Evaluating Significant Math Discourse in a Learning Environment

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Math Cognition as Math Discourse

To mathematicians since Euclid, math represents the paradigm of creative intellectual activity. Its methods set the standard throughout Western civilization for rigorous thought, problem solving and argumentation. Many schools teach math in part to instill in students a sense of deductive reasoning. Yet, too many students—and even some math teachers—end up saying that they "hate math" and that "math is boring" or that they are "not good at math" (Boaler, 2008; Lockhart, 2009). They have somehow missed the intellectual math experience—and this may limit their lifelong interest in science, engineering and technology. According to a recent "cognitive history" of the origin of deduction in Greek mathematics (Netz, 1999), the primordial math experience in 5th and 4th Century BCE was based on the confluence of labeled geometric diagrams (*shared visualizations*) and a language of written mathematics (*asynchronous collaborative discourse*), which supported the rapid evolution of math cognition in a small community of math discourse around the Mediterranean, profoundly extending mathematics and Western thinking.

The vision behind the research described in this paper is to foster *communities of math discourse* in networks of math teachers, in classrooms of K-12 math students and in online communities associated with the Math Forum. We want to leverage the potential of networked computers and dynamic math applications to catalyze groups of people exploring math and experiencing the intellectual excitement that Euclid's colleagues felt—refining and testing emerging 21st Century media of *collaborative math discourse* and *shared math visualization* to support math discourse in both formal and informal settings and groupings.

The learning sciences have transformed our vision of education in the future (Sawyer, 2006; Stahl, Koschmann & Suthers, 2006). New theories of mathematical cognition (Bransford, Brown & Cocking, 1999; Brown & Campione, 1994; Greeno & Goldman, 1998; Hall & Stevens, 1995; Lakatos, 1976; Lemke, 1993; Livingston, 1999) and math education (Boaler, 2008; Cobb, Yackel & McClain, 2000; Lockhart, 2009; Moss & Beatty, 2006), in particular, stress collaborative knowledge building (Bereiter, 2002; Scardamalia & Bereiter, 1996; Schwarz, 1997), problem-based learning (Barrows, 1994; Koschmann, Glenn & Conlee, 1997), dialogicality (Wegerif, 2007), argumentation (Andriessen, Baker & Suthers, 2003), accountable talk (Michaels, O'Connor & Resnick, 2008), group cognition (Stahl, 2006) and engagement in math discourse (Sfard, 2008; Stahl, 2008a). These approaches place the focus on problem solving, problem posing, exploration of alternative strategies, inter-animation of perspectives, verbal articulation, argumentation, deductive reasoning and heuristics as features of *significant math discourse* (Maher, Powell & Uptegrove, 2010; Powell, Francisco & Maher, 2003; Powell & López, 1989).

To learn math is to participate in a mathematical discourse community (Lave & Wenger, 1991; Sfard, 2008; Vygotsky, 1930/1978) that includes people literate in and conversant with topics in mathematics beyond basic arithmetic. Learning to "speak math" is best done by sharing and discussing rich math experiences within a supportive math discourse community (Papert, 1980; van Aalst, 2009). By articulating thinking and learning in text, students make their cognition public and visible. This calls for a reorientation of the teaching profession to facilitate dialogical student practices as well as requiring content and resources to guide and support the student discourses. Teachers and students must learn to adopt, appreciate and take advantage of the visible nature of collaborative learning. The emphasis on text-based collaborative learning can be well supported by computers with appropriate computer-supported collaborative learning (CSCL) software, such as that prototyped in the Virtual Math Teams (VMT) Project (Stahl, 2009).

A Learning Environment for Math Discourse

In order to support our vision of significant mathematical discourse, we have integrated an online environment for synchronous and asynchronous communication (VMT) with a system for exploring dynamic mathematics (GeoGebra). We have described this dual system elsewhere (Stahl, 2012; Stahl et al., 2010). We attempt to support the combination of *collaborative math discourse* and *shared math visualization* by allowing small groups of students to engage in text chat while they are exploring a dynamic math workspace together. We have created a multi-user version of GeoGebra and integrated it with chat (as well as wiki and shared whiteboard) communication with the VMT functionality. This is designed to pool the advantages of dynamic math visualization with collaborative learning and math discourse.

