Teaching Mathematics, Teaching Students, Teaching Mathematics to Students

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Today's talks

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* Overview of Teachers Promoting Change Collaboratively (TPC^2)

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* Overview of teachers' interactions in PLC settings and implications for MKT

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Mathematical Knowledge for Teaching (secondary Mathematics)

Envisioned Structure of Intervention

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* Focus on PLCs as sources and support of teacher reflection

Actual ~~Envisioned~~ Structure of Intervention

* Three courses over 3 years on conceptualizing the secondary mathematics curriculum
* Weekly Reflecting on Practice meetings—4-6 teachers in school-based groups; led by project facilitator; facilitation fades over 3 years.
* Two 25-teacher cohorts over 4 years (with leakage)
* Technology support (laptop computers, video cameras, computer projectors)
* Creation of case studies and modules to support teacher reflection on curriculum and on teaching

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Why the Change in Plans?

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Because of what we learned about teachers' MKT as the project unfolded.

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* Two cohorts instead of 4 because we argued (and NSF agreed) that it was more important to understand these teachers' MKT than to recruit new teachers
* Technology because focus on mathematical visualization demanded it.

Courses

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- Three courses on reconceptualizing secondary curriculum as springboard to calculus and differential equations
- Core concepts: Quantity, variable, variation, covariation, and function
- Incorporated research on understanding mathematical ideas with study of the mathematics itself

Examples

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* Reconceptualize similarity, angle measure, and trig function so that they form a coherent scheme of ideas
* Extend notion of covariation to functions defined parametrically and to functions of several variables

Examples

* Shape learning goals so that students build powerful personal,
* basic meanings that they can use and re-use to create new mathematics
* defined parametrically and to functions of several variables

Professional Learning Communities

* Called "Reflecting on Practice Sessions". To be a PLC is an achievement, not a process.
* Agendas focused on teachers' examination of their instruction
* Each RPS videotaped and summarized; two years of meetings coded (to be discussed)

Professional Learning Communities

A Mistake

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Analysis of PLC Participation

Coding schemes

Teachers' Beliefs

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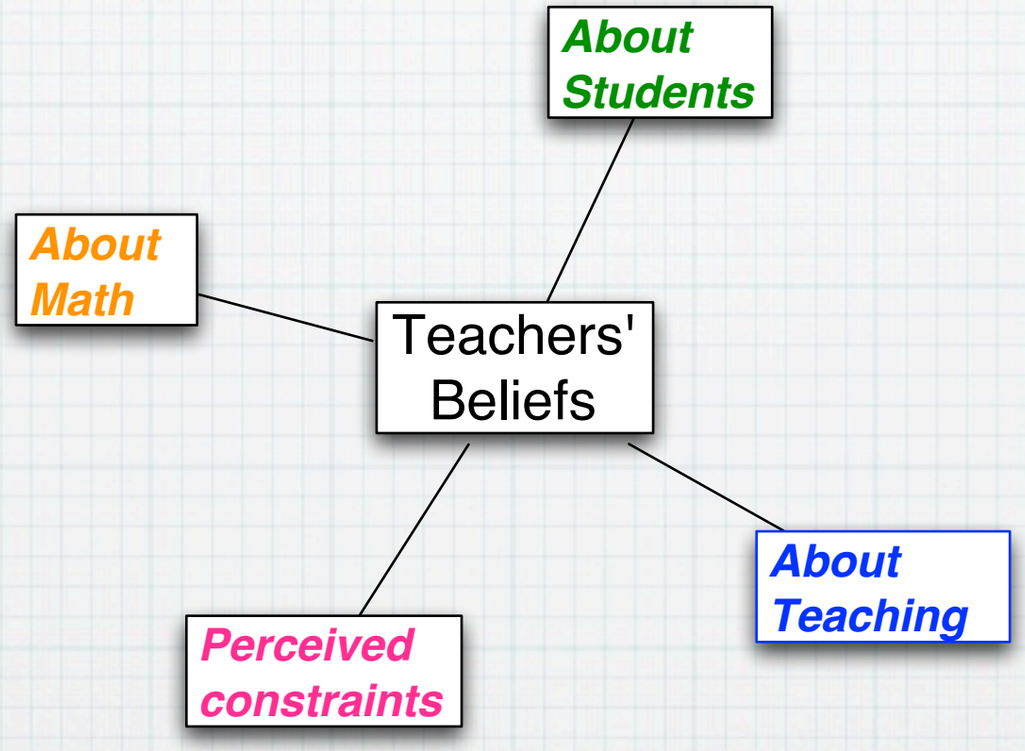

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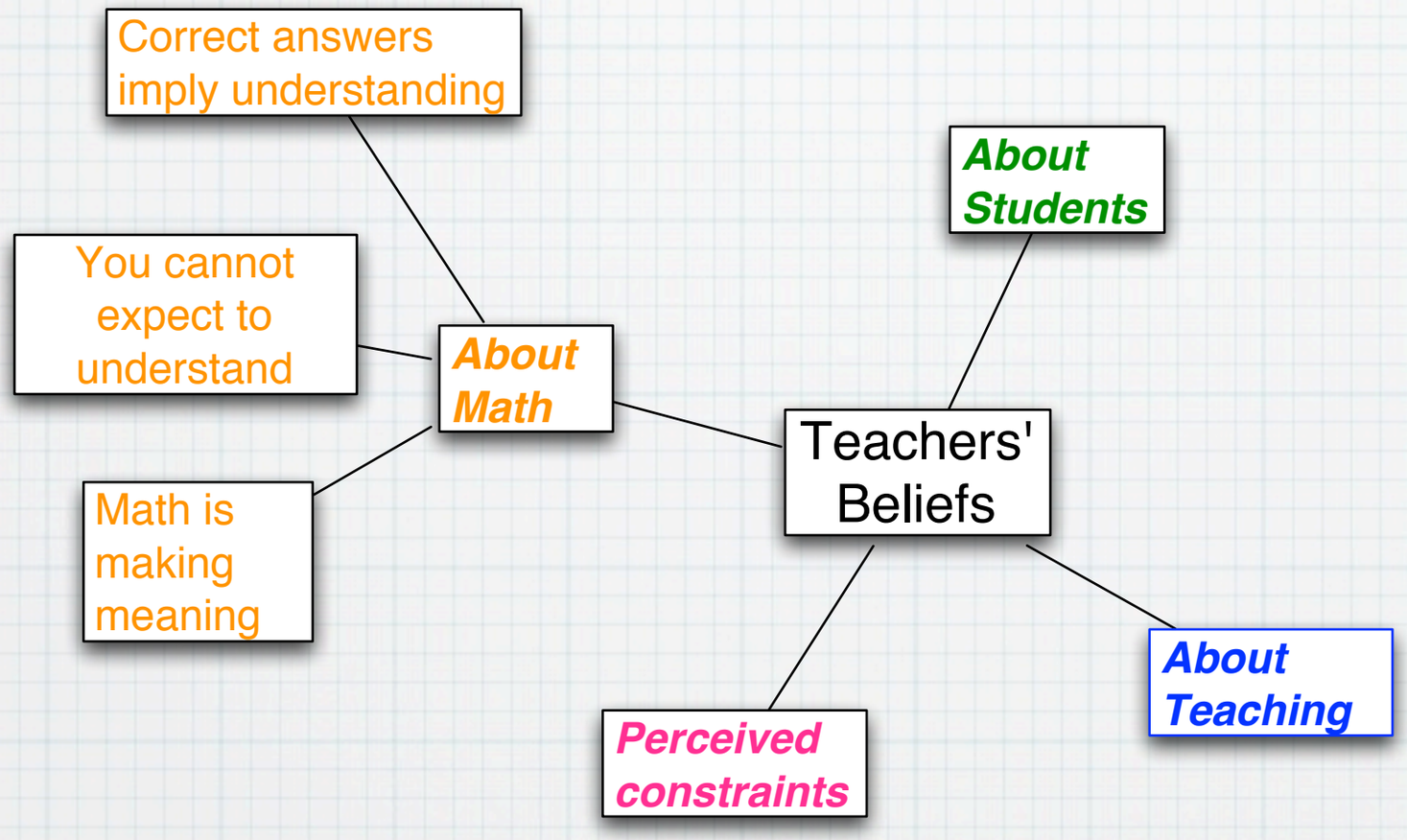

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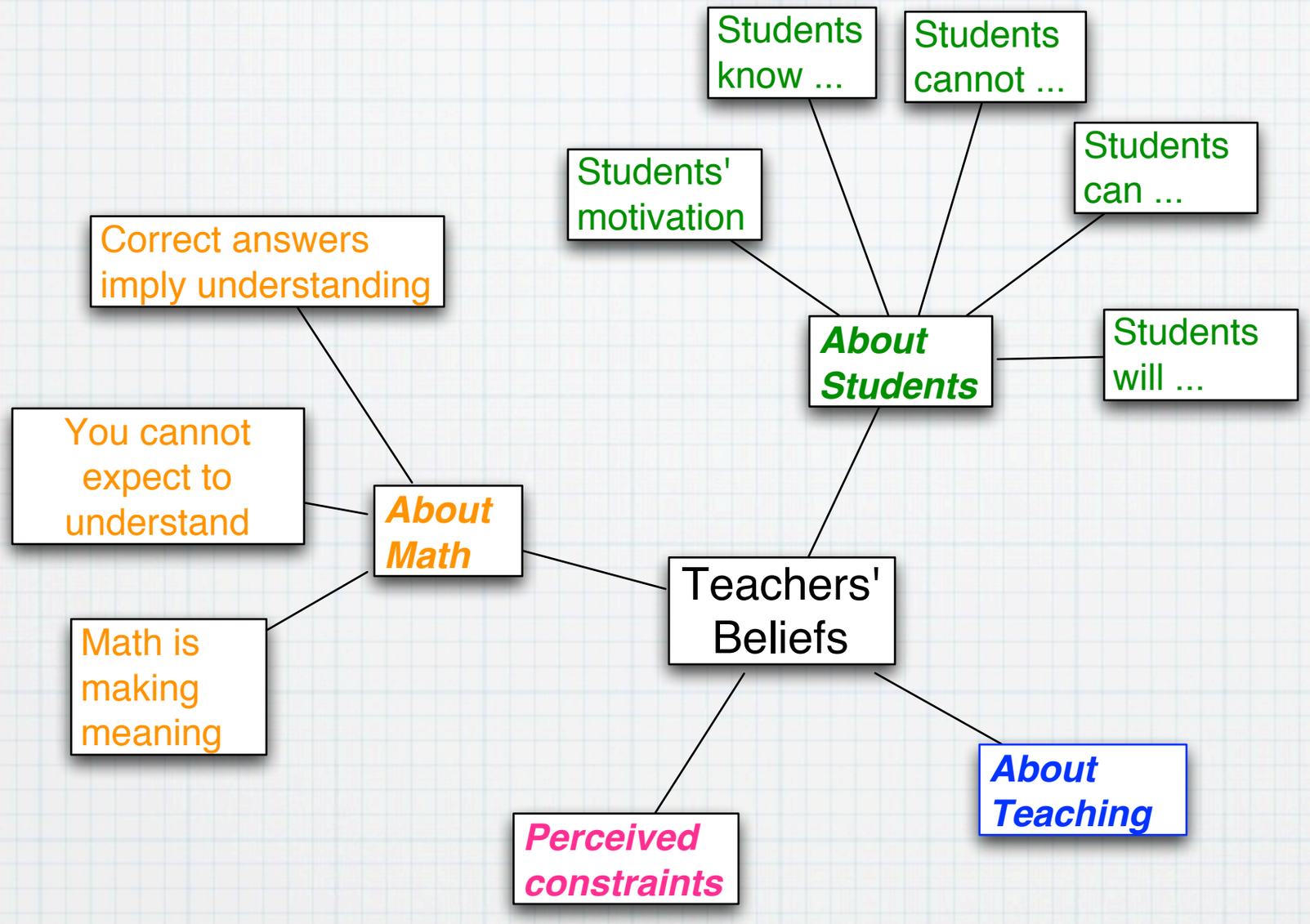

