the class. |

[a]Or begins simultaneously with the SMC in the situation where the teacher response occurs in a way that does not interrupt the thinking, such as writing down what the student is saying on the board as they are saying it.

Recall that one way that the TRC captures the *how* of the teacher response is through the Mathematical Alignment coding category. In order to operationalize this coding, we drew on two constructs from our earlier work (Leatham et al., 2015), Student Mathematics and the related Mathematical Point, and introduced the construct Teacher Mathematical Understanding. These constructs are defined in Figure 12. In practice, we first articulate the Student Mathematics, a cleaned-up version of the SMC. We then use the Student Mathematics to articulate the

Mathematical Point, the mathematical understanding that is "closest" to the Student Mathematics (Van Zoest, Stockero et al., 2016). Finally, when possible, we articulate the mathematical understanding that the teacher response appears to be moving toward—the Teacher Mathematical Understanding. This allows us to compare the Teacher Mathematical Understanding to the Mathematical Point of the SMC and hence code the Mathematical Alignment of the teacher response.

***Figure 12***
*Definitions of Key Constructs Used in Operationalizing the TRC*

| Construct | Definition |
|---|---|
| Student Mathematics | "A clearly articulated statement of an inference of what a student has expressed mathematically in the [SMC]" (Van Zoest et al., 2017, p. 36). |
| Mathematical Point | The mathematical understanding that (1) the class could move toward acquiring if they were to engage in making sense of the SMC that (2) is most closely related to the Student Mathematics of the SMC (Van Zoest, Stockero et al., 2016). |
| Teacher Mathematical Understanding | The mathematical understanding the teacher response appears to be moving towards. |

### *Introducing the Selected Transcript*

We have chosen this particular transcript (see Figure 13) because many readers are likely familiar with it from the foundational and widely used Smith and Stein (2011) book. We anticipate that this familiarity will help the reader understand the context quickly and hence focus on the TRC analysis. Additionally, this teaching episode aligns reasonably well with the Core Principles outlined in Figure 1. In Smith and Stein (2011), the episode is used to illustrate several questioning moves that create rich mathematical discussions that are driven by student contributions. Applying the TRC to this episode provides additional details about the teacher responses that help to better understand and characterize productive mathematics instruction.

In this lesson from a fourth-grade classroom, the teacher's goal is for students to build on their prior knowledge about the areas of rectangles to construct a formula for the area of a right triangle. In particular, the teacher wants students to notice

> that the areas of right triangles can be found by either embedding the triangle within a rectangle and then finding the area of the rectangle and dividing it by 2 [i.e., *bh/2*], or by dividing one of the sides of the triangle by 2 and multiplying it by the other side of the triangle (the canonical formula for the area of a right triangle: $A = ½ bh$). (Smith & Stein, 2011, p. 63)

Prior to the class discussion, students had been working in small groups with premade cardboard right triangles (with legs of lengths 6 and 8 units) to find a rule for the area of a triangle. The episode begins after students had time to work together, and the teacher has requested that a student share their rule for finding the area of a triangle. Figure 13 provides the episode of whole-class discussion with some minor modification from the original transcript.[3]

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[3] We have removed individual lines numbers, and instead numbered conversational turns to better reflect our units of analysis (see Figure 11). We have shaded the students' turns to distinguish them from the teacher's and used brackets to denote non-verbal actions. In line 26, the teacher's full turn includes both responding to the preceding SMC (not italicized, coded) and making a clear shift to an activity that is not responding to that SMC (italicized, not coded). To the right of each SMC we have articulated the Student Mathematics (SM) and Mathematical Point (MP) of that contribution to support the application of the Mathematical Alignment category of the TRC. To the right of the teacher response, we have provided the TRC codes for each coding category. A slight, but important, modification we have made to the transcript occurs in lines 10,14, 20, and 24. Specifically, without access to the original classroom video, we are assuming that in these teacher responses the teacher posed a question to the whole class, waited for student hands, and then selected a volunteer to respond. To indicate this, we have added "[assumed pause]" to those teacher responses and coded what precedes the teacher calling on one student. This distinction is important because it has implications for the Actor coding of the teacher response.