Researching Discourse Practices

Our research centers on measurements of *group* math discourse rather than on assessment of *individual* learning of math content—in accordance with the socio-cultural view that effective individual math learning can be an indirect product of *participation* in group math discourse (Lave & Wenger, 1991; Sfard, 1998; 2008; Stahl, 2006; Vygotsky, 1930/1978). Vygotsky's notion of the *zone of proximal development* suggests that students may be able to engage in mathematical work within groups at a level that they will not be able to engage in for a couple years as individuals—and that such group work can be essential for the individual development in the long run (Vygotsky, 1930/1978, pp. 84-91). As a result, there is a need to assess the educational effectiveness of group interactions as such, beyond pre/post tests of the individuals.

In addition, the striking finding within CSCL research of *productive failure* (Barron, 2003; Kapur & Kinzer, 2009; Patak et al., 2011; Schwartz, 1995) shows that there can be a paradoxical inverse relationship between measures of successful learning by small groups versus by the individual members of those groups because of group processes that reveal deep mathematical relationships but that do not lead immediately to high test scores of the individuals. For these reasons, we evaluate engagement in mathematics *in terms of the quantity and quality of the math discourse* that takes place during the small-group problem-solving interactions, looking for increases for groups as they participate and in successive project years as our teaching model, collaboration technology and curricular resources are iteratively developed.

The analysis of significant math discourse is a task and goal for students using the system, for their teachers assessing their learning as well as for researchers studying collaborative math education. Reflection on interaction logs by teachers and students primarily involves trying to follow the problem-solving path of participants and to notice critical collaboration moves. They will be encouraged to look for examples of accountability to the group, to standards of math reasoning and to the characteristics of their math objects. They will look for instances where someone poses a productive inquiry that initiates effective group exploration—or where the group fails to come up with a useful proposal or fails to take up a proffered proposal. Examples will be culled and shared on the community wiki.

Formative evaluation is a constant process built into the design of our work. As a design-based research effort, our project involves designing and exploring an iteratively refined solution—and by documenting its impact on the quantity and quality of math discourse by teachers and students. The interlocking components of the project will be reviewed at weekly project team meetings. Team meetings include interaction analysis data sessions (Jordan & Henderson, 1995; Stahl, 2010), in which the group collaboratively discusses new data from logs of teachers or students—and makes design decisions for refining the co-evolving components of our research. The project team discusses what seems to be working and what does not. It decides what to modify for the next iteration. Our ultimate goal is to *increase the quality and quantity of both teacher and student mathematical discourse*. Therefore, teacher professional development is oriented to improving the math discourse of their students.

Other research has documented the efficacy of dynamic-math visualization tools for *individual learning*; for instance, a study of geometry students in eleven Florida schools revealed a significant difference in the FCAT mathematics scores of students who were taught geometry using Geometer's Sketchpad compared to those who used the traditional method—regardless of differences based on SES or gender (Myers, 2009). Our project has a different focus. We have developed coding schemes and analysis approaches oriented to the *group unit of analysis* based on conversation analysis of adjacency pairs and longer sequences (Sacks, 1962/1995; Schegloff, 2007; Stahl, 2009, Chs. 20, 22, 23, 26; 2011b; Stahl et al., 2011). This approach serves both quantitative and qualitative analysis, by simultaneously specifying the structure of meaningful discourse moves and providing countable categories of group interaction units, in order to document changes over time—comparing discourse characteristics in selected time slices within teams or across cohorts.

The project will automatically produce raw data in the form of log files of participant online interactions. The log files are anonymous, but allow tracking of individual users through consistent login handles. The VMT environment is instrumented to capture all user actions in the chat and whiteboard—this has been extended to multi-user GeoGebra. A database of all sessions is automatically maintained and provides spreadsheet logs in handy formats and Replayer files. Software tools will be used for automated and manual log analysis of discourse measures and their evolution during training. While low-level group processes (e.g., number, length and rate of chat postings and drawing actions in different time slices) can be tracked automatically and analyzed statistically, higher-level math-discourse processes have to be interpreted manually. Raw and coded logs are maintained in a database to facilitate analysis of changes over time for groups across sessions and across successive cohorts of participants.