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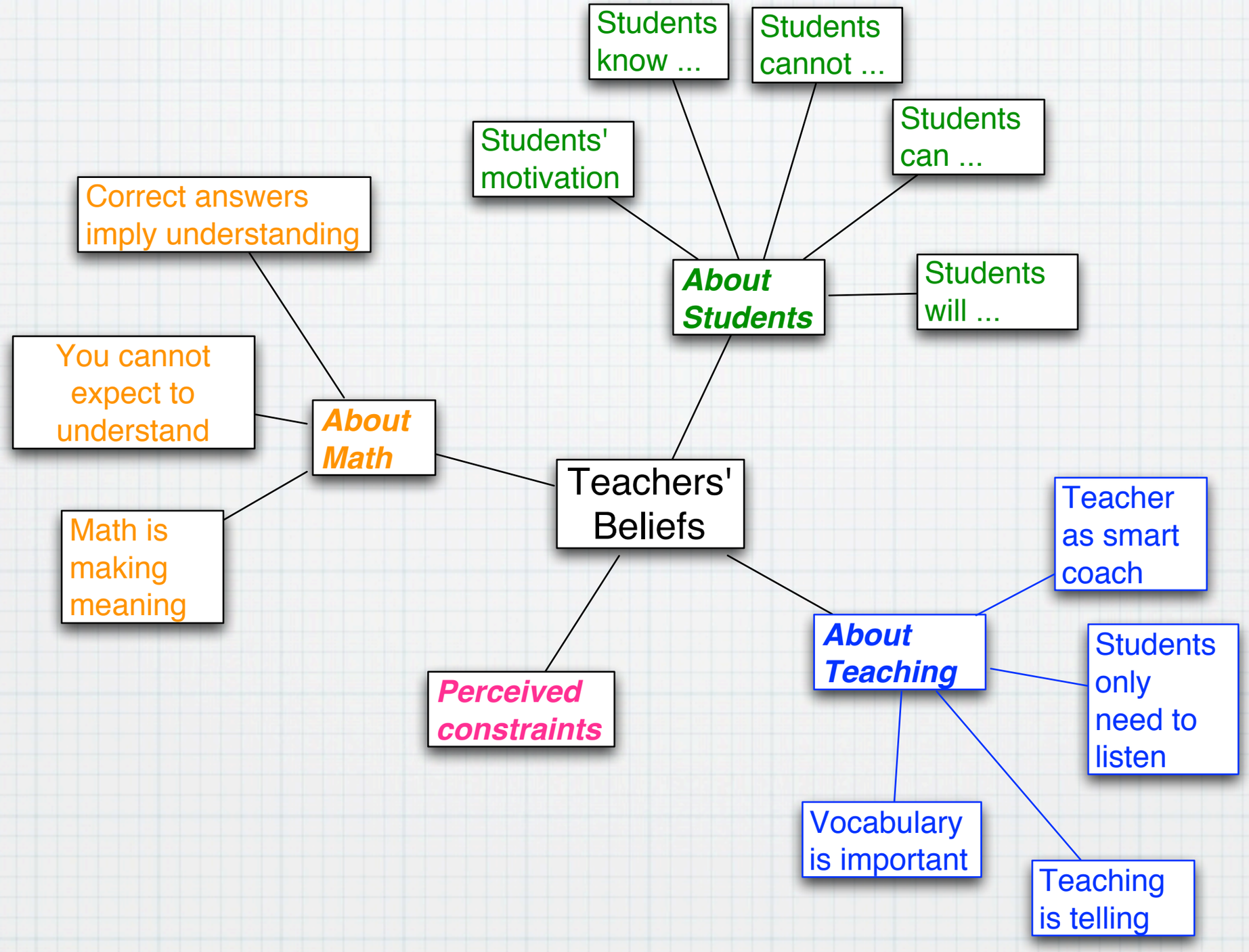

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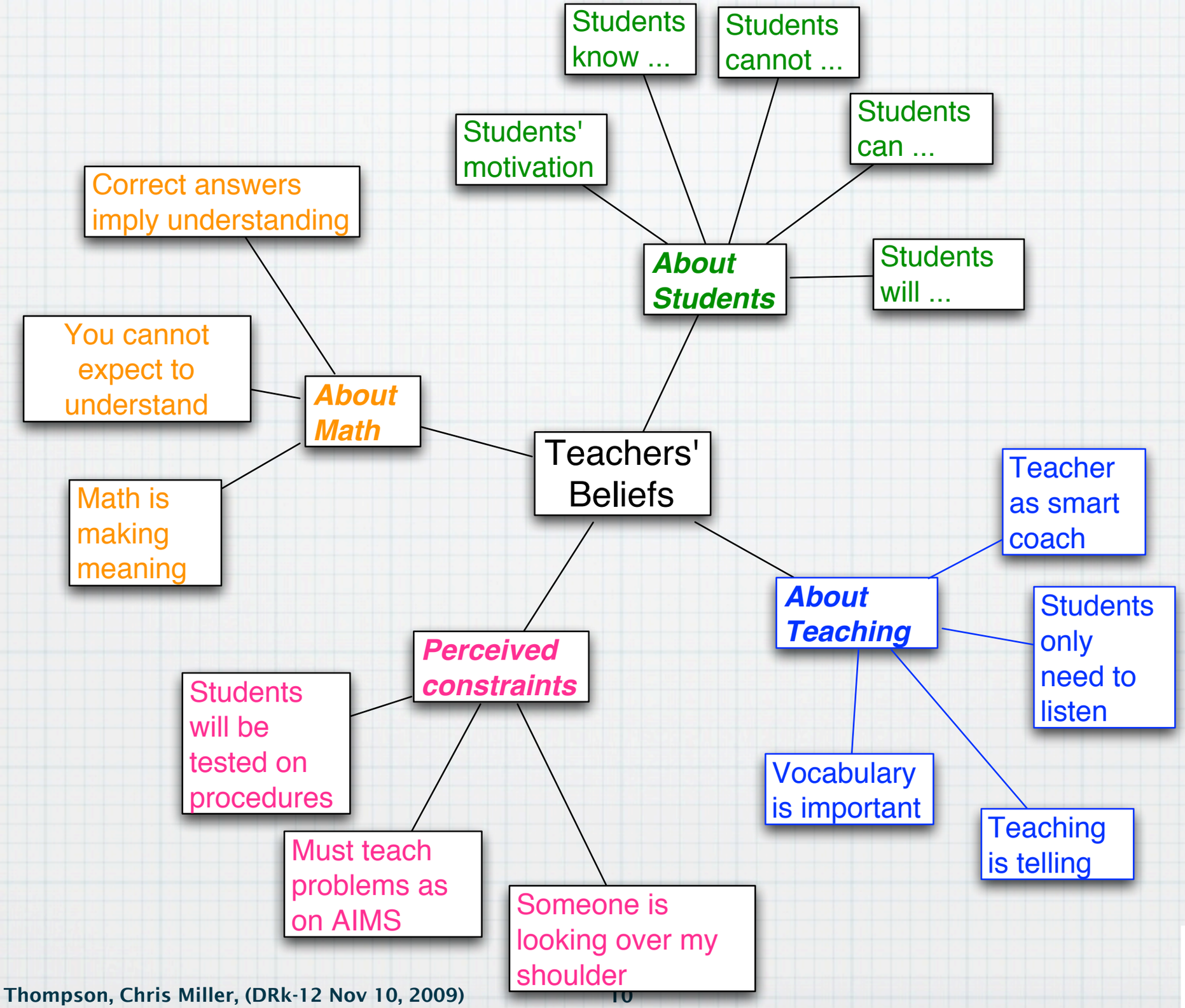

Teacher Knowledge

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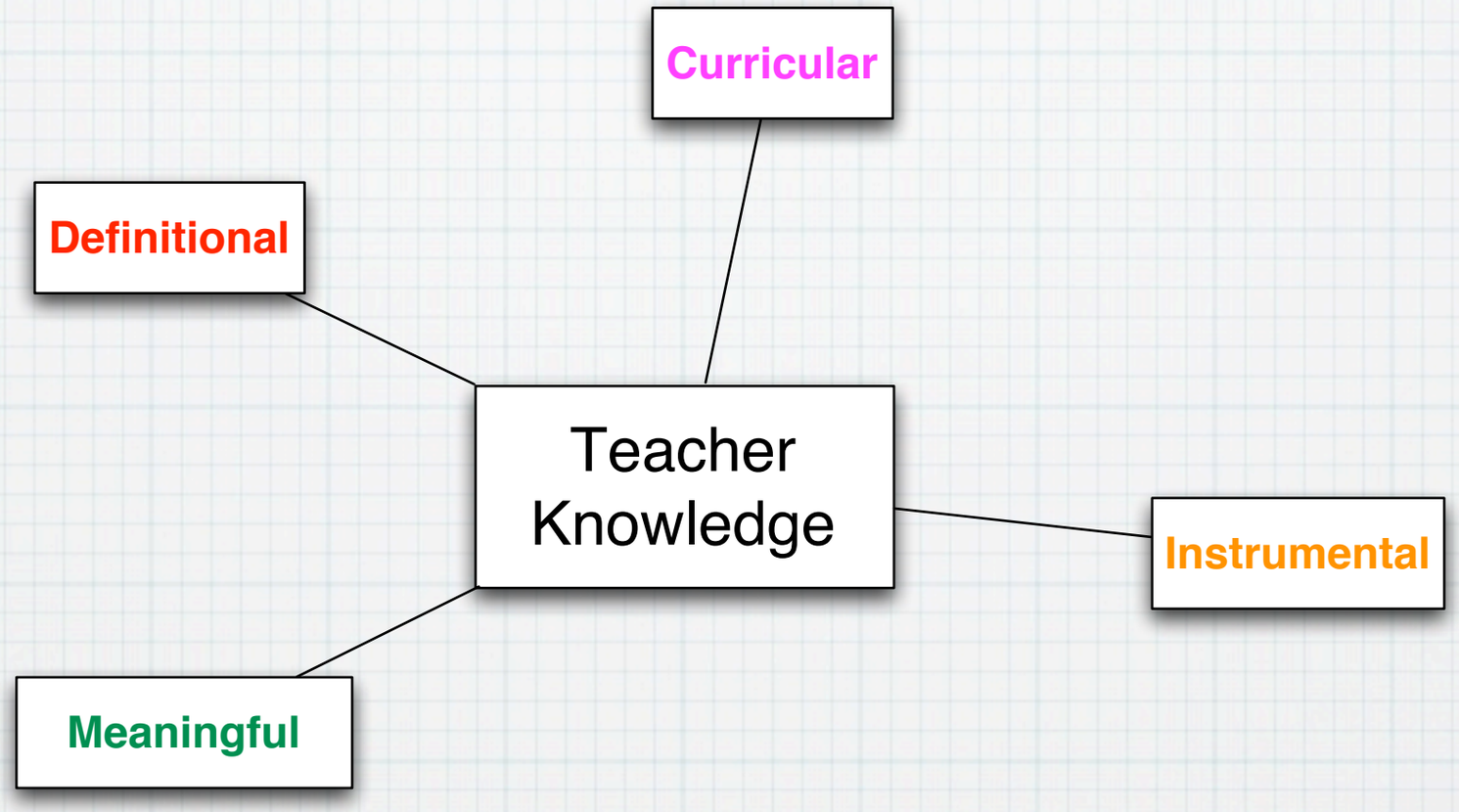

