Proportions Playground Tasks

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Unless otherwise noted, these have all been used in PD with practicing middle school teachers. Our three areas of focus for the PD were: quantity, covariation, and constant. All of the tasks were written with these three ideas in mind. While we didn't set out to explicitly focus on precision of language, that is also a key idea underlying many of these tasks.

Buildings Toy

Task 1

What other metaphors might this be? What kinds of situations could this be used to explore?

Goals: help teachers get used to the tool and thinking about contexts that could be used to write tasks for the Buildings Toy

Task 2

Can we create two buildings that are proportional to each other? What can be proportional? Are there ways we can use the toy to test whether there are proportional relationships?

Goals: thinking about what we mean by proportional – it has to be a comparison of measurable quantities, so "buildings" cannot be proportional, but heights and widths can be. This is an introduction to what a comparison of quantities means. Also, thinking about the relationship between similarity and proportion can be brought out through discussion here, if desired.

Task 3

A planned community has a building code that requires that new apartment buildings need to be 4 apartments tall for every 2 apartments they are wide.

- Try to build some buildings in the Toy that follow the requirements. Describe what they are like.
- Could a building be built following these rules that is an odd number of units wide? How do you know?
- Could a building be built following these rules that is an odd number of units tall? How do you know?

Goals: this can be about unit rate and its relationship to equivalent expressions of that unit rate (e.g., can 2 tall and 1 wide be accepted?) This is also about the difference between mathematical possibilities and real-world possibilities. While fractional amounts are entirely possible, they do not work with the context.

Task 4

A student in Mr. Martin's class said, "My building's easy! I just add 3 columns and 2 rows each time.' Could this student be describing a proportional situation?

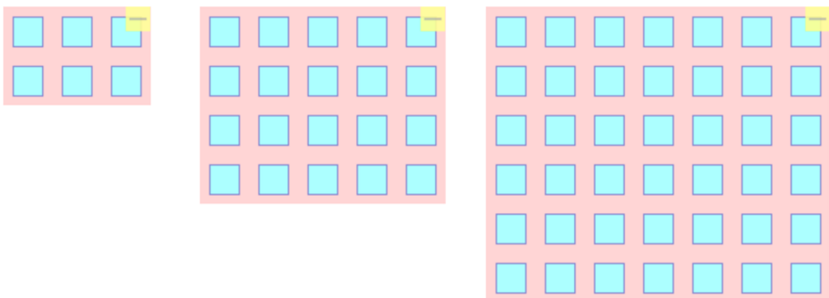
Goals: thinking about the ambiguity of students' answers and what it means for something to be proportional.

<u>Task 5</u>

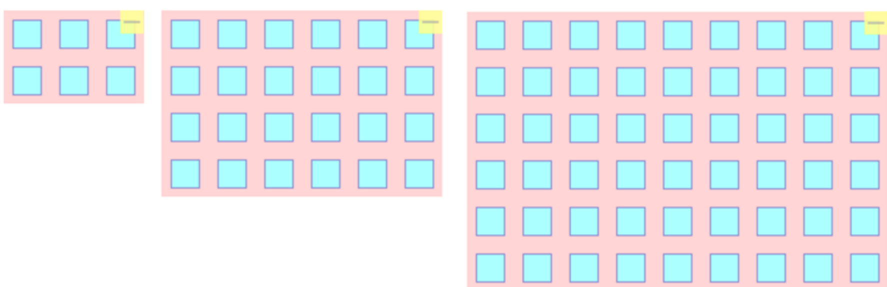
Ms. Lindsey asked her students to use the Buildings Toy to show a 3:2 relationship.

- Which of the sets of images shows a 3:2 relationship?
- What relationship does a set of images show if it's not showing a 3:2 relationship?
- Do any of these solutions show a proportional relationship between the buildings?