**Figure 13**

*Transcript from Smith and Stein (2011, pp 64-65)*

| | | | | | Recognition | | |
|---|---|---|---|---|---|---|---|
| Line Number | Speaker | Instance | Actor | Action | Student Actions | Student Ideas | Mathematics |
| 1 | Tammy | [Inaudible at first] … When we got … We had two of them here. We had length times width divided by two. | **SM:** Our rule or formula for the area of a triangle is length times width divided by two.<br>**MP:** The area of a triangle is base times height divided by two. | | | | |
| 2 | Teacher | [Records on an overhead transparency ($l$ x $w$)/2.] Where are you coming up with this? | *Same Student* | *Justify* | *Explicit* | *Core* | *Indeterminate* |
| 3 | Tammy | Because when you cut the square in half, that's half, and, like, when you get, like, 36, 'cause that's a whole square and half of it's 18, so, like, if you had another - any square - any square, and you did, um, the length times the width, and then you divided that in half, you'd get your answer. | **SM:** We found our rule or formula by cutting our square in half. A whole square was 36 and half of it is 18. For any square, the length times the width is the area of the square and if you divided it in half you would get the area of the right triangle [made by cutting the square on a diagonal].<br>**MP:** The area of a triangle is length times width divided by two. | | | | |
| 4 | Teacher | How do you know to divide? Where are you getting this dividing by two? I'm curious about where you're coming up with that. | *Same Student* | *Justify* | *Implicit* | *Core* | *Indeterminate* |
| 5 | Tammy | When we started with a whole square, it was 36. But then you have to cut it in half for a triangle. | **SM:** We knew how to divide by two because when we started with a whole square, it was 36. But then you have to cut it in half for a triangle.<br>**MP:** A square cut on the diagonal gives two right triangles that each have half the area of the square. | | | | |
| 6 | Teacher | Why do you… I'm wondering why you need to do that? | *Same Student* | *Justify* | *Implicit* | *Core* | *Indeterminate* |
| 7 | Tammy | Cause, it, um, so we could have a triangle. So we know how many halves. And in each one, we had 18 in each of our squares. | **SM:** We need to cut the square in half to get a triangle. We know how many halves, and in each one we had 18, in each of our squares.<br>**MP:** A right triangle makes up half of a rectangle so you can find the area of the rectangle and divide it by 2 to find the area of the triangle. | | | | |
| 8 | Teacher | Ok, so you are saying that the triangle takes up one-half of the square and that since you could find the area of the square, you just took one-half of it to find the area of the triangle. Right? | *Teacher* | *Clarify* | *Implicit* | *Core* | *Core* |
| 9 | Tammy | Right. | **SM:** We are saying that the triangle takes up one-half of the square and since we could find the area of the square we just took one-half of it to find the area of the triangle.<br>**MP:** A right triangle makes up half of a rectangle so you can find the area of the rectangle and divide it by 2 to find the area of the triangle. | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | Teacher | Is there another way that … Can someone tell me, or share with me, another way that we could write the same formula to see if it would still work? [assumed pause] Quinn. | *Whole Class* | *Collect* | *Not* | *Peripheral* | *Indeterminate* |
| 11 | Quinn | Um, half times… um, half of length times width. | **SM:** Another formula for area of a triangle is 1/2(l x w)<br>**MP:** (a x b)÷c is the same as 1/c (a x b) because of the associate and inverse properties of multiplication. | | | | |
| 12 | Teacher | Is that the same thing? | *Same Student* | *Connect* | *Implicit* | *Core* | *Core* |
| 13 | Quinn | Yeah. | **SM:** Yes, 1/2(l x w) is the same as (l x w)/2.<br>**MP:** (a x b)÷c is the same as 1/c (a x b) because of the associate and inverse properties of multiplication. | | | | |
| 14 | Teacher | [Teacher records 1/2(*l* x *w*).] [assumed pause] David? | *Teacher* | *Repeat* | *Implicit* | *Peripheral* | *Indeterminate* |
| 15 | David | Yeah, because when you write 2, it's just another way of saying "half." | **SM:** Yes, 1/2(l x w) is the same as (l x w)/2, because when you write 2, it's another way of saying "half".<br>**MP:** The term "half" represents the ratio equivalent to 1/2. | | | | |
