Designing a Learning Environment to Promote Math Discourse

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Mathematics education in the future faces enormous opportunities from the availability of ubiquitous digital networks, from innovative educational approaches based on theories of collaborative learning and from rich resources for interactive, online, dynamic math exploration.

The fact that more and more teachers and students are learning online—with distance education, online masters programs, home schooling, online high schools, etc.—makes the incorporation of virtual collaborative learning environments a natural trend.

This paper reports on the design of a virtual learning environment that integrates synchronous and asynchronous media with an innovative multi-user version of a dynamic math visualization and exploration toolbox. This VMT-GeoGebra environment is designed to support the production of significant math discourse.

An Online Math Collaborative-Learning Environment

The VMT-GeoGebra learning environment integrates two forms of technology to support math learning with collaborative and interactive tools:

- Computer-supported collaborative learning (CSCL) software and
- Dynamic mathematics (software that allows users to manipulate geometric diagrams, equations, etc.).

(a) CSCL provides virtual-learning environments in which teams of students can interact synchronously and asynchronously to build knowledge together. This student-centered approach has many advantages, including increased motivation, sharing of skills, engaging in significant discourse and practicing teamwork. The system reported here extends the Virtual Math Teams (VMT) environment, which has already been prototyped and tested (Stahl, 2009b).

(b) Dynamic math (such as Geometer's Sketchpad, Mathematica, Cabri or GeoGebra) has profoundly impacted math education (Goldenberg, 1995; Hoyles & Noss, 1994; King & Schattschneider, 1997; Laborde, 1998; Myers, 2009; Scher, 2002), with Geometer's Sketchpad and GeoGebra used in many US classrooms and globally. Yet, research on math education has not analyzed how students use dynamic math tools in sufficient detail (compare Cakir, Zemel & Stahl, 2009; Stahl, 2009b). GeoGebra (http://www.geogebra.org) is an open-source system for dynamic geometry, algebra and beginning calculus—including trigonometry, conics, matrices, graphing and Euclidean constructions. It offers multiple representations of objects in its graphics, algebra and spreadsheet views which are all dynamically linked, making GeoGebra a particularly flexible tool for exploration. The VMT-GeoGebra system provides the first multi-user version of dynamic math, so that student teams can explore math collaboratively; it integrates the GeoGebra dynamic math tools into the larger VMT virtual collaborative-learning environment with text chat and wiki to support persistent discourses about math—that can be shared, reflected on and researched.[1]