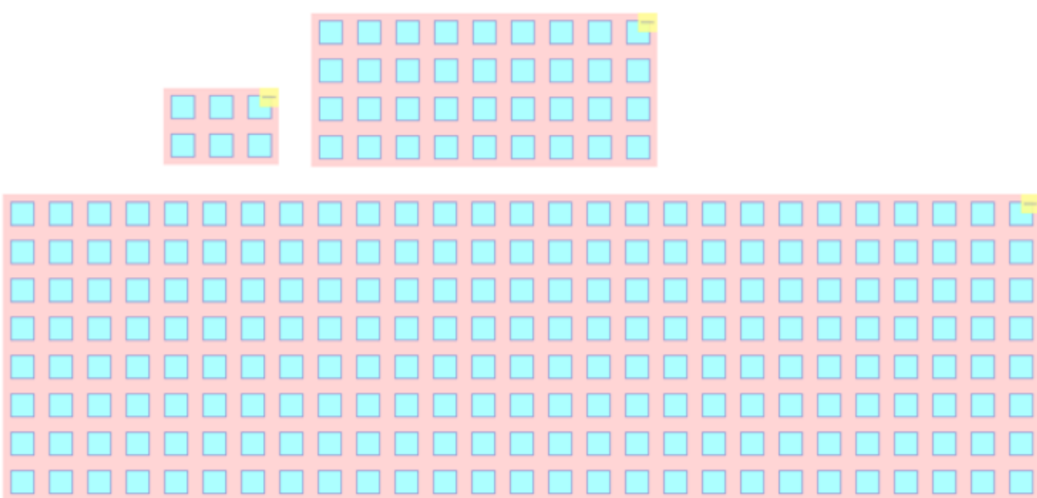
Solution 1



Solution 2



Solution 3



Goals: Thinking about what has to covary to maintain a 3:2 relationship and what has to stay constant.

Bars Toy

Task 1

Mystery: Which of the situations being shown by the Bars is proportional (if any)?

Goals: What does "constant" mean? What kind of constant relationship defines a proportion? What has to stay constant? Also, an opportunity to talk about constant difference, constant of proportionality, and constant rate of change.

Task 2

Which of the Bars scenarios matches each of the following word problems?

- Sarah is decorating cakes for a bake sale with her friends. If 3 friends work together, they can decorate all their cakes in 5 hours. How long would it take 6 friends to decorate the same cakes (assuming they work at the same pace)?
- Calvin runs 3 miles for every 5 miles his brother runs. How far will Calvin have run when his brother has run 10 miles?
- Two faucets each allow 3 gallons of water to be run in 10 minutes. A bucket was placed at each faucet. One of the buckets started empty and the other started with 2 gallons of water in it. How much is in each bucket after 15 minutes?

Goal: Contextualizing the mathematics to make it more realistic and relevant.

Wiggle Image Toy

Task 1

Mystery! After being challenged to create a proportional image in Wiggle Images, Chris was puzzled. She explained: I pulled it out to be twice as wide, but the picture looks funny. And, when I make it 3 times as wide, it's even stranger?

- What is happening to Chris' picture?
- How can Chris fix the picture so it looks right?
- What is Chris keeping constant and what is changing?

Goal: Thinking about what has to remain constant in a proportional relationship

Space Cactus Toy

Task 1

You are a scientist working in a science lab where a special "space cactus" is being developed as a tool for storing water on Mars. You track the growth of a particular cactus by measuring its height each week for several weeks. Data for each cactus can be captured using the Cactus Toy.

Use the Cactus Toy to tell us the story of one of the variations of cactus you are developing in your lab.

Goals: Playful engagement with a complex toy. Working with the toy features. Thinking about how to create a visual to tell a story.

Task 2

Create data for a new cactus in the science lab.