| 16 | Teacher | Oh, when I say "two" … Anytime that I say "two," it's the same as saying "half"? | *Same Student* | *Clarify* | *Explicit* | *Core* | *Core* |
| 17 | David | No, when you say "length times width *divided by two*." | **SM:** No, I mean when you say you say length times width divided by two, that 2 is the same as saying half of length times width.<br>**MP:** Dividing by a is the same as multiplying by 1/a. | | | | |
| 18 | Teacher | Oh, "*divided by two*." | *Teacher* | *Repeat* | *Explicit* | *Core* | *Core* |
| 19 | David | It's just like saying "multiplied by half." | **SM:** Saying length times width divided by 2 is just like saying length times width multiplied by 1/2..<br>**MP**: Dividing by a is the same as multiplying by 1/a. | | | | |
| 20 | Teacher | What if I did this? I'll put it in red so you can see it. What if I did this? (T writes: 1/2(*l* x *w*) next to (*l* x *w*)/2 on the overhead transparency) Do those mean the same thing? I need some people who haven't participated to help me out. Do you think that those mean the same thing? [assumed pause] Louis? | *Whole Class* | *Connect* | *Implicit* | *Core* | *Core* |
| 21 | Louis | I think that you could come up with 18 with either one 'cause that's the same thing as the other one. | **SM:** I think you could have come up with 18 using either 1/2(l x w) or (l x w)/2 because that's the same thing as the other one.<br>**MP**: a x b÷c is the same as 1/c(a x b) because of the associate and inverse properties of multiplication. | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 22 | Teacher | How do you know that? | *Same Student* | *Justify* | *Implicit* | *Core* | *Indeterminate* |
| 23 | Louis | I think it's the same because, um, half of length times width equals 18. Half of 36 is 18. I think it is the same because, um, it's just another way of saying "36 divided by 2." | **SM**: 1/2(l x w) is the same as (l x w)/2, because 1/2 of length time width equals 18. Half of 36. is 18. I think it is the same because, um, it's just another way of saying 36 divided by 2.<br>**MP**: a x b÷c is the same as 1/c(a x b) because of the associate and inverse properties of multiplication. | | | | |
| 24 | Teacher | Does everyone agree with Louis? [assumed pause] What about you, Jason? | *Whole Class* | *Evaluate* | *Not* | *Core* | *Indeterminate* |
| 25 | Jason | I agree with him that it's the same thing. We have one-half of length times width. It's just the opposite of the other thing. It just was length times width, and then we divided by 2. | **SM**: I agree with him that 1/2(l x w) is the same as (l x w)/2. We have one-half of length times width which is just the opposite of length times width and then divided by 2.<br>**MP**: Dividing a by b is equivalent to multiplying a by 1/b because b and 1/b are multiplicative inverses. | | | | |
| 26 | Teacher | OK. So we could represent the formula as either ($l$ x $w$)/2 or 1/2($l$ x $w$). *I am going to move to the next problem. How can we represent the area of this triangle as a rule or formula? Angela? [Draws a right triangle with a height of 6 and a base of 8 on grid paper at the overhead.]* [a] | *Teacher* | *Clarify* | *Explicit* | *Core* | *Core* |
| 27 | Angela | You could make it a square and then take half of it: 48/2 = 24. The triangle is 24 squares. | **SM:** You could inscribe the triangle within a square and then take half of the area of the square. So 48/2= 24. The area of the triangle is 24 squares.<br>**MP**: A right triangle makes up half of a rectangle so you can find the area of the rectangle and divide it by 2 to find the area of the triangle. | | | | |
| 28 | Teacher | [the teacher motions the student to come to the board] | *Same Student* | *Develop* | *Not* | *Core* | *Indeterminate* |
| 29 | Angela | [The student draws a 6 x 8 rectangle around the triangle.] | **SM:** The student inscribes the triangle within a 6 by 8 rectangle.<br>**MP**: A right triangle makes up half of a rectangle so you can find the area of the rectangle and divide it by 2 to find the area of the triangle. | | | | |
| 30 | Teacher | [The teacher writes (l x w)/2 next to the students' drawing.] | *Teacher* | *Develop* | *Explicit* | *Peripheral* | *Core* |