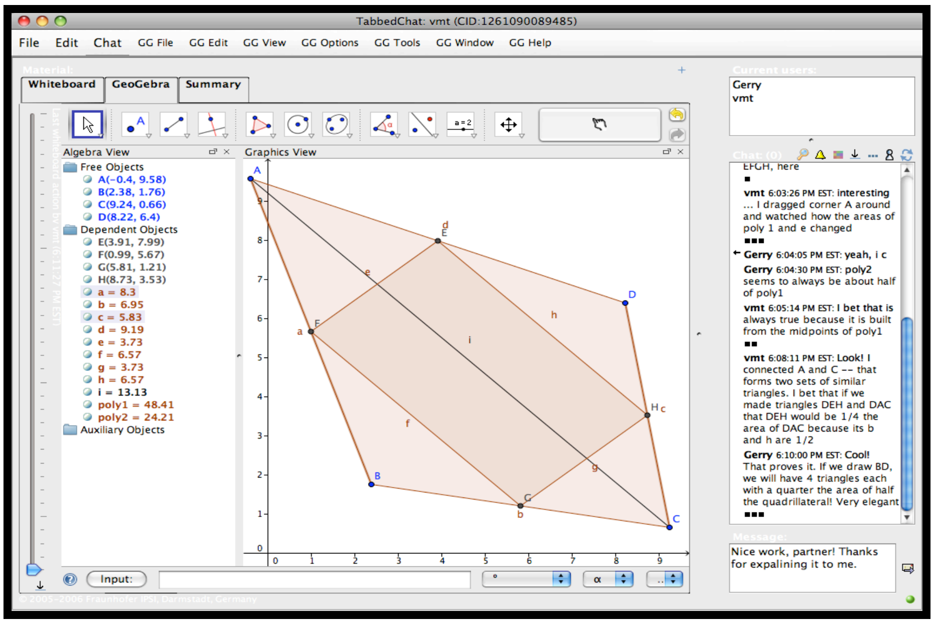


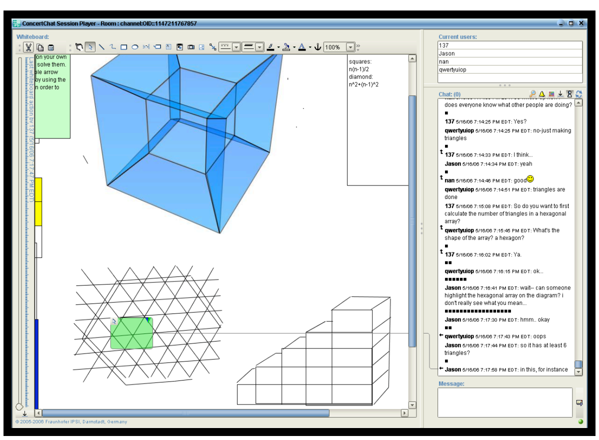
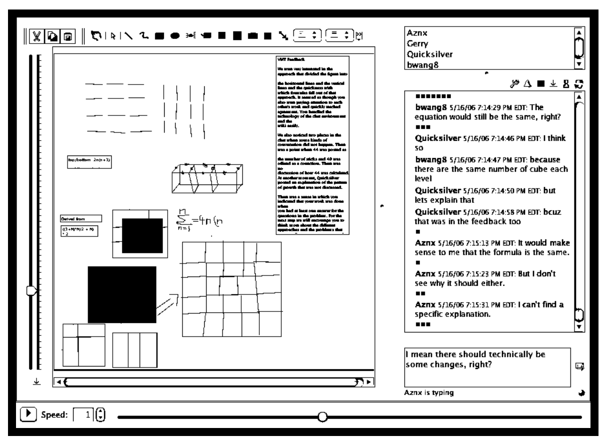
Figure 1. A demo (not real student interaction data) GeoGebra construction created and discussed collaboratively in a multi-user prototype of the learning environment, based on the VMT system. The VMT system includes (not shown here): a Lobby with social networking and tools for teachers, integration with a wiki, and Web browsers.

The VMT-GeoGebra system grew out of the successful Virtual Math Teams (VMT) Project. The VMT Project developed an open-source virtual learning environment for math students between 2003 and 2010. The system integrated a social-networking portal, synchronous text chat, a shared whiteboard, an asynchronous wiki, a referencing tool, mathML expressions and a web browser. Student actions and chat postings are automatically logged, they can be replayed for reflection, assessment and analysis by students, teachers and researchers. Over a thousand student-hours of piloted usage were logged. A qualitative micro-analytic approach to interaction analysis was developed based on ethnomethodologically inspired conversation analysis (Garfinkel, 1967; Sacks, 1962/1995; Stahl, 2009a; 2009c; Zemel, Çakir & Stahl, 2009). A large number of publications have appeared from the project (see

[1] For a demo of the prototype system, go to http://vmt.mathforum.org/VMTLobby. Log in as "guest" with password "guest". The Lobby should open showing the List of All Rooms. Select Project "VMT Research". Click on "Apply filters". Open "Geometry". Open "Polygons". Click on "GeoGebra Demo Room" Eventually a JavaWebStart chat room should open. Explore its different tabs and functions.

http://GerryStahl.net/vmt/pubs.html), including 2 books (Stahl, 2006; 2009b) and 8 doctoral dissertations (Çakir, 2009; Litz, 2007; Merges, 2010; Mühlpfordt, 2008; O'Hara, 2010; Sarmiento-Klapper, 2009; Wee, 2009; Zhou, 2010).

The VMT Project pioneered the study of online collaborative math discourse—both its nature and modes of computer support for it. The 28 studies in (Stahl, 2009b) present some of the most important of the 169 publications related to the project. They include a number of case studies of interactions in the VMT environment by middle-school, high-school and junior-college students, which analyze: how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face; how the structure of text chat interaction differs from spoken conversation; how the media of graphical diagrams, textual narratives and symbolic representations can be intimately interwoven to build deep math understanding; how deictic referencing is important to establishing shared understanding; how students co-construct a joint problem space; how collaborative meaning making and knowledge building are accomplished in detail; how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and how a methodology based on interaction analysis can be used for a science of group cognition. (See Figures 2 and 3.)