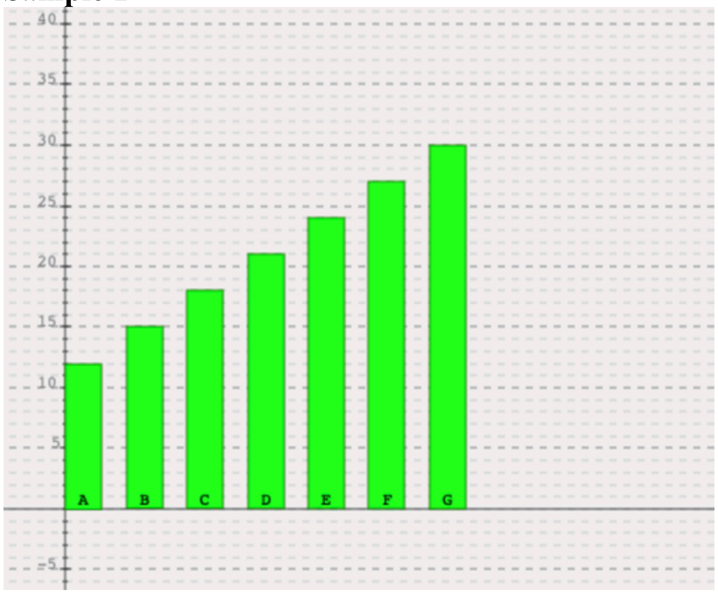
- What would the data collected for a space cactus look like if there was a constant relationship?
- Is this relationship proportional? Why or why not? How do we have to interpret the graphic to see whether there is a proportional relationship?
- Tell the story of this cactus' growth over time.

Goals: Modeling a constant relationship and examining whether that relationship is proportional. Understanding the kind of relationship that is proportional. Thinking about how to use a visual to tell a story.

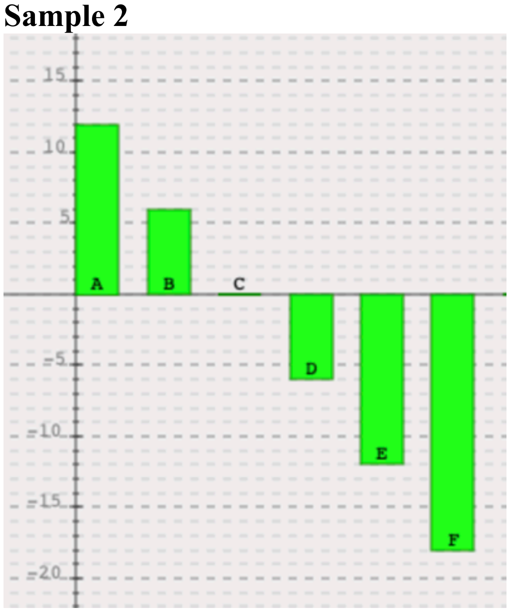
Task 3

Here are some cactus stories created from the cacti in the lab. Determine whether each is proportional. If so, explain what you are looking at that lets you know it's a proportion (e.g., what assumptions need to be made to interpret it as a proportion?)