[a] Turn 26 includes the teacher response to the instance of student thinking in line 25 (not italicized). This italicized portion of the teacher's turn is not responding to the instance of SMT in line 25 since the teacher clearly states she is moving on. Thus, it is not considered part of the teacher response to the SMT in turn 25.

### ***Discussion of this Application of the TRC***

*Analysis of Actor*

Looking only at the pattern in conversational turns in this discussion might lead to the conclusion that the teacher and students participate equally (15 teacher turns and 15 student turns). Looking at the Actor codes, however, allows us to foreground more nuanced information about who the teacher responses engage. In this excerpt, the teacher responses engaged students 10 of the 15 times, predominantly re-engaging the *Same Student(s)* (7 of 15, 47% of responses), and periodically engaging the *Whole Class* (3 of 15, 20% of responses). By contrast, the *Teacher* was the actor in only 5 of the 15 (33%) responses. Thus, the Actor coding allows us to see an important, but subtle aspect of what it means for the teacher to be a productive facilitator of discussion—that teacher responses engage a variety of student actors and that the *Teacher* is also occasionally involved in publicly considering the SMC.

This variation in actors is in contrast to how some teachers might interpret what it means to facilitate discussions. As documented by researchers, some teachers consider facilitation as constantly re-engaging the whole class and being as hands-off as possible (Stein et al., 2008), while others see it as consistently going back to the same student, rather than turning ideas over to the rest of the class (Schleppenbach et al., 2007; Stockero et al., 2017). In fact, teachers sometimes believe that instruction based on student thinking requires that the teacher refrain from providing any substantive input at all (Baxter & Williams, 2010; Chazan & Ball, 1999). The Actor codes in this excerpt reveal that productively facilitating classroom discussions likely involves giving many, varied actors (including the teacher) the opportunity to engage with the SMC; a pattern that, again, is not visible when only considering who is speaking.

*Analysis of Action*

We identified 7 of the 14 TRC Action Codes in the 15 teacher responses in the transcript: *Justify* (4 teacher responses), *Clarify* (3), *Connect* (2), *Repeat* (2), *Develop* (2), *Collect* (1), and *Evaluate* (1). Forty percent (6 of 15) of the teacher responses use actions (i.e., *Justify* and *Develop*) that engage the actor in digging deeper into the student ideas. A third (5 of 15) of the responses were coded *Repeat* and *Clarify*, actions intended to make the ideas accessible and clear to the class. The high frequency of these types of codes highlights the potential nature of productive discussion: that there is a focus on conceptual understanding of student ideas rather than a focus on simply getting the right answer—something likely characterized by more frequent occurrences of *Evaluat*e, *Correct*, or *Validate* Actions.

Examining the sequence of Actions in this transcript reveals other interesting patterns. At the beginning of the discussion the teacher responses surface *Justifi*cations of the first idea put forth by a student (lines 2, 4, and 6), providing the class with not only an answer but also with an underlying "why" to discuss. After making sure the SMC is clear *(Clarify*, line 8), the discussion is expanded by the *Collect*ion of other ideas (line 10). These ideas are then *Connect*ed (line 11) to the original student's idea, followed by moves to ensure the connections are clear to students (*Repeat* and *Clarify*; lines 14, 16, and 18) before the *Connect*ion is presented again in writing on the board (line 20). The connection between ideas is then explored more deeply through a series of probing Actions (lines 22-30), including *Justify* and *Develop*. The patterns in these actions provide insights into nuanced, potentially productive rhythms of classroom discourse. In this excerpt we see that after new ideas are surfaced and elaborated they are clarified to ensure that all students are able to follow the ideas and are then connected to other ideas already on the table, thus supporting the development of mathematical ideas rather than simply the sharing of multiple ideas. Similar to Lampert and colleagues' (e.g., Lampert et al., 2010; Lampert & Graziani, 2009) instructional activities or Kelemanik and colleagues' (2016) routines for reasoning, patterns such as this could serve as a template to support novices as they learn to enact the complex practice of facilitating classroom discourse.

*Analysis of Student Recognition*

In this section we first discuss two teacher responses in the transcript to highlight the independence of the Student Actions and Student Ideas subcategories of the Student Recognition category. We then highlight patterns that emerge in applying these codes to this transcript and what those patterns might suggest about classroom discourse, subtleties that are seldom captured in existing tools for analyzing teacher responses.

The student in line 1 shares her rule for the area of a triangle as "length times width divided by two," to which the teacher response (line 2) includes both a non-verbal and a verbal component. For Student Actions, we code this teacher response as being *Explicit* since the non-verbal component of the response represents the student's exact words symbolically. This response is *Core* to the Student Ideas since it both represents the student's rule and asks (verbally) for justification of that rule. By contrast, the teacher response to line 30 also *Explicitly* takes up a student's action by having them record their idea on the board. After the student inscribes the triangle on the board within a 6 by 8 rectangle, the teacher writes (l x w)/2 next to the student's drawing (line 30). The teacher response of writing the rule (i.e., an abstraction) from the student's concrete example is likely to require a big enough leap in logic and understanding that the student may perceive it as related to, but not the main idea they presented.