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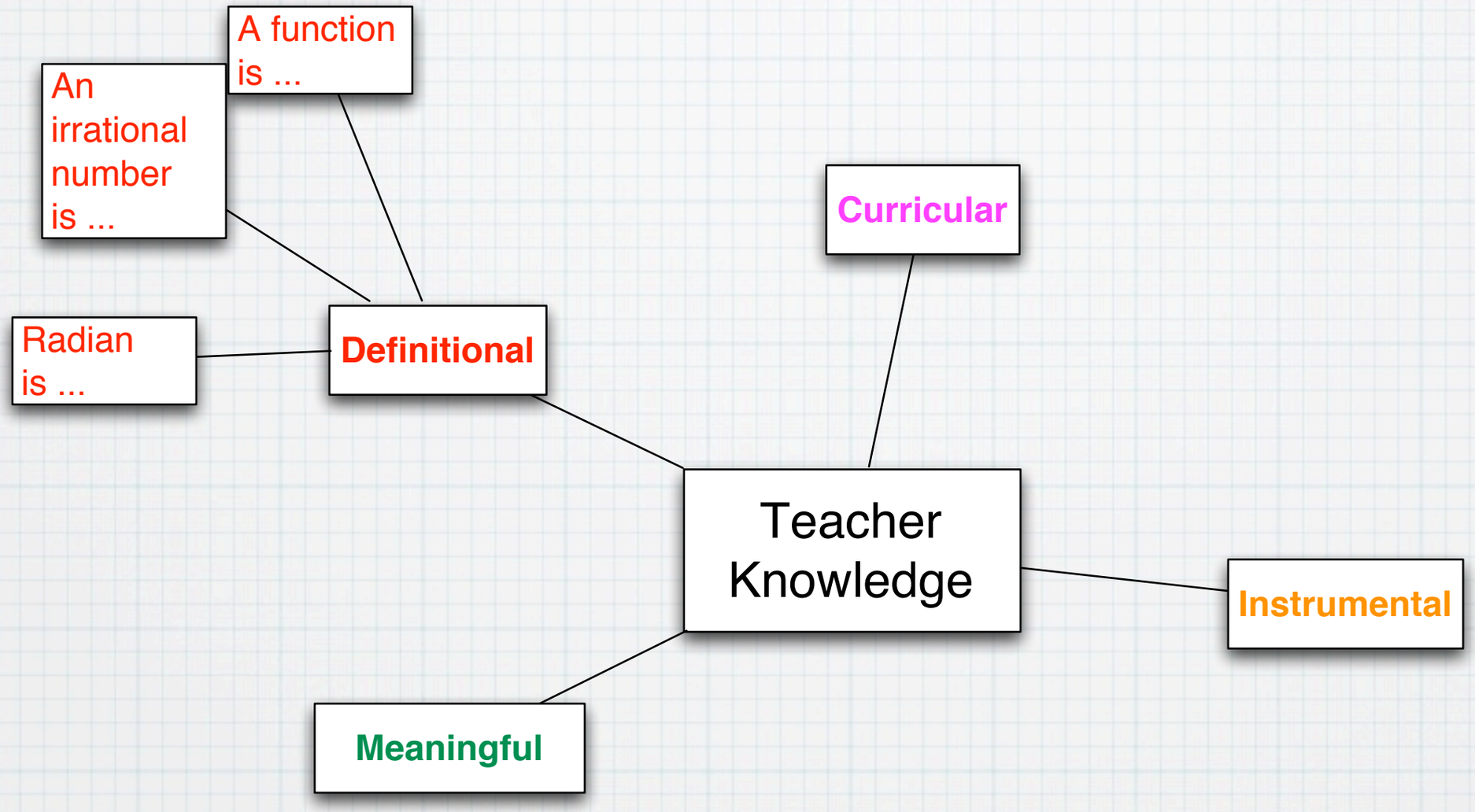

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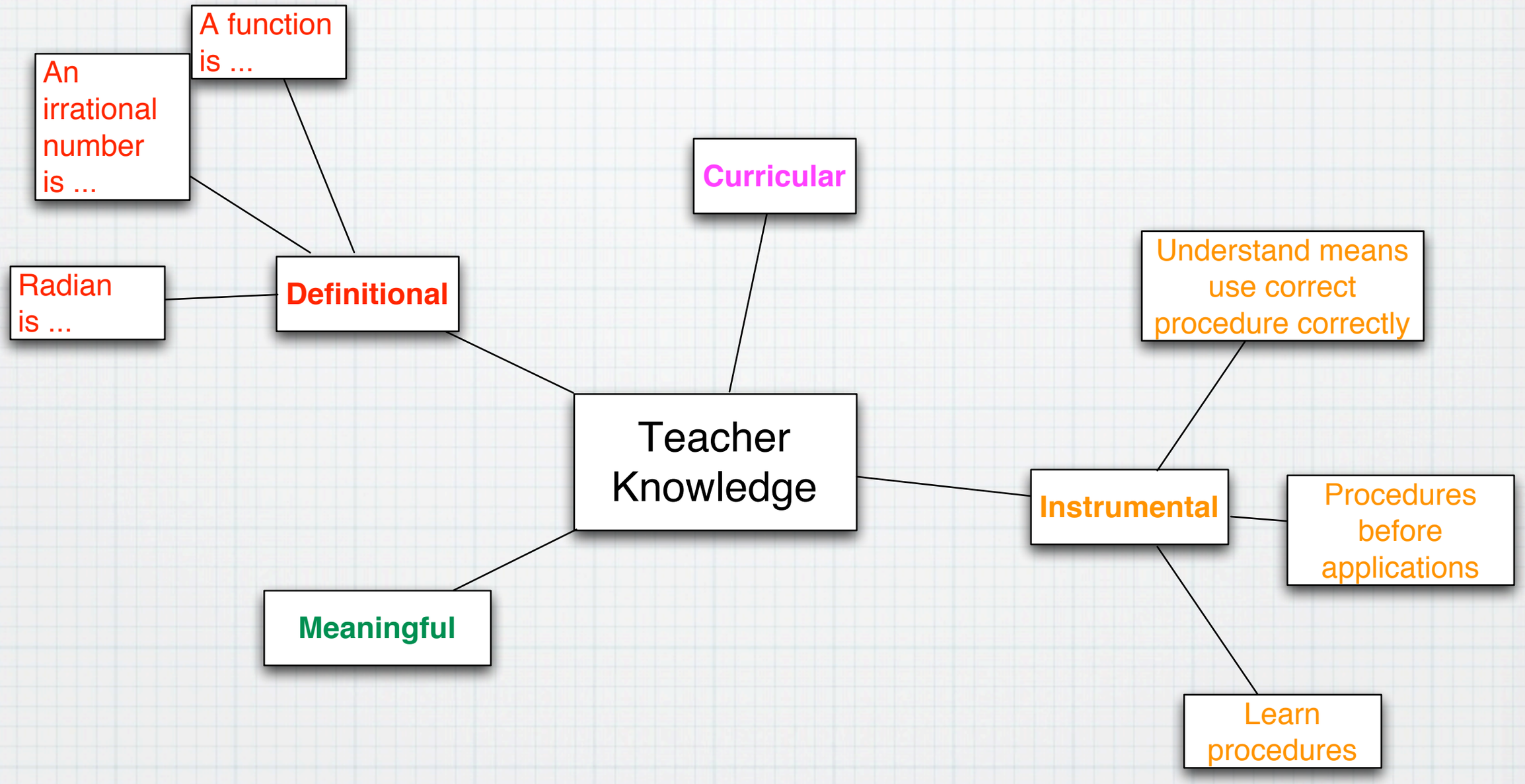