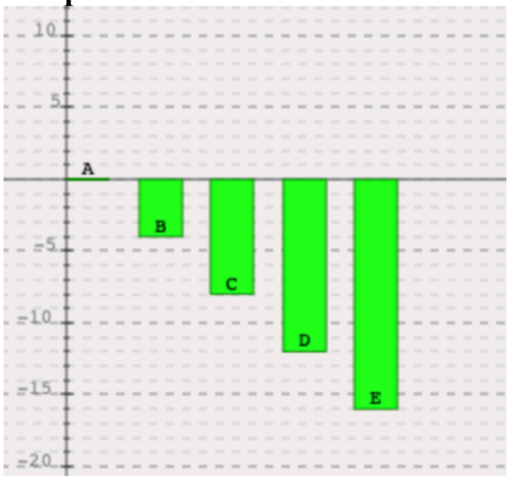
Sample 1



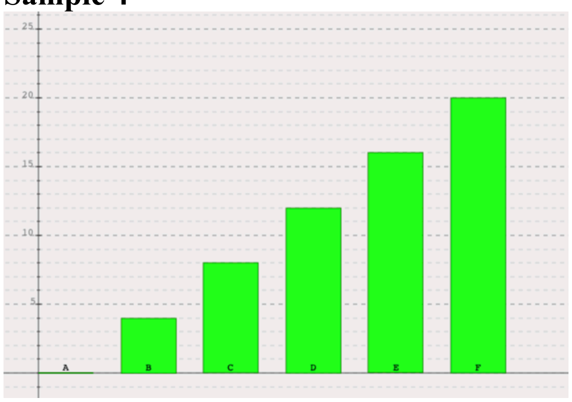
Sample 2



Sample 3



Sample 4



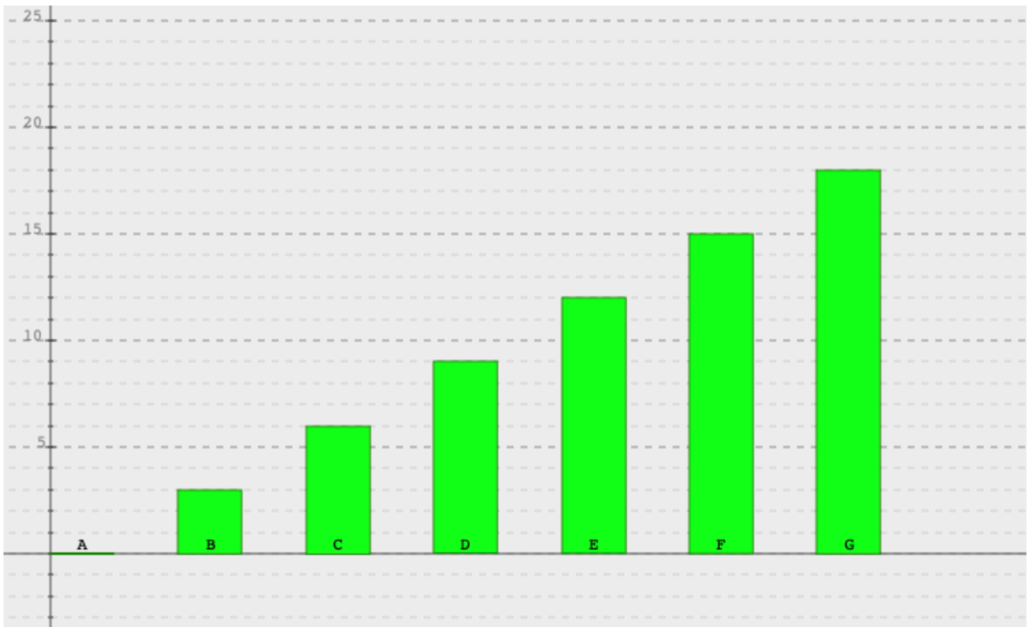
Goals: Visualizing proportional relationships (constant and covariation). Thinking about the definition of a proportion and recognizing that they don't have to fit into Q1.

<u>Task 4 (Not yet tried in PD)</u>

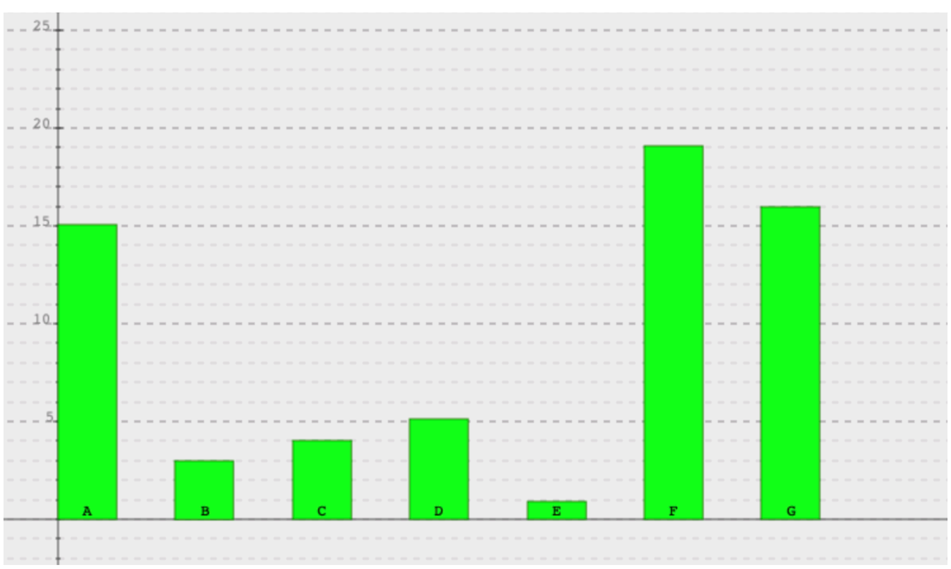
For each of the following data records of space cacti from the lab, how tall should each be at its next two measures? How do you know?