Quantitative analysis—based largely on the coding of discourse moves in teacher and student VMT logs—will track changes in key measures of significant math discourse. Discourse will be coded and measured along the following dimensions: (1) volume of discourse and level of participation, (2) percentage of on-task math discourse, (3) use of representations, (4) integration of chat and drawing, (5) use of accountable talk moves, (6) adoption of socio-mathematical norms and practices, (7) speaking meaningfully with explanation and argumentation, (8) involvement in posing, exploring and solving problems and (9) additional dimensions to be developed based on project experience.

The theory of math learning through participation in math discourse (Sfard, 2008; Stahl, 2008b) specifies important mathematical discourse moves, such as encapsulation, reification, saming, routines, deeds, explorations and rituals. The theory of accountable talk (Michaels, O'Connor & Resnick, 2008; Resnick, 1988) specifies discourse moves that promote accountability to the group, to standards of math reasoning and to the characteristics of the math objects. Speaking meaningfully in math discourse "implies that responses are conceptually based, conclusions are supported by a mathematical argument and explanations include reference to the quantities in the problem context [as opposed to a focus on merely] describing the procedures and calculations used to determine the answer" (Clark, Moore & Carlson, 2008, p.298).

Socio-mathematical norms include what counts as an acceptable, a justifiable, an easy, a clear, a different, an efficient, an elegant and a sophisticated explanation (Yackel, 1995; Yackel & Cobb, 1996). Mathematical practices emerge from interaction, are taken up by participants and are applied repeatedly (Medina, Suthers & Vatrapu, 2009; Stahl, 2011a). These dimensions of significant math discourse are associated with typical sentences and discourse moves that can be identified by coders. A coding scheme will be validated with acceptable inter-rater reliability, as in (Stahl, 2009, Chs. 22, 23; 2011b).

Detailed interaction analysis of selected cases will show *how* the math discourse actually evolves. Quantitative analysis can establish the statistical significance of changes in learning outcomes, but it generally does not provide much insight into the mechanisms of change; these mechanisms will become visible in detailed case studies in which the specifics of the interactions can be studied. By combining quantitative and qualitative analysis of discourse transformations, the project evaluation will determine how the online interaction involves engagement in significant mathematical discourse.

References

Andriessen, J., Baker, M., & Suthers, D. (Eds.). (2003). *Arguing to learn: Confronting cognitions in computer-supported collaborative learning environments*. Dordrecht, Netherlands: Kluwer Academic Publishers. Computer-supported collaborative learning book series, vol 1

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences. 12*(3), 307-359.

Barrows, H. (1994). *Practice-based learning: Problem-based learning applied to medical education*. Springfield, IL: SIU School of Medicine.

Bereiter, C. (2002). *Education and mind in the knowledge age*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Boaler, J. (2008). *What's math got to do with it? Helping children learn to love their most hated subject: And why it is important for America.* New York, NY: Viking.

Bransford, J., Brown, A., & Cocking, R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Research Council. Web: http://books.nap.edu/html/howpeople1/.

Brown, A., & Campione, J. (1994). Guided discovery in a community of learners. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice.* (pp. 229-270). Cambridge, MA: MIT Press.

Clark, P. G., Moore, K. C., & Carlson, M. P. (2008). Documenting the emergence of "speaking with meaning" as a sociomathematical norm in professional learning community discourse. *Journal of Mathematical Behavior. 27*, 297–310.

Cobb, P., Yackel, E., & McClain, K. (2000). *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design*. Mahwah, NJ: Lawrence Erlbaum Associates.

Greeno, J. G., & Goldman, S. V. (1998). *Thinking practices in mathematics and science leraning*. Mahwah, NJ: Lawrence Erlbaum Associates.