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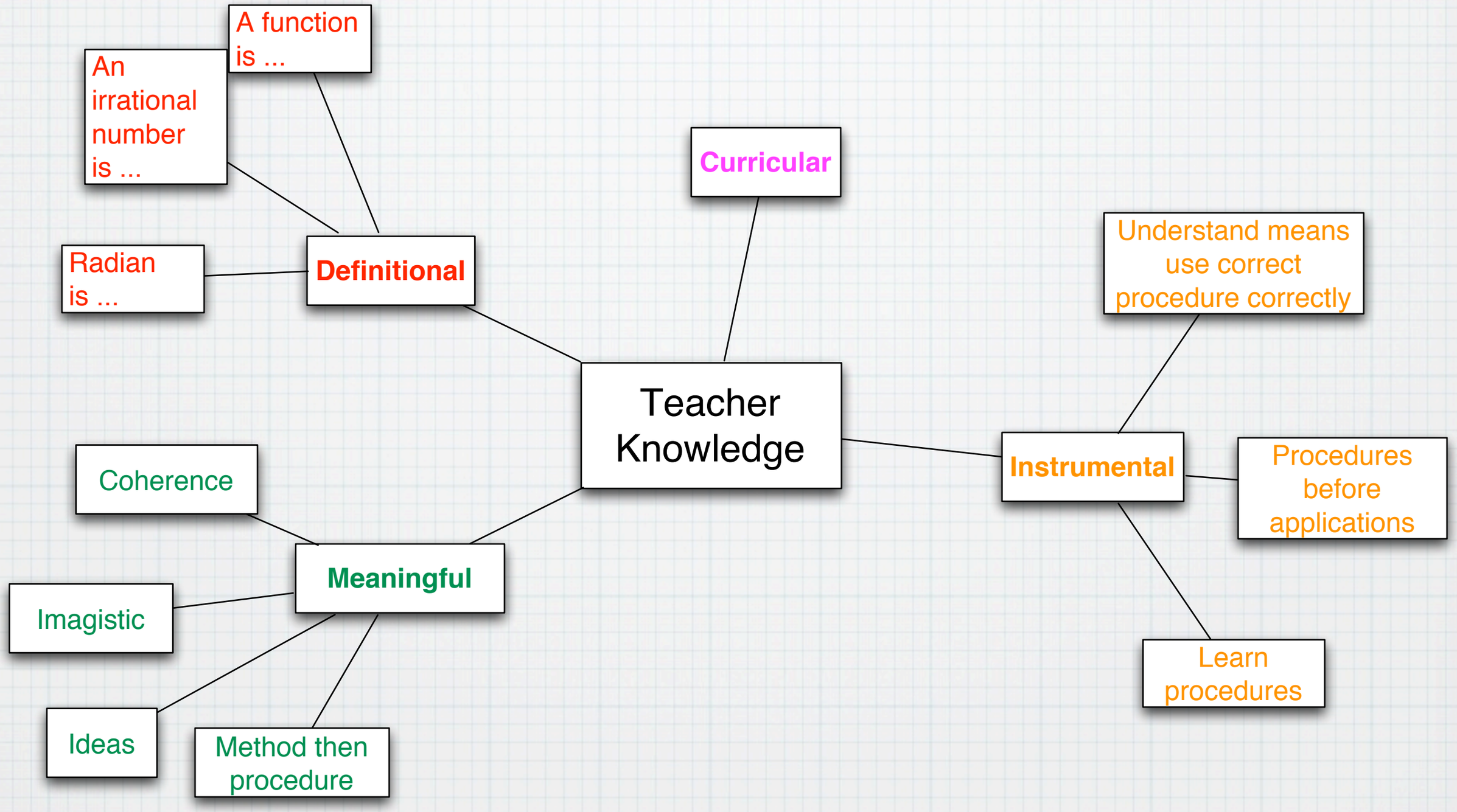

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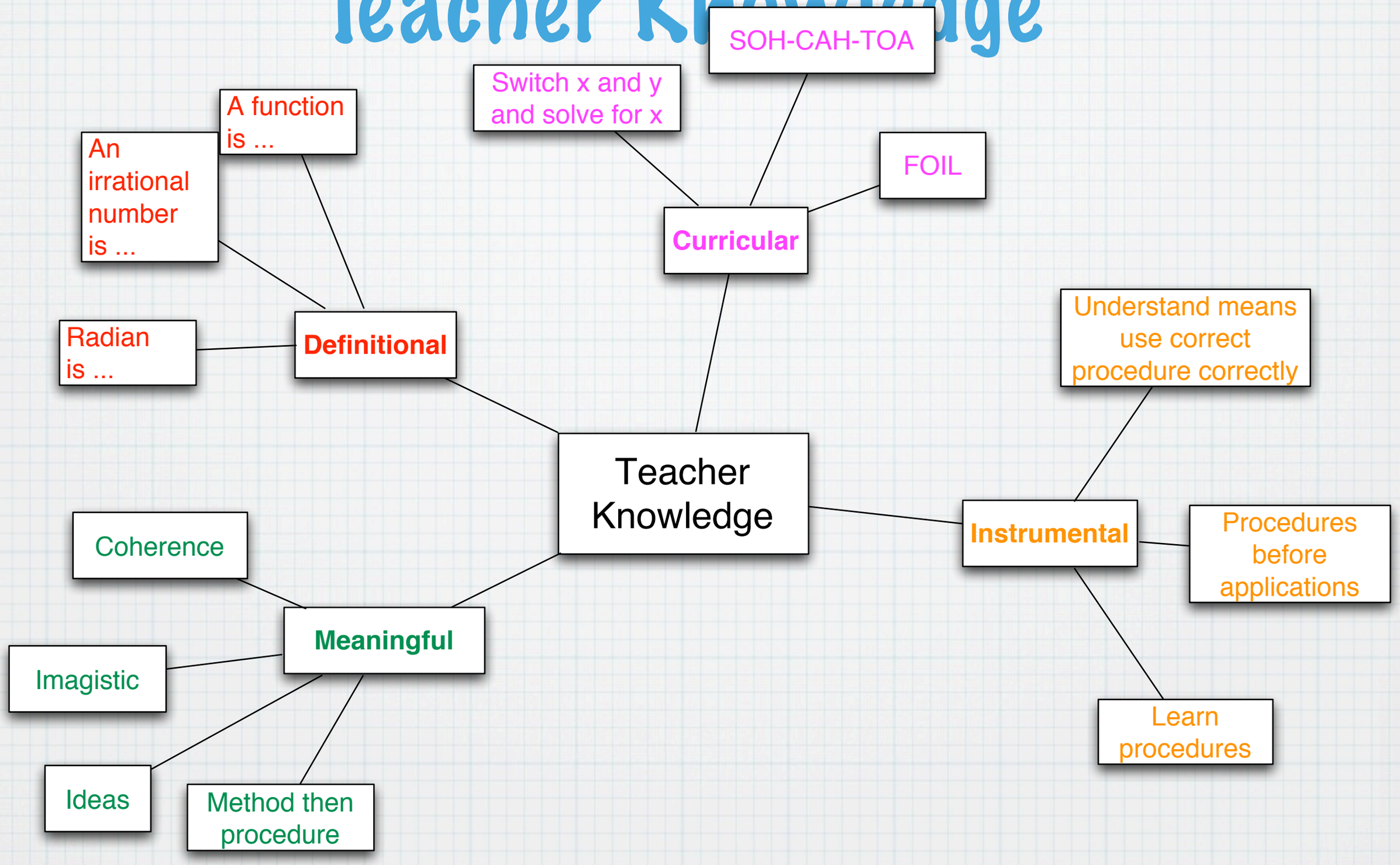
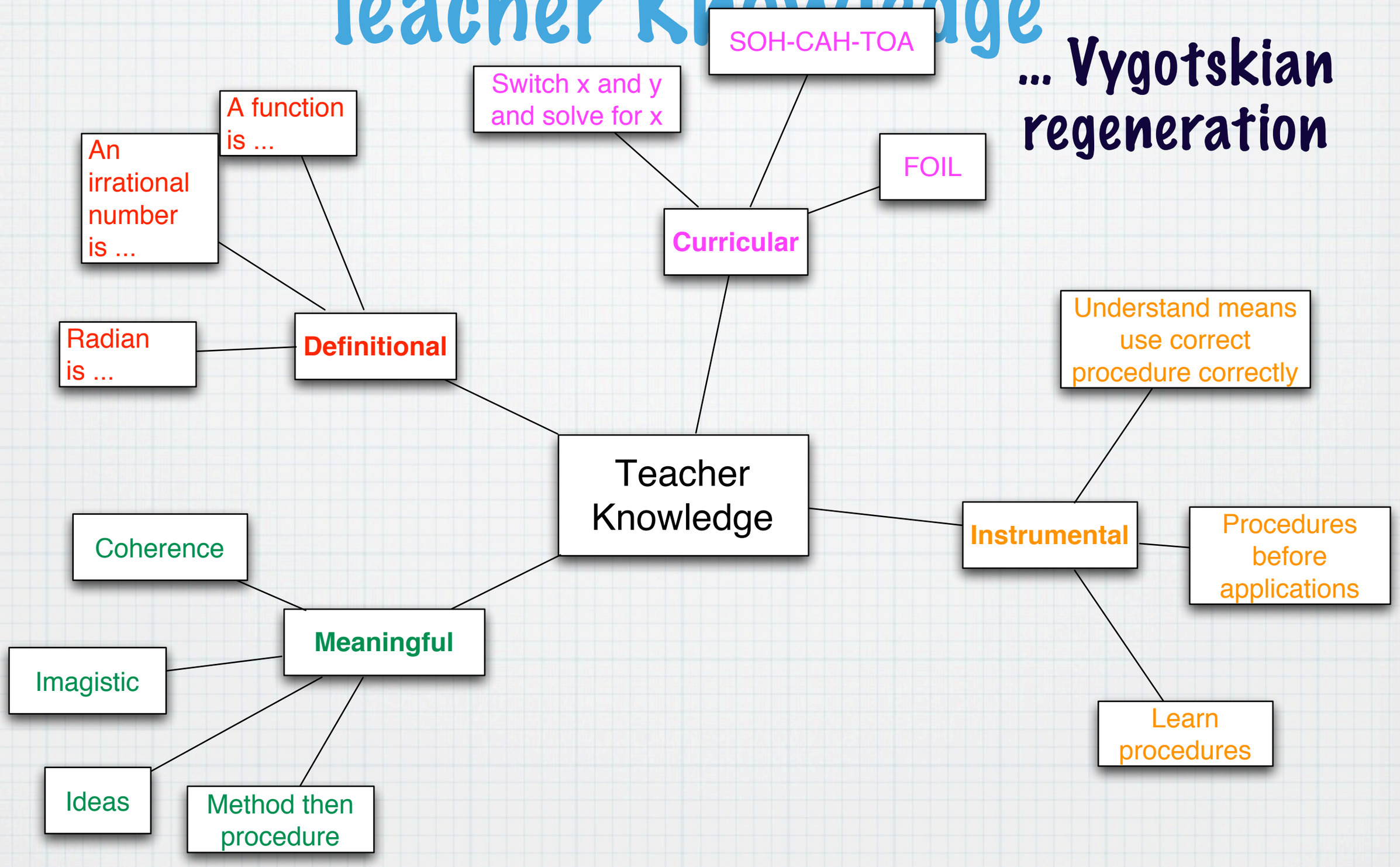