It is thus coded *Peripheral* for Student Ideas. These two contrasting teacher responses highlight the independence of the Student Actions and Student Ideas subcategories and demonstrate that a high level of responsiveness in one category does not automatically mean the same level of responsiveness in the other. Thus, the disentangling of these two aspects of Student Recognition accomplished by a scheme like the TRC allows researchers to not only detect whether the teacher is being responsive to student mathematical thinking but to see the subtly different ways in which responsiveness occurs.

Looking across the Student Recognition coding of the transcript reveals patterns that are likely important for productive classroom discourse. The coding shows that teacher responses are not only frequently coded as *Implicit* (seven responses; 47%) or *Explicit* (five responses; 33%) for Student Actions, but are all also coded as either *Core* (12 responses; 80%) or *Peripheral* (3 responses; 20%) for Student Ideas. This coding indicates that most of the teacher responses both used Student Actions and were also closely aligned with Student Ideas in a way that the student would recognize their ideas in the response. These patterns are important because they illuminate ways in which teachers might think about how they use students' language and actions, as well as how they represent student ideas in their responses when using SMCs as fodder for classroom discussions. These patterns would not be apparent, however, if the coding scheme did not examine the two aspects of Student Recognition independently.

### *Analysis of Mathematical Alignment*

Of the 15 teacher responses in the transcript, eight (53%) were coded *Indeterminate* for Mathematical Alignment. Many of the responses coded *Indeterminate* (lines 2, 4, 6, 10, 14, 22, 24, and 28) take the form of questions that seek additional information from students without revealing the underlying mathematical understanding the teacher is targeting. For example, in the exchange between the student and teacher in lines 1 through 7, the teacher asks a series of questions that are clear enough for the student to answer, but vague enough that the TMUs cannot be inferred. Thus, in this opening exchange the teacher is not yet revealing the mathematics they are working toward, allowing the student to be the one to develop their idea further.

In contrast, seven (47%) teacher responses were specific enough that the underlying TMUs could be inferred and were found to be closely aligned with the relevant MPs (i.e., *Core*). For example, the teacher response in line 8 explicitly summarizes what has been discussed so far with a TMU that is almost identical to the Mathematical Point in line 7; hence, it is *Core* for Mathematical Alignment. The same is true of the teacher responses in lines 16, 20, 26, and 30.

One might notice a pattern, highlighted by the application of the TRC, in the distribution of *Indeterminate* and *Core* teacher responses. Specifically, the iterative pattern seems to be: several teacher responses for which Mathematical Alignment is coded *Indeterminate*, followed by one or more teacher response(s) coded *Core*. As suggested above, this pattern seems to follow from the teacher's recurring practice of asking questions to seek more information, followed by summarizing what has been part of the conversation so far, and then asking more questions to continue the conversation. This pattern suggests that teacher responses may be strategically open or closed depending on when in the conversation they occur and the teacher's underlying mathematical goals. This analysis demonstrates that having a tool that captures the ways teacher responses relate to the mathematics of the student contribution—the *how* of a teacher response—allows us to reveal subtle, yet important differences in teacher responses.

*Exploring Combinations of Coding Categories*

We now illustrate what a coding scheme such as the TRC allows us to notice about this episode by considering interactions among the *who*, *what* and *how* of teacher responses. We discuss two observations about these interactions, noting that we are not providing a rigorous analysis of the transcript, but rather, are illustrating the analytical potential that exploring combinations of TRC categories might afford.

The first observation comes from examining the combination of the Actor and Action codes. In particular, this combination begins to shed light into the kinds of actions in which different classroom players might be engaged. In this transcript, it turns out that all of the *Justifi*cation is being asked of the *Same Student(s)* (lines 2, 4, 6 and 22). Two of the three teacher responses coded as *Clarify* actions (lines 8 and 26), as well as the responses coded *Repeat* (lines 14 and 18) are all done by the *Teacher*. Only two responses—one coded as *Connect* (line 20) and one coded as *Evaluate* (line 24)—engage the *Whole Class*. This small sample suggests that there might be patterns in *who* (the actor) does *what* (the action) that might illuminate important nuances in different classrooms' mathematical discussions and might reveal more opportunities for the teacher to engage the whole class in the discussion.