Hall, R., & Stevens, R. (1995). Making space: A comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The cultures of computing.* Oxford, UK: Blackwell Publishers.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences. 4*(1), 39-103. Web: http://lrs.ed.uiuc.edu/students/c-merkel/document4.HTM.

Kapur, M., & Kinzer, C. (2009). Productive failure in CSCL groups. *International Journal of Computer-Supported Collaborative Learning. 4*(1), 21-46. Doi: http://dx.doi.org/10.1007/s11412-008-9059-z.

Koschmann, T., Glenn, P., & Conlee, M. (1997). Analyzing the emergence of a learning issue in a problem-based learning meeting. *Medical Education Online. 2*(1). Web: http://www.utmb.edu/meo/res00003.pdf.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge, UK: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Lemke, J. L. (1993). *Talking science: Language, learning and values*. Norwood, NJ: Ablex.

Livingston, E. (1999). Cultures of proving. *Social Studies of Science. 29*(6), 867-888.

Lockhart, P. (2009). *A mathematician's lament: How school cheats us out of our most fascinating and imaginative art forms*. New York, NY: Belevue Literary Press.

Maher, C. A., Powell, A. B., & Uptegrove, E. B. (Eds.). (2010). *Combinatorics and reasoning: Representing, justifying and building isomorphisms*. New York, NY: Springer.

Medina, R., Suthers, D. D., & Vatrapu, R. (2009). Representational practices in VMT. In G. Stahl (Ed.), *Studying virtual math teams.* (ch. 10, pp. 185-205). New York, NY: Springer. Web: http://GerryStahl.net/vmt/book/10.pdf. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3_10.

Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in the Philosophy of Education. 27*(4), 283-297.

Moss, J., & Beatty, R. (2006). Knowledge building in mathematics: Supporting collaborative learning in pattern problems. *International Journal of Computer-Supported Collaborative Learning. 1*(4), 441-465. Doi: http://dx.doi.org/10.1007/s11412-006-9003-z.

Myers, R. Y. (2009). *The effects of the use of technology in mathematics instruction on student acheivement.* Unpublished Dissertation, Ph.D., Curriculum and Instruction, Florida International University. Miami, FL. Web: http://digitalcommons.fiu.edu/etd/136.

Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas*. New York, NY: Basic Books.

Patak, S., Kim, B., Jacobson, M. J., & Zhang, B. (2011). Pedagogical trajectories of structure and their impact on learning electricity with agent-based models. *International Journal of Computer-Supported Collaborative Learning. 6*(1)

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior. 22*(4), 405-435.

Powell, A. B., & López, J. A. (1989). Writing as a vehicle to learn mathematics: A case study. In P. Connolly & T. Vilardi (Eds.), *The role of writing in learning mathematics and science.* (pp. 157-177). New York, NY: Teachers College.

Resnick, L. B. (1988). Treating mathematics as an ill-structured discipline. In R. Charles & E. Silver (Eds.), *The teaching and assessing of mathematical problem solving.* (pp. 32-60). Hillsdale, NJ: Lawrence Erlbaum Associates.

Sacks, H. (1962/1995). *Lectures on conversation*. Oxford, UK: Blackwell.

Sawyer, R. K. (Ed.). (2006). *Cambridge handbook of the learning sciences*. Cambridge, UK: Cambridge University Press.

Scardamalia, M., & Bereiter, C. (1996). Computer support for knowledge-building communities. In T. Koschmann (Ed.), *CSCL: Theory and practice of an emerging paradigm.* (pp. 249-268). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schegloff, E. A. (2007). *Sequence organization in interaction: A primer in conversation analysis*. Cambridge, UK: Cambridge University Press.

Schwartz, D. (1995). The emergence of abstract representations in dyad problem solving. *Journal of the Learning Sciences. 4*(3), 321-354.

Schwarz, B. B. (1997). Understanding symbols with intermediate abstractions: An analysis of the collaborative construction of mathematical meaning. In L. B. Resnick, R. Saljo, C. Pontecorvo & B. Burge (Eds.), *Discourse, tools, and reasoning: Essays on situated cognition.* (pp. 312-335). Berlin, Germany: Springer.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher. 27*(2), 4-13.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.