Teacher Knowledge ... Vygotskian regeneration



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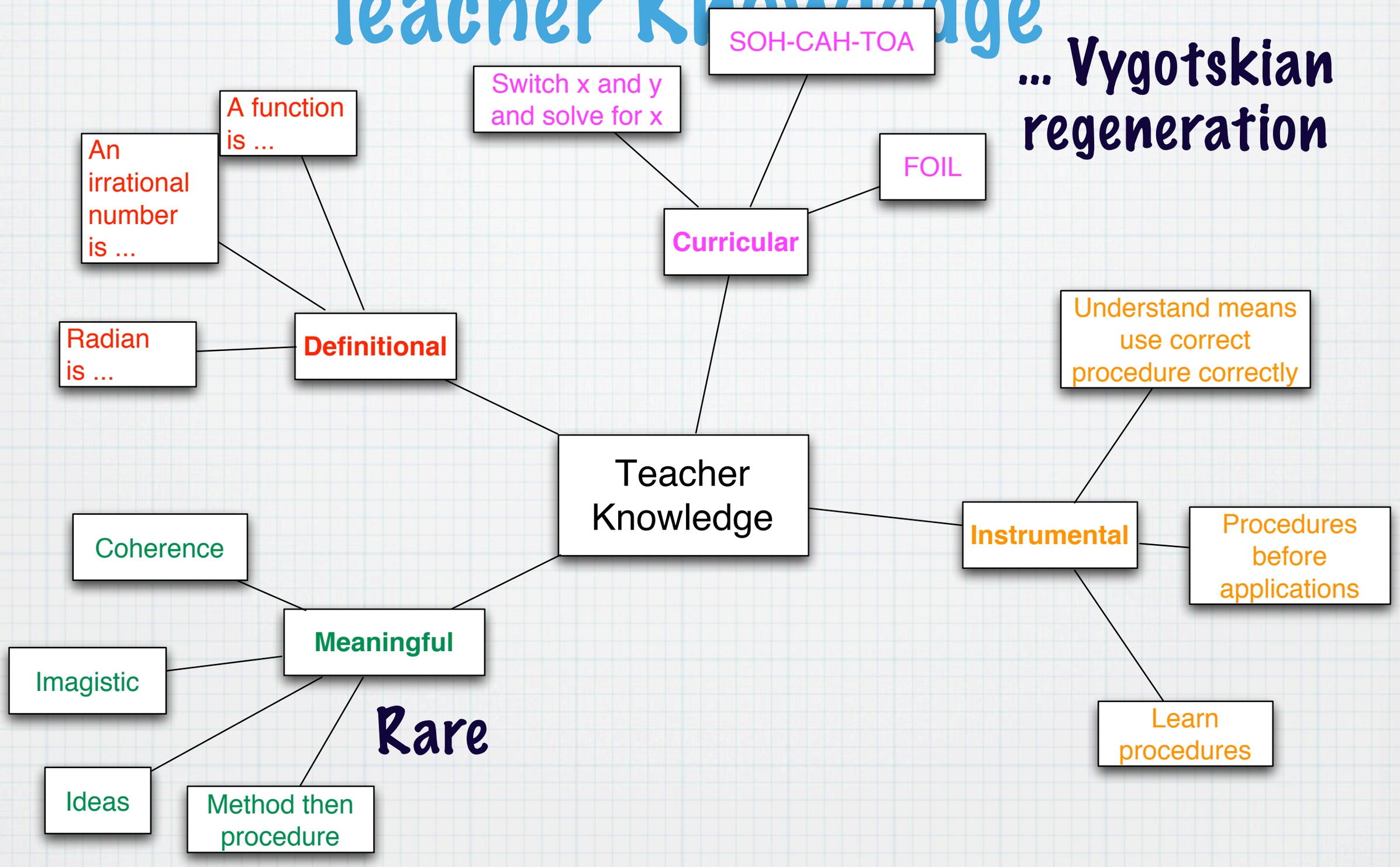

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* More—in the sense that it is an internalized, generalized, fuzzy image of mathematics that rests upon what teachers learned, and upon the orientations they developed, while in school

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Curricular Knowledge

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* More—in the sense that it is an internalized, generalized, fuzzy image of mathematics that rests upon what teachers learned, and upon the orientations they developed, while in school
* Less—in the sense that it is not knowledge OF a curriculum. It does not entail a developmental progression of ideas.

Teacher Knowledge

	Fall '05	Spr '06
Definitional	17%	27%
Meaning	9%	7%
Curricular	44%	36%
Instrumental	30%	30%

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Meaning	9%	7%
Curric/Instr	74%	66%

Orientation to Students' Mathematics

Decentering

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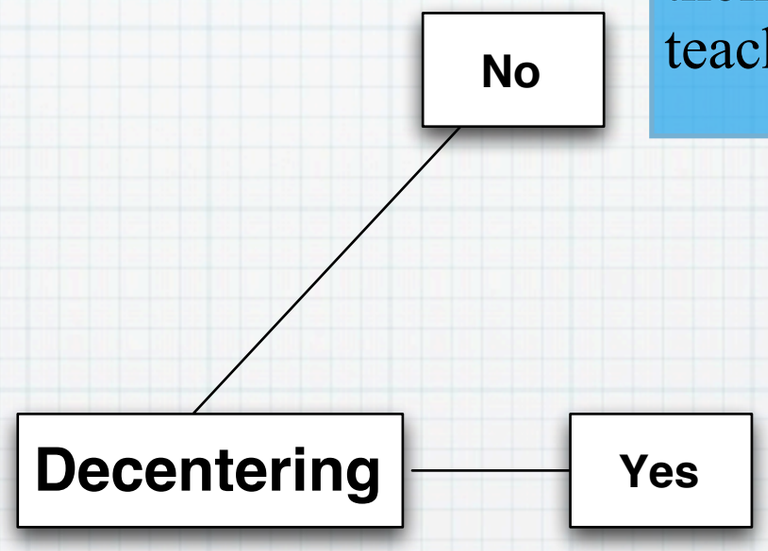
Decentering

Given the opportunity, does a teacher consider how students are thinking and the implications their ways of thinking have for appropriate instruction?

Orientation to Students' Mathematics

Teacher gauges students' understanding in terms of their mastery of the teacher's mathematics..



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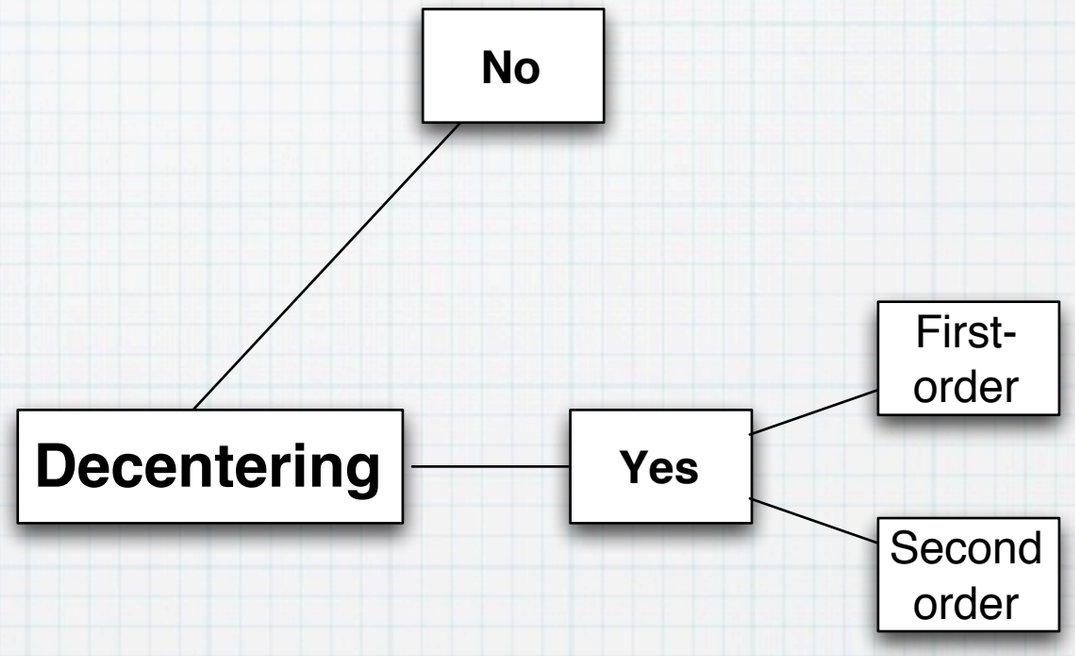

Orientation to Students' Mathematics

Teacher Decenter	Fall 05	Spr 06
Yes	4%	3%
No	96%	97%

What We Learned ...

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Teachers had internalized an incoherent curriculum
to the point that it was impossible for them to conceive of
someone learning it meaningfully

What We Learned ...

We and they had vastly different images of reformed teaching

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Listening to Students

Teachers thought that listening to students meant (a) determining whether students got it right, (b) to have opportunities to give students positive feedback ("romantic constructivism");

We understood that listening to students meant making models of students thinking and adjusting your behavior according to your understanding of how the student interpreted what you said

What We Learned ...

We and they had vastly different images of reformed teaching

Goals of Instruction

Teachers thought that the goal of instruction was to make learning simple and uncomplicated;

We thought that the goal of instruction was to help students learn to deal with complexity and sophistication.

What We Learned ...

We and they had vastly different images of reformed teaching

Conceptions of Students

Teachers thought that their students were unable and unwilling to engage in sophisticated ways of thinking

We took for granted that students not only are capable of such thinking but find it more enjoyable.

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Student Engagement

Teachers understood "student engagement" to mean whether students are paying attention and on task

We understood it to mean whether students were thinking about ideas and reasoning with them

What We Learned ...

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Instructional Sequences

Teachers were thinking about what to do next

We were thinking about how to build upon what students just learned

What We Learned ...

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Role of Meaning

Teachers thought of "meaning" as frosting on top of skill

We thought of "meaning" as absolutely foundational for skill

What We Learned ...

We and they had vastly different images of reformed teaching

Interconnectedness of teaching, learning, and curriculum

Teachers saw curriculum, teaching, and learning as isolated activities.

We saw curriculum, teaching, and learning as tightly related and interdependent.

One Teacher's Transformation of MKT

Augusta

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- High School Math Teacher
  - 3 years experience
  - Algebra I (Topics 1-2) Class
  - Video Taped everyday for entire school year
  - Debriefing/Planning sessions video/audio taped

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- Support
  - Curriculum co-constructed with PI
  - Planning supported by weekly meetings with PI
  - Debriefing with PI after every class
  - Videographer
  - Computer and projector