The second observation comes from considering the combination of the Recognition-Student Ideas and Mathematical Alignment categories. With this combination, we hypothesized that teacher responses that stay close to the ideas in the SMC are also likely to be closer to the underlying Mathematical Point of that SMC. In this classroom episode, the two most frequently occurring code combinations are *Core-Indeterminate* (6 occurrences) and *Core-Core* (6 occurrences). The occurrences of these two seemingly incongruent pairs actually correspond to the ebb and flow of the classroom discussion described above—the teacher responses seem to follow the pattern of a series of open questions intended to elicit more about the student idea followed by a response that explicitly states the main idea of the preceding exchange. Thus, an analysis of this combination of categories could lead to a better understanding of the flow of conversation in responsive mathematics discussions that orient students to each other's ideas.

Although this analysis is only of a small snippet of classroom instruction, it provides a window into the types of relationships that might exist in this classroom among the *who*, *what*, and *how* of teacher responses. One could imagine different patterns in these combinations of coding categories in classrooms where the teaching is less responsive to students. The explicit attention to, and disentangling of, the various facets of teacher responses in a coding scheme like the TRC provides insight into subtleties resulting from variations in teacher responses to student mathematical contributions that occur during classroom instruction.

**Conclusion**

We have argued that progress in the area of research on mathematics teacher responses could be enhanced were the field to attend more explicitly to the *who*, *what*, and *how* facets of those responses, as well as to the units of analysis of the student contribution, the teacher response, the teacher response referent, and the context. We described the TRC to illustrate how such attention might play out, and then applied it to an excerpt of classroom mathematics discourse to demonstrate the affordances of this approach. We conclude by making several further observations about the potential versatility and power in articulating units of analysis and developing and applying tools that attend to the *who*, *what* and *how* when conducting research on teacher responses.

Explicit attention in tool development to choices related to the units of analysis discussed in this paper could provide greater flexibility for the field to apply the resulting tools. For example, a tool that leaves as variable the choice of teacher response referent could be used to examine teacher responses in relation to any student contribution. In this paper we illustrated how a tool such as the TRC could be used to examine teacher responses to the student contribution that immediately preceded the response. The teacher response referent, however, could be adapted to consider when teachers respond to prior contributions that are still (or that they want to keep) on the table. For example, in our work on Mathematical Opportunities in Student Thinking (MOSTs; see BuildingonMOSTs.org) we are interested in teachers' responses to MOSTs—"student thinking worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36). In that case, the student contribution of interest is the immediately preceding MOST, which may or may not be the immediately preceding contribution.

One could also imagine researchers defining the teacher response referent to be another particular type of contribution (such as a conjecture or a solution). The purpose of such research might be to see how those student contributions are returned to in teacher responses throughout the lesson. Thus, there are a variety of ways that researchers could vary the teacher response referent, as well as the other units of analysis, to match their research focus.

Tools that disentangle the *who, what,* and *how* facets of teacher responses could allow researchers to examine a variety of issues in the mathematics classroom. For example, issues of equity and positioning could be examined using a tool such as the TRC in at least two different ways. One way is by looking at the demographics of the student whose thinking is being shared in relation to the teacher response to that thinking. An example would be using the tool to notice important differences in teacher responses to different students' contributions, such as when the Action in response to some students' contributions is to *Dismiss* or *Correct* or simply *Validate* while the Action in response to other students' contributions is to *Develop* or *Justify*. One could also use the Student Recognition category to illuminate how the content of different students' contributions is taken up. For example, is the Student Recognition-Student Ideas always *Core* with some groups of students and *Other* with different groups of students? Similar relationships could be examined with the Student Recognition-Student Actions, Mathematical Alignment, and Actor coding categories. A second way issues of equity and positioning could be examined is by looking at the demographics of the Actor who is being invited to engage with the student contribution and the Action they are being asked to take. For example, some students may be regularly asked to *Justify* or *Develop* their peers' contributions and other students may only be asked lower level *Literal* questions about those contributions.