Goals: Thinking about the structure of proportions and how that looks. Using constant relationships to predict additional data. Becoming more precise about different constant relationships.

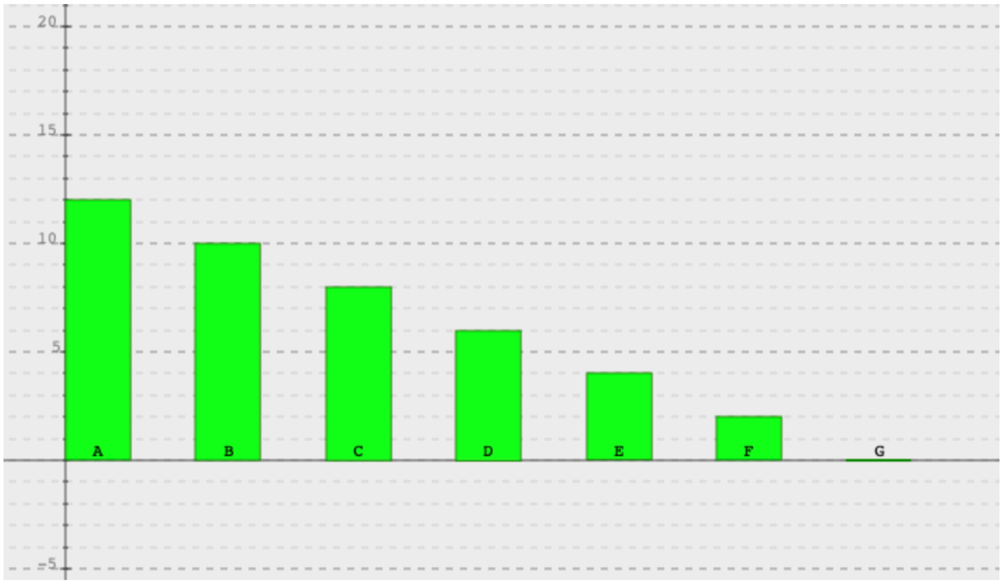
Cactus A



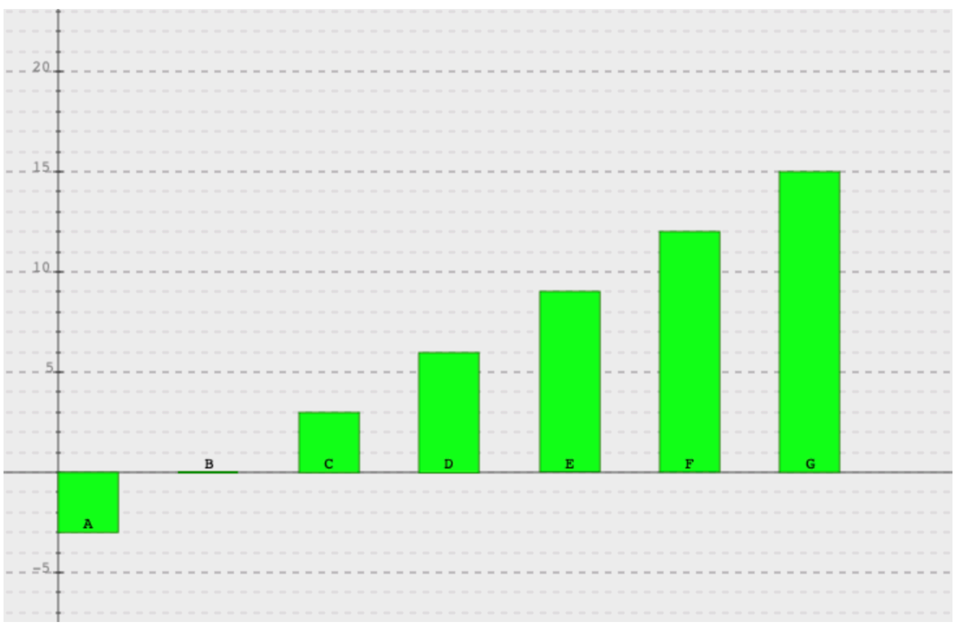
Cactus B



Cactus C



Cactus D



Critters Toy

Task 1

Mystery: Are any of these relationships constant?

- The size of one kitty compared to the other?
- The size of one kitty belly compared to that kitty's overall size?
- The distance between eyeballs for the orange kitty compared to the gray kitty?
- The amount of white space?
- The amount of white space compared to the overall size?

How can we use the Critters Toy to prove our answers?

Goals: What varies and what stays the same both between the kitty images and within either one of the images.

Task 2

Mr. Jackson's student wrote about the critters situation saying that he knew the kitties were proportional because they increased the height and width by the same amount.

- Is the student right?
- What might the student have been thinking about to say this?
- What might a clearer description of the relationship between the kitties be?

Goals: Examining "constant" and what kinds of language might productively describe the constant relationship between the kitties. Looking at the similarities and differences between additive interpretations and multiplicative interpretations.

Task 3

Another student in Mr. Jackson's class said that to keep the kitties proportional, you'd just have to dilate them.

- Is the student right?
- What might the student have been thinking about to say this?
- What additional information might you want from the student?

Goals: Examining "constant" and what kinds of language might productively describe the constant relationship between the kitties. Thinking about the relationship between dilation and maintaining proportionality.

Task 4

A third student in Mr. Jackson's class said that to create proportional kitties, you have to increase the size at the same rate.

- Is the student right?
- What might the student have been thinking about to say "at the same rate"?