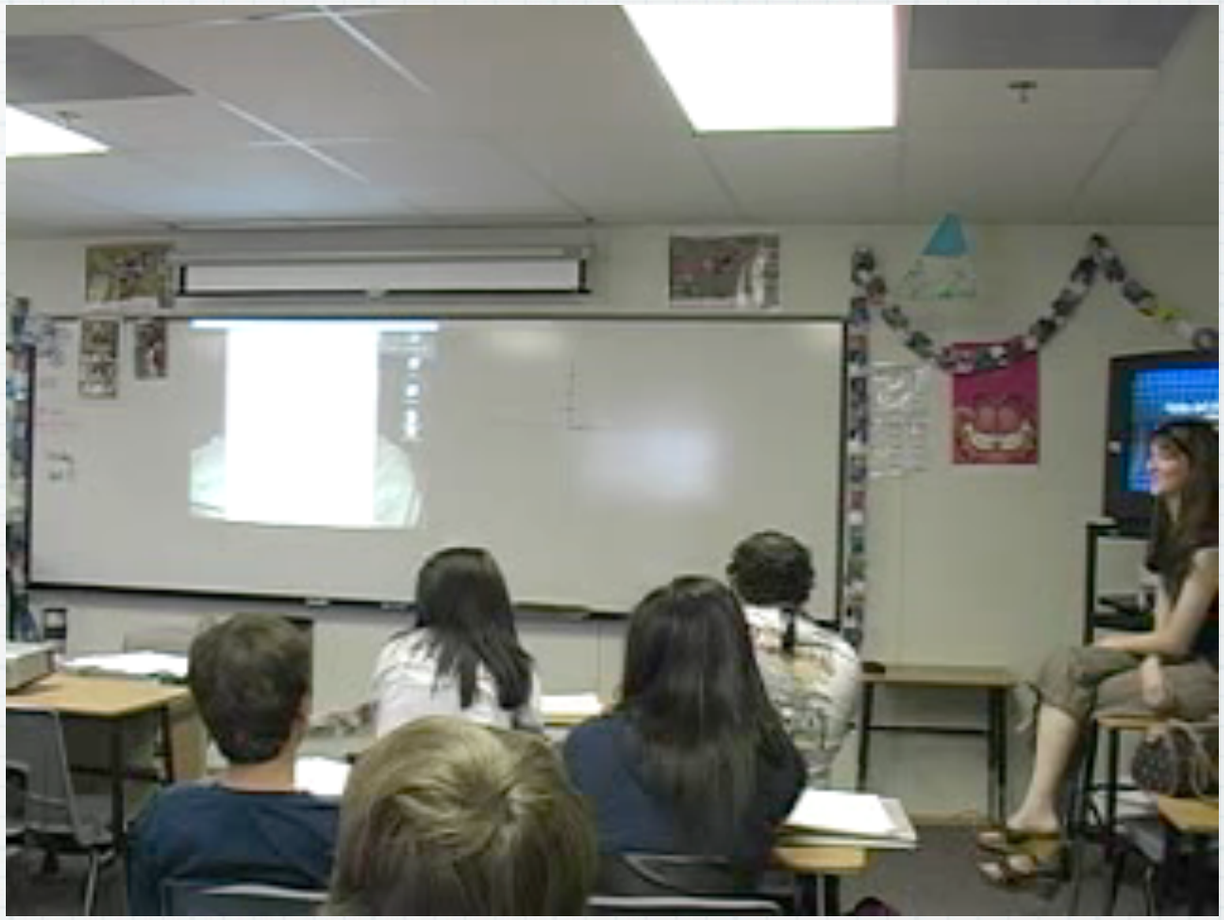
Observed Change

Classroom Discussion In August

Classroom Discussion In April

Observed Change

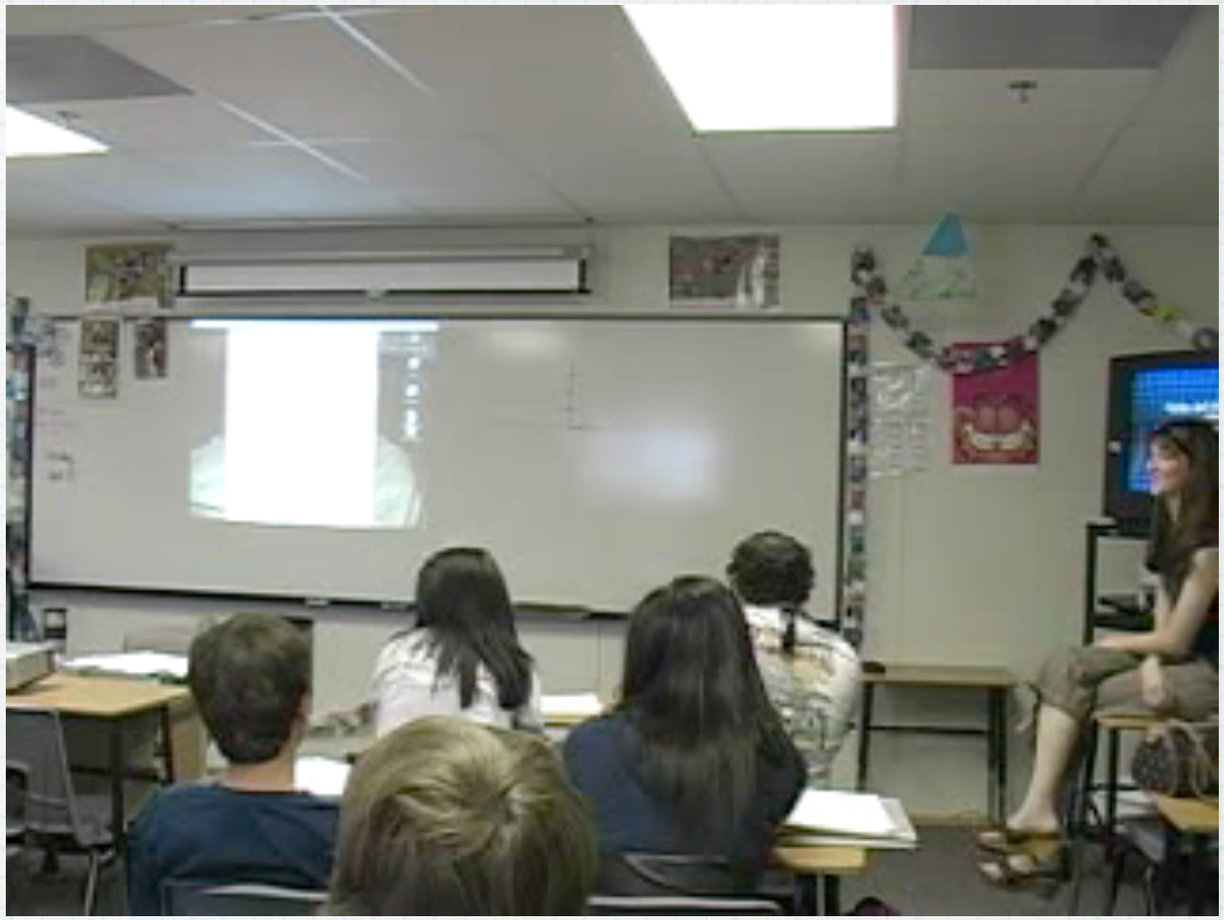
Classroom Discussion
In August



Classroom Discussion
In April

Observed Change

Classroom Discussion
In August



Classroom Discussion
In April

Changes in Augusta's MKT

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* Augusta's understanding of the mathematics

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* Augusta's image of her students' mathematical capability

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* Augusta's understanding of the mathematics
* Augusta's image of her students' mathematical capability
* Augusta's image of the role of teaching in students' learning what she intends

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  - Unclear in her own thinking as to the ways of thinking she wants students to have
    - Indicator
      - Statements about generalizing, inequalities, properties, etc.

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      - What questions are used for assessment

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- Image of Teaching
  - Teaching students to solve problems
    - Indicator
      - Leading through each step
      - Statements about student thinking
      - Mathematical discourse

January

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  - Reflects on her ways of thinking
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      - Is able to adjust to students' reasoning during the lesson

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- Image of Teaching
  - Realizes (with prompting) and reflects on how her teaching impacts what the students learn
    - Indicator
      - Discussions during debriefing followed by adjustments in teaching

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      * Collaborated with colleagues in PLC to create coherent unit on quadratics focused on the idea of ROC

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- Image of Students' Capabilities
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    - Indicator
      - Complex ideas and reasoning in class
      - What questions are used for assessment
- Image of Teaching
  - Facilitator of classroom discourse that promotes students' ways of thinking
    - Indicator
      - More open ended questions focused on imaging mathematical ideas/situations
      - Students actively participating

Augusta's Class Questions

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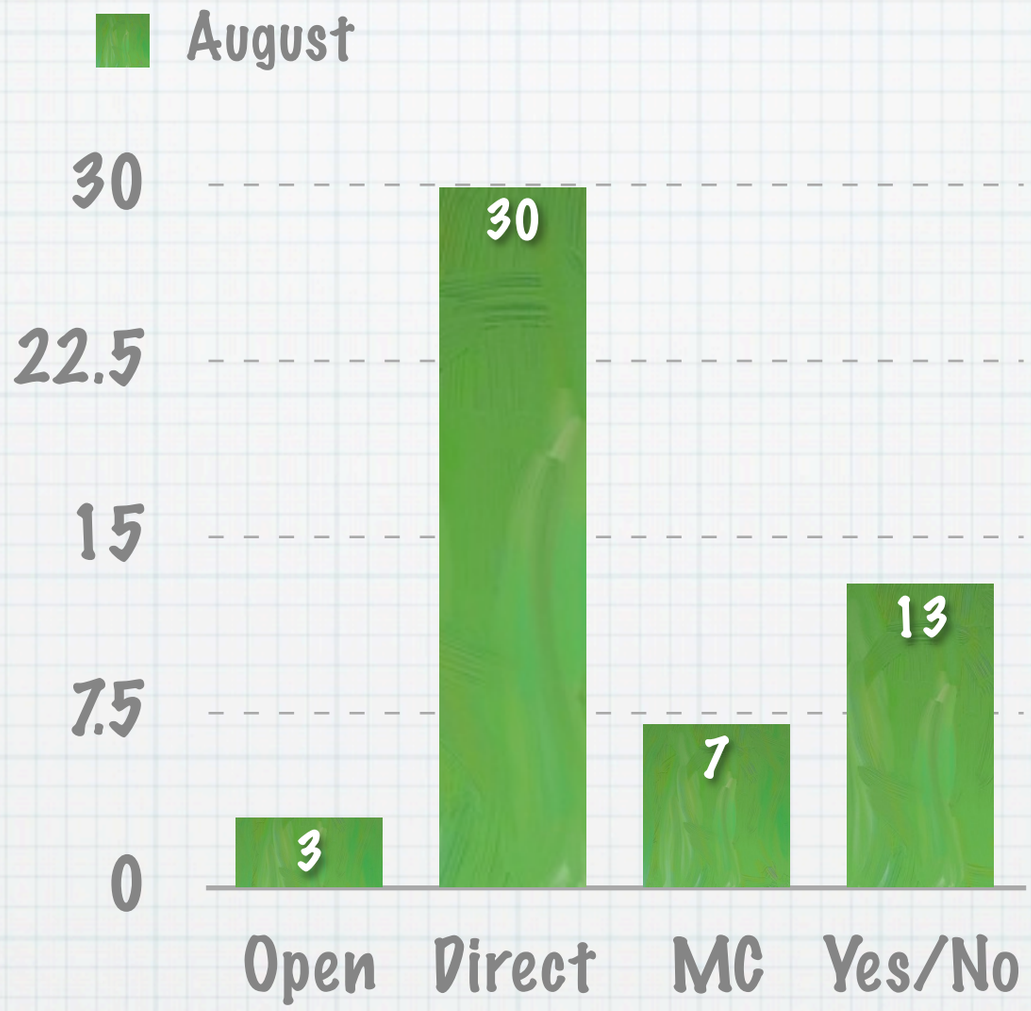