Attention to the *how* facet of teacher responses seems critical to any attempt to capture the responsiveness (e.g. Bishop et al., 2016) of the teacher responses as they seek to productively use student contributions, yet this facet is currently the least explicit and most underrepresented in the literature. Tools could capture this facet by adopting or adapting, for example, the combination of the Recognition-Student Actions and Recognition-Student Ideas coding categories of the TRC. Such codes could allow researchers to explore questions such as whether teacher responses take up Student Actions by routinely using students' words but do or do not truly focus on the Student Ideas in the student contribution. The Mathematical Alignment could also provide an indication of the extent to which the mathematical direction the teacher is taking actually builds on and responds to the mathematics of the student contribution. Thus, attending to the *how* facet could allow researchers to examine coded data in multiple ways to attend in more subtle ways to the responsiveness of teacher responses.

Our long-term goal of better understanding teachers' productive use of high-leverage SMCs, what we call *building* on MOSTs (Leatham et al., 2015; Van Zoest, Peterson et al., 2016), led us toward a fine-grained examination of how teachers respond to SMCs in-the-moment. A complex teaching practice such as building on MOSTs is difficult to study and often requires the practice to be decomposed in order to "articulate, unpack and study" it (Boerst et al., 2011, p. 2859). We have found that decomposing and studying teacher responses by explicitly attending to certain facets and units of analysis has provided insights that were not possible without doing so. We hope that our providing a common language for these facets and units can benefit the field as we work together to understand the complex practice of productively responding to student mathematical thinking.

## Acknowledgements

1. This work was supported in part by the U.S. National Science Foundation (NSF) under Grant Nos. 1220141, 1220357, 1220148, 1720613, 1720566, and 1720410. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.
2. Preliminary versions of the ideas in this paper were presented at the 41st Conference of the International Group for the Psychology of Mathematics (Peterson et al., 2017) and the 2018 American Educational Research Association Annual Conference (Peterson et al., 2018).

3. The authors thank Napthalin Atanga, Rachel Bernard, Elizabeth Fraser, Alicia Heninger, Carlee Madis, Mary Ochieng, Kylie Palsky, Joshua Ruk, and Amanda Seiwell for their contributions to developing the Teacher Response Coding Scheme (TRC).

**References**

Baxter, J. A., & Williams, S. (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education*, *13*(1), 7-26.

Bishop, J. P., Hardison, H., Przybyla-Kuchek, J., & Hassay, E. (2020). Leveraging student thinking to foster productive discussions. *Mathematics Teacher: Learning and Teaching PK-12, 113*(12), 995-1002.

Bishop, J. P., Przybyla-Kuchek, J., & Hardison, H. (2018, April 13-17). A case study of teacher moves in mathematically responsive interactions. *Paper presented at the 2018 Annual Meeting of the American Educational Research Association.* Retrieved April 16, 2018, from the AERA Online Paper Repository.

Bishop, J., Hardison, H., & Przybyla-Kuchek, J. (2016). Profiles of responsiveness in middle grades mathematics classrooms. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1173-1180). The University of Arizona.

Boerst, T. A., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, *113*(12), 2844-2877.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education*, *27*(1), 174-186.

Chapin, S. & O'Connor, C. (2007). Academically productive talk: Supporting student learning in mathematics. In W. G. Martin, M. Strutchens, & P. Elliott (Eds.), *The learning of mathematics, 69th NCTM yearbook* (pp. 113-128). National Council of Teachers of Mathematics.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, grades K-6*. Math Solutions.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics*, *19*(2), 2-10.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*(3), 401-429.

Drageset, O. G. (2014). Redirecting, progressing, and focusing actions—a framework for describing how teachers use students' comments to work with mathematics. *Educational Studies in Mathematics, 85*(2), 281-304.

Drageset, O. G. (2015). Different types of student comments in the mathematics classroom. *The Journal of Mathematical Behavior, 38*(1), 29-40.

Ellis, A., Özgür, Z., & Reiten, L. (2019). Teacher moves for supporting student reasoning. *Mathematics Education Research Journal, 31*(2), 107-132. doi:10.1007/s13394-018-0246-6

Even, R., & Gottlib, O. (2011). Responding to students: Enabling a significant role for students in the class discourse. In Y. Li & G. Kaiser (Eds.), *Expertise in mathematics instruction* (pp. 109-129). Springer.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper elementary mathematics classrooms. *The Elementary School Journal*, *102*(1), 59-80.

Kelemanik, G., Lucenta, A., & Creighton, S. J. (2016). *Routines for reasoning: Fostering the mathematical practices in all students*. Heinemann.