- What stays the same for the rate to be considered the same?

Goals: Examining "constant" and what kinds of language might productively describe the constant relationship between the kitties. Thinking about what we mean by rate as it seems to be used to describe a number of things.

Tasks Designed to Connect the Content Learning to Teaching

Task 1 (Bars Toy, scenario 2 – the proportional scenario)

- How can we figure out the values of the two bars in between the values we've recorded?
  - Using a ratio table?
  - Using Bars?
- What standards does this help us meet?
- How does the Bars Toy help us answer this task?
- How does finding the values in between the whole numbers help promote thinking about the relationship as multiplicative rather than additive?

Goals: Quantities covary at a constant rate. The relationship between the quantities is constant while the magnitude of the quantities changes. Covariation is the restriction on this relationship. We intend this to be used to tie to ratio table representation and to think about the relationship between ratio tables and Bars as well as the affordances of each representation.

Task 2 (Bars Toy, scenario 2)

Consider the following two questions:

- A teacher sets up groups with 3 boys to 5 girls. She has 32 students in her class. How many groups can she make?

- A teacher sets up groups with 3 boys to 5 girls. If there are 32 students in the class, how many girls are in the class?

- What's constant and what's changing in each scenario?
- What are the quantities? Does it matter that there are 3 quantities?
- What do you need to know to answer this question?
- Does the Bars Toy help answer this question?
- What standards do we meet with this task?

Goals: Identifying quantities. Thinking about what is constant in a proportional relationship. Thinking about how we ask questions.

Task 3 (Bars Toy)

Fix these tasks! In a recent PD class, when we asked teachers to write real-world situations related to the Bars Toy, they gave us the following responses.

- How does each response need to be fixed to more accurately describe the situation?
- Which Bars scenario does each describe?