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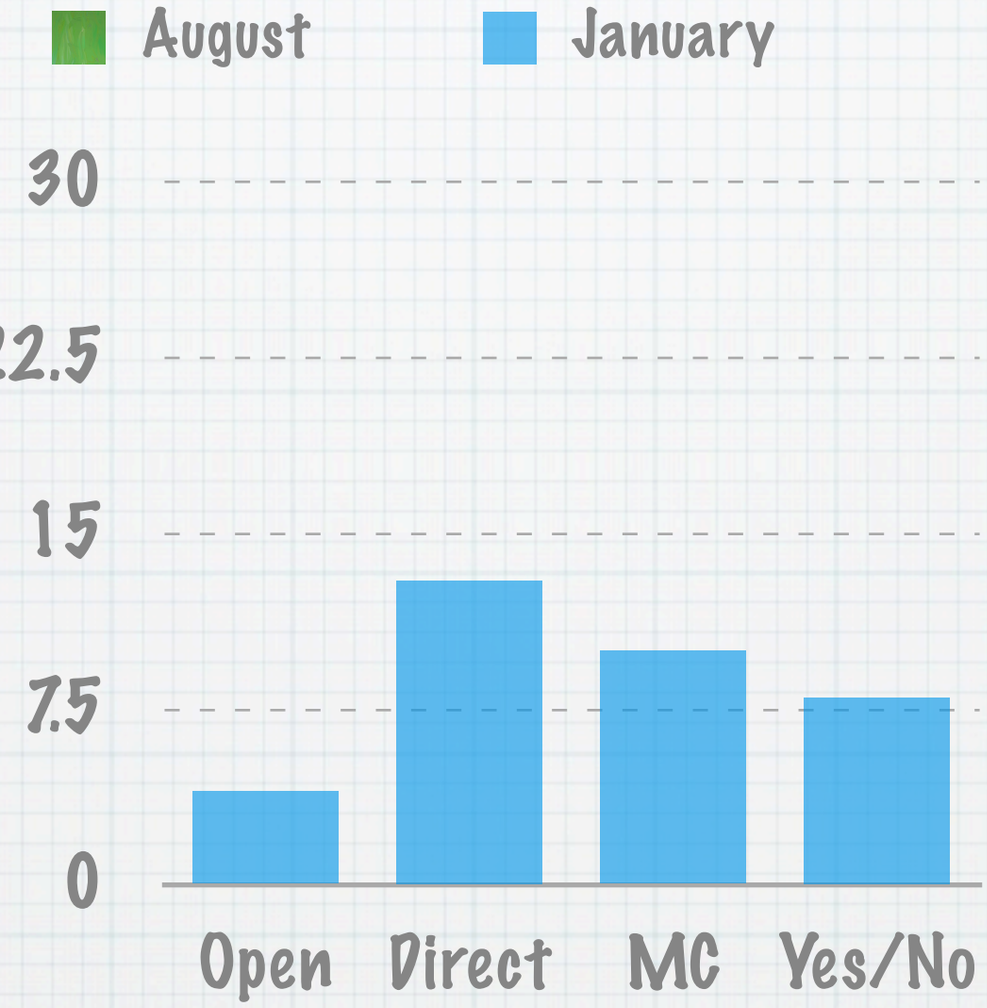

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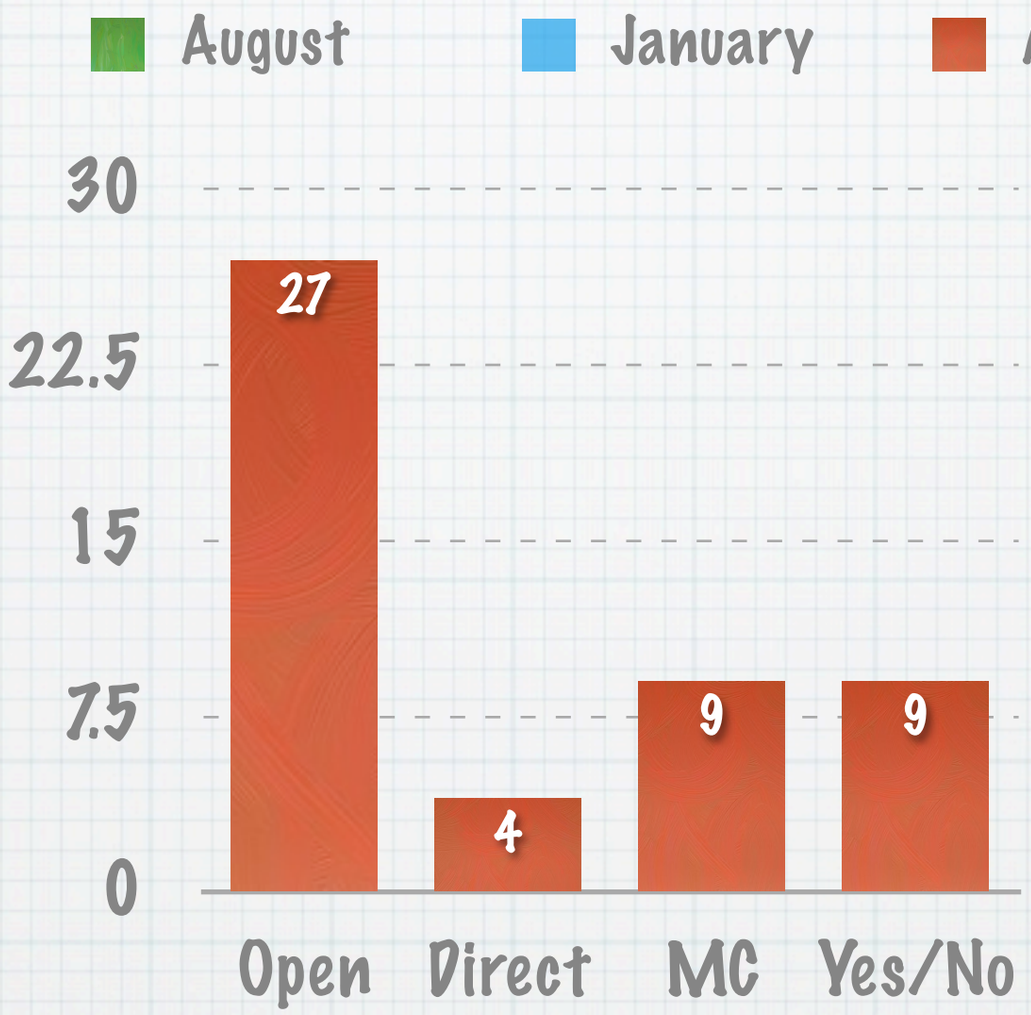

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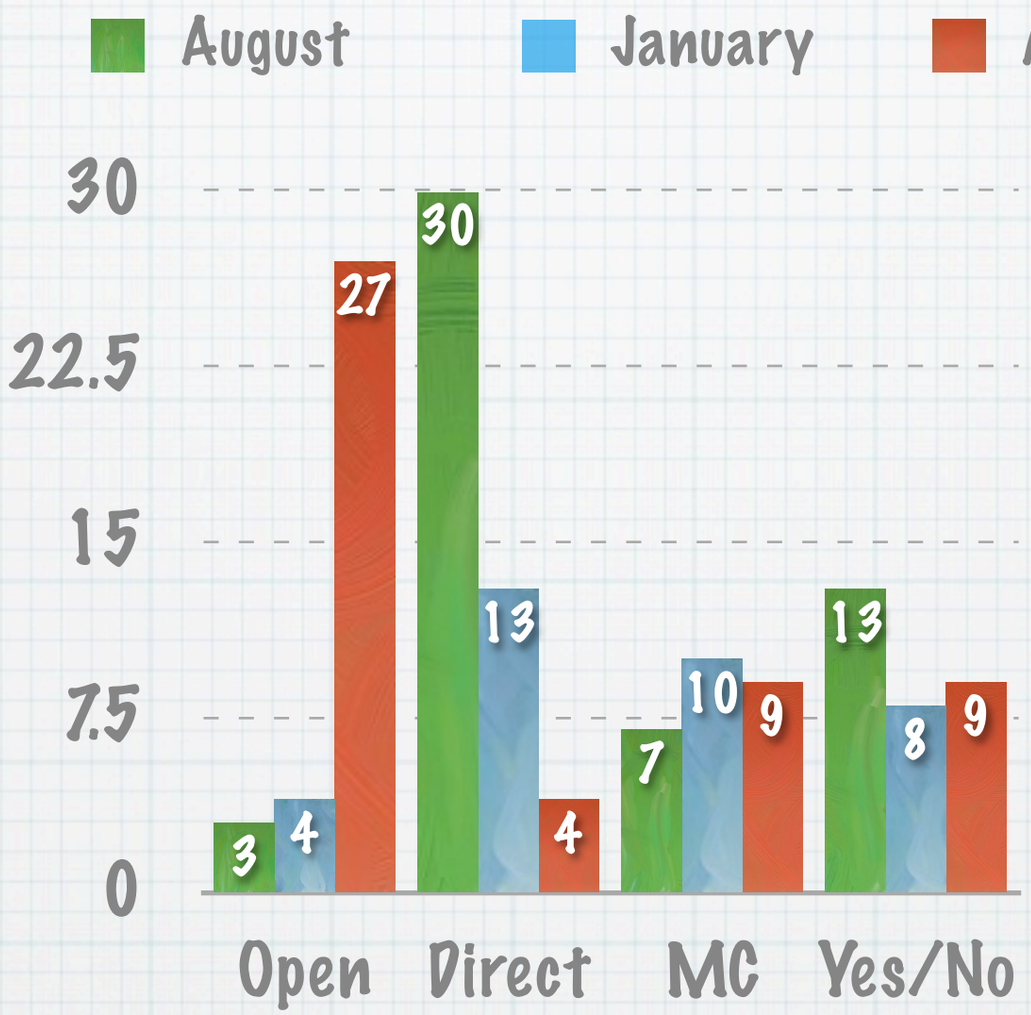

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Drivers of Change

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* Pat's pushing
  * The mathematics
  * Questioning (nature AND purpose)
  * Expectations of students' abilities