Stahl, G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge, MA: MIT Press. 510 + viii pages. Web: http://GerryStahl.net/mit/.

Stahl, G. (2008a). Book review: Exploring thinking as communicating in CSCL. *International Journal of Computer-Supported Collaborative Learning. 3*(3), 361-368. Web: http://GerryStahl.net/pub/Sfardreview.pdf. Doi: http://dx.doi.org/10.1007/s11412-008-9046-4.

Stahl, G. (2008b). Thinking as communicating: Human development, the growth of discourses and mathematizing. *International Journal of Computer-Supported Collaborative Learning. 3*(3), 361-368. Web: http://ijcscl.org/?go=contents. Doi: 10.1007/s11412-008-9046-4.

Stahl, G. (2009). *Studying virtual math teams*. New York, NY: Springer. 626 +xxi pages. Web: http://GerryStahl.net/vmt/book. Doi: http://dx.doi.org/10.1007/978-1-4419-0228-3.

Stahl, G. (2010). Group cognition as a foundation for the new science of learning. In M. S. Khine & I. M. Saleh (Eds.), *New science of learning: Cognition, computers and collaboration in education.* (pp. 23-44). New York, NY: Springer. Web: http://GerryStahl.net/pub/scienceoflearning.pdf.

Stahl, G. (2011a). Social practices of group cognition in virtual math teams. In S. Ludvigsen, A. Lund, I. Rasmussen & R. Säljö (Eds.), *Learning across sites: New tools, infrastructures and practices.* (pp. 190-205). New York, NY: Routledge. Web: http://GerryStahl.net/pub/cmc.pdf.

Stahl, G. (2011b). *The structure of collaborative problem solving in a virtual math team.* Paper presented at the iConference 2011. Seattle, WA. Web: http://GerryStahl.net/pub/iconf2011.pdf.

Stahl, G. (2012). *Designing a learning environment for promoting math discourse.* Paper presented at the the 12th International Conference on Mathematical Education. Web: http://GerryStahl.net/pub/icme_design.pdf.

Stahl, G., Koschmann, T., & Suthers, D. (2006). Computer-supported collaborative learning: An historical perspective. In R. K. Sawyer (Ed.), *Cambridge handbook of the learning sciences.* (pp. 409-426). Cambridge, UK: Cambridge University Press. Web: http://GerryStahl.net/elibrary/global.

Stahl, G., Ou, J. X., Weusijana, B. K., Çakir, M. P., & Weimar, S. (2010). Multi-user GeoGebra for virtual math teams. *GeoGebra: The New Language For The Third Millennium. 1*(1), 117-126. Web: http://GerryStahl.net/pub/geogebra_romania.pdf.

Stahl, G., Zhou, N., Cakir, M. P., & Sarmiento-Klapper, J. W. (2011). *Seeing what we mean: Co-experiencing a shared virtual world.* Paper presented at the international conference on Computer Support for Collaborative Learning (CSCL 2011). Hong Kong, China. Proceedings pp. 534-541. Web: http://GerryStahl.net/pub/cscl2011.pdf, http://GerryStahl.net/pub/cscl2011.ppt.pdf, http://youtu.be/HC6eLNNIvCk.

van Aalst, J. (2009). Distinguishing knowledge-sharing, knowledge-construction, and knowledge-creation discourses. *International Journal of Computer-Supported Collaborative Learning. 4*(3), 259-287. Doi: http://dx.doi.org/10.1007/s11412-009-9069-5.

Vygotsky, L. (1930/1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Wegerif, R. (2007). *Dialogic, education and technology: Expanding the space of learning*. New York, NY: Kluwer-Springer.

Yackel, E. (1995). Children's talk in inquiry mathematics classrooms. In P. C. H. Bauersfeld (Ed.), *The emergence of mathematical meaning: Interaction in classroom cultures.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education. 27*(4), 458-477.