Kilpatrick, J., Martin, W. G., & Schifter, D. (Eds.). (2003). *A research companion to* Principles and standards for school mathematics. National Council of Teachers of Mathematics.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In L. Kucan & M. K. Stein (Eds.), *Instructional explanations in the disciplines* (pp. 129-141). Springer.

Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning. *The Elementary School Journal*, *109*(5), 491-509.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88-124.

Leatham, K. R., Stockero, S. L., Peterson, B. E., & Van Zoest, L. R., (2020, February 6-8). *Understanding and developing skills needed to build on student mathematical thinking*. Presentation at the 24th Annual Conference of the Association of Mathematics Teacher Educators, Phoenix, AZ.

Lineback, J. E. (2015). The redirection: An indicator of how teachers respond to student thinking. *Journal of the Learning Sciences*, *24*(3), 419-460.

National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* National Council of Teachers of Mathematics.

Ozgur, Z., Reiten, L., & Ellis, A. B. (2015). On framing teacher moves for supporting student reasoning. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1062-1069). Michigan State University.

Peterson, B. E., Van Zoest, L. R., Rougée, A. O. T., Freeburn, B., Stockero, S. L., & Leatham, K. R. (2017). Beyond the "move": A scheme for coding teachers' responses to student mathematical thinking. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 17-24). International Group for the Psychology of Mathematics Education.

Peterson, B. E., Van Zoest, L. R., Rougée, A. O. T., Freeburn, B., Stockero, S. L., & Leatham, K. R. (2018, April 13-17). The teacher response coding scheme: Disentangling teacher responses to student mathematical thinking. *Paper presented at the 2018 Annual Meeting of the American Educational Research Association.* Retrieved November 4, 2019, from the AERA Online Paper Repository. htpps://doi.org/10.302/1315199

Robertson, A. D., Atkins, L. J., Levin, D. M., & Richards, J. (2016). What is responsive teaching? In A. D. Robertson, R. Scherr, & D. Hammer (Eds.). *Responsive teaching in science and mathematics* (pp. 1-35). Routledge.

Robertson, A.D., Scherr, R. & Hammer, D. (Eds.). (2016). *Responsive teaching in science and mathematics* (pp. 1-35). Routledge.

Scherrer, J., & Stein, M. K. (2013). Effects of a coding intervention on what teachers learn to notice during whole-group discussion. *Journal of Mathematics Teacher Education*, *16*(2), 105-124.

Schleppenbach, M., Flevares, L. M., Sims, L. M., & Perry, M. (2007). Teachers' responses to student mistakes in Chinese and U.S. mathematics classrooms. *The Elementary School Journal*, *108*(2), 131-147.

Selling, S. K. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. *Journal for Research in Mathematics Education*, *47*(5), 505-551.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions.* National Council of Teachers of Mathematics.

Spangler, D. A., & Wanko, J. J. (Eds.). (2017). *Enhancing classroom practice with research behind* Principles to actions. National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313-340.

Stockero, S. L., Peterson, B. E., Ochieng, M. A., Ruk, J. R., Van Zoest, L. R., & Leatham, K. R. (2019). Teachers' initial responses to high leverage instances of student mathematical thinking. In M. Graven, H. Venkat, A. A. Eissen, & P. Vale (Eds.), *Proceedings of the 43rd Annual Conference of the International Group for the Psychology of Mathematics Education* (pp. 335-342). University of Pretoria.

Stockero, S. L., Van Zoest, L. R., Freeburn, B., Peterson, B. E., & Leatham, K. R. (2020). Teachers' responses to instances of student mathematical thinking with varied potential to support student learning. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00334-x.

Stockero, S. L., Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Rougée, A. O. T. (2017). Teachers' responses to a common set of high potential instances of student mathematical thinking. In E. Galindo, & J. Newton (Eds.), *Proceedings of the 39th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1178-1185). Hoosier Association of Mathematics Teacher Educators.

Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Stockero, S. L. (2016). Conceptualizing the teaching practice of building on student mathematical thinking. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1281–1288). University of Arizona.

Van Zoest, L. R., Stockero, S. L., Leatham, K. R., & Peterson, B. E. (2016). Theorizing the mathematical point of building on student mathematical thinking. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 323-330). PME.

Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning*, *19*(1), 33-54.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, *30*(2), 171.