1. It's 3 parts fruit juice and 5 parts ginger ale. So, for whatever recipe you want to do, it's a ratio of 3 to 5.
2. They're both racing at a constant speed, but red is moving faster than blue. So, we could ask questions about how far this person has gotten at different times.

3. Jack and Jill both got some water and brought it down the hill. Jill's bucket carried 3 gallons at a time and Jack's bucket carried 5 gallons at a time.
4. It's like when you go into the carnival and you pay 25 cents to get in then you pay for each ride.

Debrief Questions:

- What are the quantities in each situation?
- What is covarying?
- What is constant?

Goals: Thinking about constant, covariation, and quantity, thinking about how situations need to be expressed for students to understand them, matching models to stories.

Task 4a (Buildings Toy)

Let's revisit the apartment scenario from Tuesday to create a task for your classroom:

A planned community has a building code that requires that new apartment buildings need to be __(x)__ apartments tall for every _(y)___ apartments they are wide.

1. Consider which of these pairs of numbers might be best for your students to explore.
   a) x=5 and y=6 b) x=4 and y=6 c) x=3 and y=5
2. What makes a number pair the best?
3. What questions could you ask about the buildings using this number pair that would help your students meet the standards?

Debrief Questions:

- What are we comparing?
- How would we set up this proportion?
- What scale factors are possible with the number pair you chose? Can you scale both up and down?
- Do ratio table with each number pair to explore unit rates.

Goals: Help teachers think about the relationship between the quantities and what pedagogical affordances those number pairs offer.

Task 4b (Buildings Toy)

Now that we selected a pair of numbers…

- How can we have students explore the apartment context so you can find out whether they understand that proportional relationships are multiplicative?

Considerations to address:

- What task do we pose?
- How do we make sure they understand the task?
- What data do we have them capture?

- What representations do we have them use?

Goals: Thinking about the constant relationship in a proportion and how to help students develop an understanding of it.

Task 4c (Buildings Toy)

If your students are thinking additively rather than multiplicatively, how will you challenge their ideas?

- What additional question(s) do you ask?
- What additional task(s) do you pose?
- How can we use the buildings toy to challenge an additive idea?

Debrief Questions

- What is covarying?
- What is constant?
- How can we use the data they captured?

Goals: Planning a strategy for addressing a known misconception about covariation.

Task 5 (NOT Used in PD yet)

Goals: Provide a variety of contexts for thinking about what's constant in a proportional situation and what other constant relationships might look like. Modeling situations with representations.

Task Sorting

For each of the following tasks, determine:

- What kind of relationship does the task present?
- What is the constant in each example?
- Can you model the relationship? (In words, graphs, and/or symbols)
- How can the constant help you determine that kind of relationship?

Task 1

Erik is putting oil in his scooter. When he had 3/4 of a liter in the reservoir, he realized that the tank was 1/2 full. How many liters will the fully filled reservoir hold?

Task 2

For a specific gas, a scientist made the following pairs of measurements:

Pressure	0.1	0.25	0.5	1	1.25	3

Volume	50	20	10	5	4	5/3

What kind of relationship does this gas exhibit? Can you predict the pressure for any volume?

Task 3
Yesterday, I used 0.75 liters of paint for a painting $6m^2$. Today, I will paint 18 m^2. How much paint do I need?

Task 4
A movie ticket costs $7 today. Inflation is expected to grow by 4 percent a year for the next several years. Predict the cost of the tickets if this inflation rate continues for (a) 5 years, (b) *n* years.

Task 5
Andrew has a recipe for cookies that calls for 3 cups of brown sugar to make 7 batches.

a) How many batches can Andrew make with one cup of sugar?
b) How many cups of brown sugar does Andrew need for one batch?

Task 6
If two monkeys can jump across a ravine six feet wide, then how wide a ravine can four monkeys jump across?

Task 7
The locomotive of a train is 12m long. If there are 4 cars connected to the locomotive, the train is 52m long. If there are 8 cars connected to the locomotive, how long will the train be?

Task 8
Sue and Julie were running equally fast around a track. Sue started before Julie. When Sue had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?