Figures 2 and 3. Images of actual student online collaborative work on patterns. In Figure 2, a student points from a chat message to a smallest hexagon pattern composed of 6 triangles illustrating VMT's unique integration of chat and whiteboard with its deictic reference tool. Figure 3 shows the Replayer tool interface across the bottom.

The VMT Project was structured as design-based research, with the technology, research and theory co-evolving through dozens of iterations. The VMT Project demonstrated both the practicality of the VMT-GeoGebra system and the need for it. While the VMT Project prototyped a rich cyber-learning environment and studied student interaction, it did not develop the range of supports that we know are needed for classroom use: robust software, problem sets, guidelines, etc. Furthermore, it did not include a dynamic-math component. The VMT-GeoGebra system extends the environment to cover these needs.

The VMT Project was widely recognized as an important example of synchronous support for online collaboration and was studied by several international researchers. The VMT Replayer allows complete replay of a user session, including all actions and system notices, as though the session was digitally video-recorded. The researcher's view is guaranteed to be identical to the user's view since it is generated from the same data as sent to a client computer. The log information is also made available in convenient textual or spreadsheet formats for student reflection and reporting as well as for researcher analysis.

In the VMT-GeoGebra system, GeoGebra version 4 has been ported into the VMT system, making the dynamic math tools fully multi-user. GeoGebra is integrated as a tab in VMT (see Figure 1 above). GeoGebra is a particularly appropriate dynamic-math application for this project because its source code is freely available as open source, there is an active international development community to support on-going development, the application supports a wide range of math from algebra and geometry construction to calculus and 3-D, GeoGebra has won international prizes, and it has been translated into about 50 languages. Like all other dynamic-math applications, GeoGebra has until now only existed as a single-user application. While users can send their static constructions to each other, display screen images, or awkwardly include a view of the GeoGebra application within other environments through screen sharing (e.g., in Blackboard, Moodle, Elluminate, etc.), only one person can dynamically manipulate the construction. The port into VMT converted GeoGebra to a client-server architecture, allowing multiple distributed users to manipulate constructions and to all observe everyone's actions in real time. Every action in the GeoGebra tab is immediately broadcast by the server to all collaborating clients (and logged in detail for replay and research).

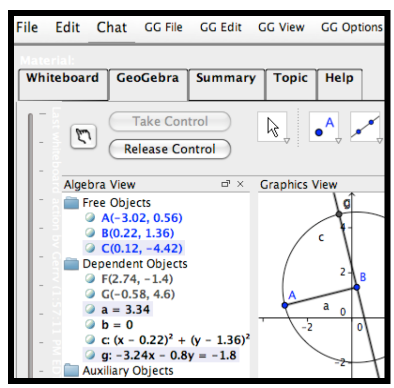


Figure 4. The GeoGebra tab with turn-taking button to avoid conflicts.

We have been exploring turn-taking mechanisms (see Figure 4) to avoid conflicts in the construction and modification of GeoGebra drawings; although it is important in synchronous chat to allow multiple users to type simultaneously, we have found that it is natural for a group to allow one member at a time to change a graphical construction and for group members to take turns editing and rearranging.

The VMT-GeoGebra system is not a walk-up-and-use simple app. It requires orientation of students to its purposes and introduction to its functionality. The system therefore includes sets of Activities, which step students through interactions with each other, with the technology and with the mathematics. Each Activity stresses the use of the chat medium to support coordination and collaboration as well as to reflect on the mathematical actions engaged in and to investigate the relationships among the dynamic math objects. These Activities are correlated with math content presented in the U.S. *Common Core State Standards for Mathematics* and in selected math textbooks.

Math teachers are trained in the use of the VMT-GeoGebra environment by having them work in it on the Activities in small groups of teachers, and reflect on their experiences and on how they might use the Activities in their classrooms.

Conclusion

Incorporation of GeoGebra in the VMT environment framework allows users to engage in text chat while manipulating the construction. Importantly, users can graphically point from a chat posting to an area of the construction that they want to index (see Figure 2)—a handy support for math discourse that is unique to VMT. They can also scroll back and forth through the history of the GeoGebra construction, animating its evolution—a powerful way to explore many mathematical relationships. In addition, a complete record of the collaborative construction is available to the participants, their teachers and project researchers, allowing them all to analyze and reflect upon the complete interaction, including the construction actions synchronized with the chat. GeoGebra in VMT provides an exciting collaborative experience and a rich dataset for research on collaborative learning of mathematics.

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