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Quadratic Functions Unit

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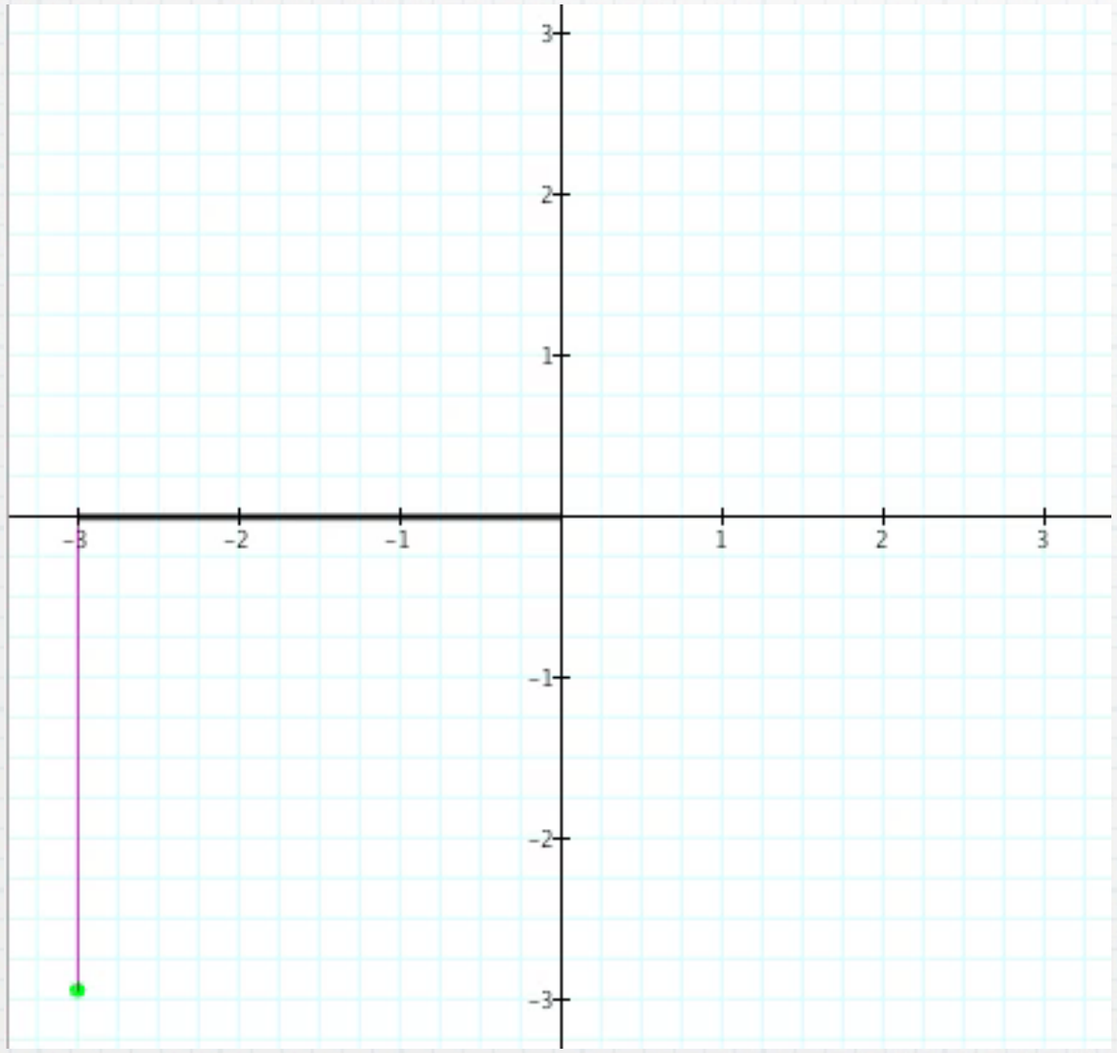

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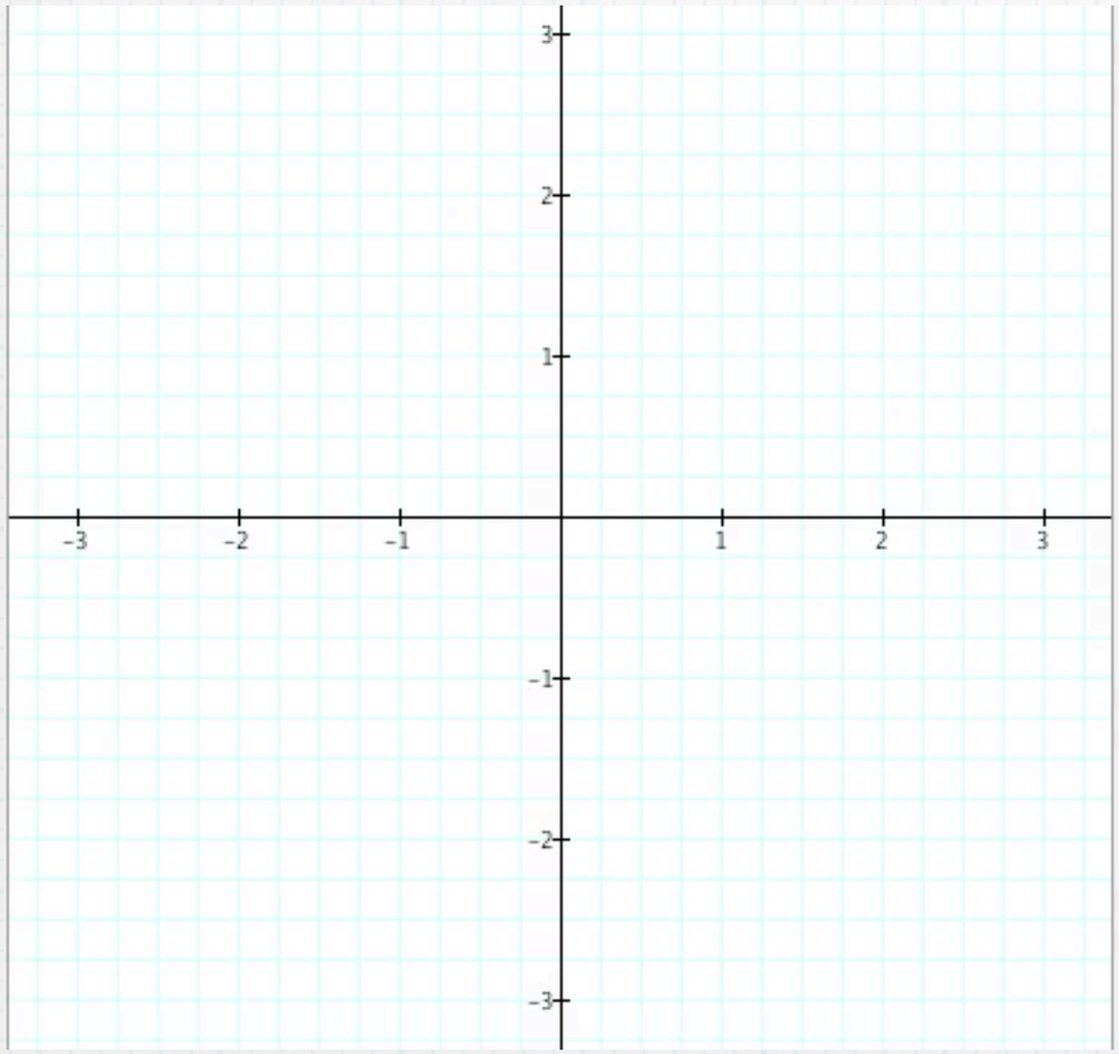

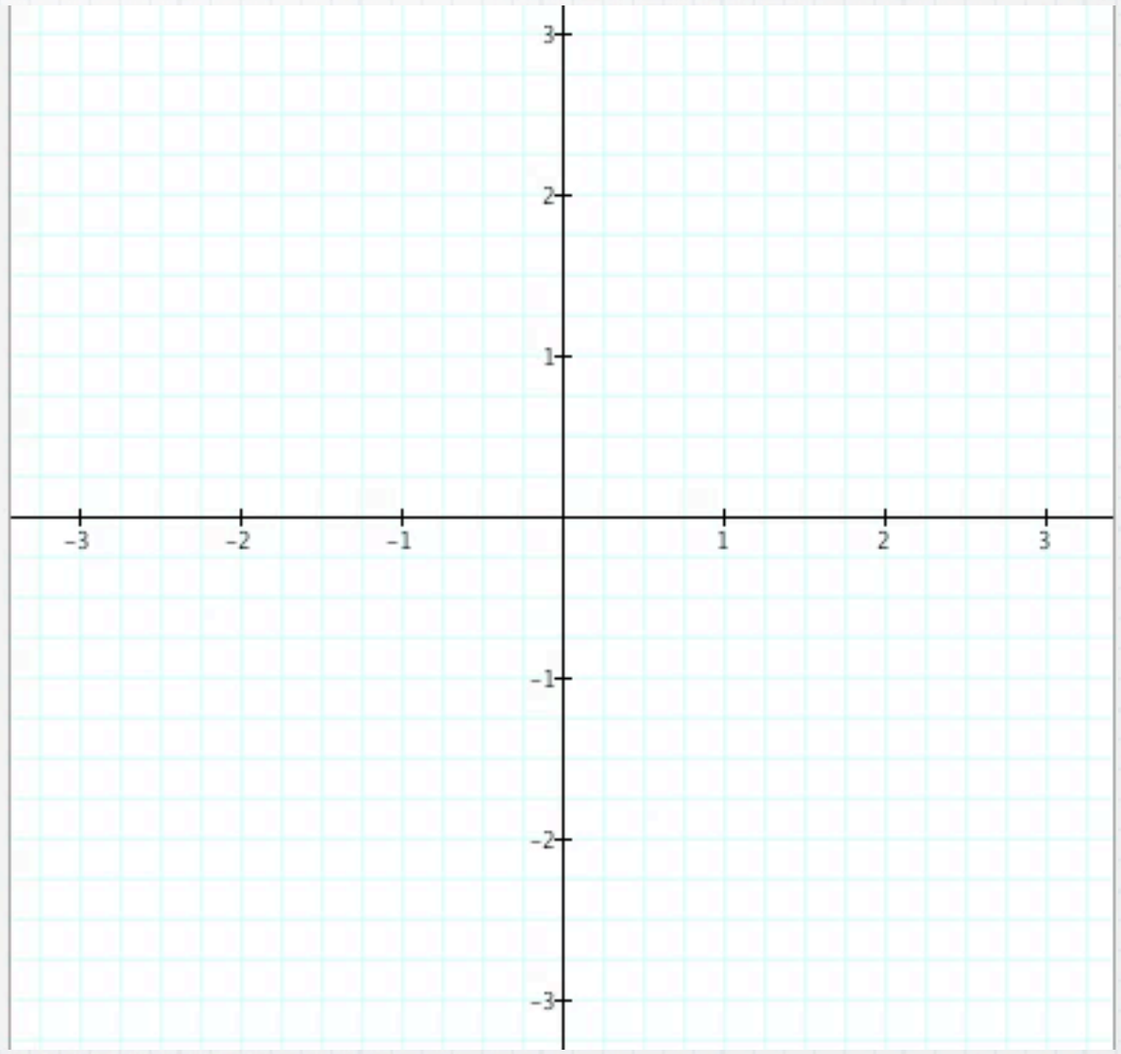

Mathematical Knowledge for Teaching

A New Perspective

Meta-criteria for Definition of MKT

* Needs to apply to all mathematics teaching, not just "good" mathematics teaching
* Needs to apply to secondary mathematics, not just elementary mathematics
* Needs to apply to ALL aspects of teaching, not just in-the-moment teaching (e.g., lesson planning, designing units, assessing learning)
* Needs to account for teachers' decision making
* Must take curriculum as problematic
* Must be clear on what we mean by "knowledge"

Mathematical Knowledge for Teaching ...

* Fractions
* Proportional reasoning
* Rate of change
* Linear function
* Exponential function
* etc.

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No such thing as generic MKT. Rather, teachers develop MKT for specific ideas, then schemes of ideas, etc.

Mathematical Knowledge for Teaching ...

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* Rate of change
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* Exponential function
* etc.

Developed in depth in Silverman & Thompson, 2008

Thank you