

# Teachers Extending Their Knowledge in Online Collaborative Learning Environments: Opportunities and Challenges

Arthur B. Powell, Rutgers University

Jason Silverman, Drexel University

Stephen Weimar, The Math Forum at NCTM



# A Tale of Two Projects





---

The Math  
Forum

# Session Overview

- Overview of the Math Forum
- Introduction to EnCoMPASS  
Analysis and Discussion
- Introduction to Virtual Math Teams  
Analysis and Discussion
- Discussion and Next Steps



The Math Forum  
PEOPLE LEARNING MATH TOGETHER

- Online resource portal and community for mathematics education since 1992
- Evolved to include a variety of services and communities
- Signature Service: Problems of the Week (PoWs) — Online Mentored Problem Solving Environment
  - Open-middle and Open-ended tasks
  - Online Submission
  - Mentoring
  - Approving



The Math Forum  
PEOPLE LEARNING MATH TOGETHER

- PoWs have been generative
- Norms/Practices at the core of resultant projects and initiatives
  - Taking student ideas seriously
  - Valuing students as mathematical thinkers with ideas
  - “Doing mathematics together” (communication and collaboration)
  - Teaching as data-driven inquiry
  - Learners are learners (symmetric PD)

# EnCOMPASS

The logo for EnCOMPASS features a yellow compass rose with a blue sine wave passing through its center, positioned over the 'CO' in the word 'COMPASS'.

---

*[mathforum.org/encompass](http://mathforum.org/encompass)*

Jason Silverman  
Wesley Shumar

Stephen Weimar  
Valerie Klein

The Math Forum

Drexel University

# EnCOMPASS



*[mathforum.org/encompass](http://mathforum.org/encompass)*

Jason Silverman  
Wesley Shumar

Stephen Weimar  
Valerie Klein

and

Cheryl Fricchione, Candice Roberts,  
Anthony Matranga and  
Melissa Sebastian

The Math Forum

Drexel University

# Problems of the Week

**Studies have shown participation in the PoWs resulted in significant gains...**

For Students:

- Problem Solving, Mathematical Practices and Student Autonomy (Renninger, Farra, Feldman-Riordan, 2000)

For Pre-Service Teachers (OMP):

- Analyze student work (noticing), Provide mathematically-rich feedback (as opposed to stock praise) (Ray & Renninger, 2006; Renninger, Ray, Luft, & Newton, 2006)



Home

Math Help

Problems &amp; Puzzles

Math Talk

Resources &amp; Tools

About The Math Forum

Store

Search

Current Problems of the Week

Problems of the Week Library

Write Math: PoWs by Standard

Other Problems and Puzzles

## Submit Math Club Mystery [Problem #4036]

### Math Club Mystery

Members of the Math Club at Morganson Middle School sent us this problem:

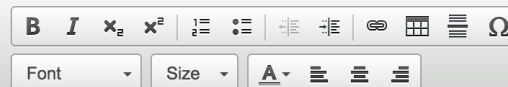
The math club took a field trip to see the movie *An Inconvenient Truth*. A total of 28 people went on the trip, including students, teachers, and parent chaperones. There were more parents than teachers.

Movie tickets cost \$7, but the students got a discount and only had to pay \$3 each. The group paid a total of \$108 to get everyone in. Determine how many students, how many teachers, and how many parents went on the trip.



Summarize your ideas in a sentence or two:

Explain your ideas and how you figured them out. [What do these buttons do?](#)



Interesting and Grounded in Best Practices...  
What *actually* enabled the successes?

# Culture and Practices of the Math Forum

## Professional Practices

- Taking student ideas seriously
- Professional Noticing: attending, interpreting, deciding
- Teaching as data-driven inquiry: conjecture; test; revise

## Culture/Norms

- Valuing students as mathematical thinkers with ideas
- “Doing mathematics together”  
communication and collaboration

# Culture and Practices of the Math Forum

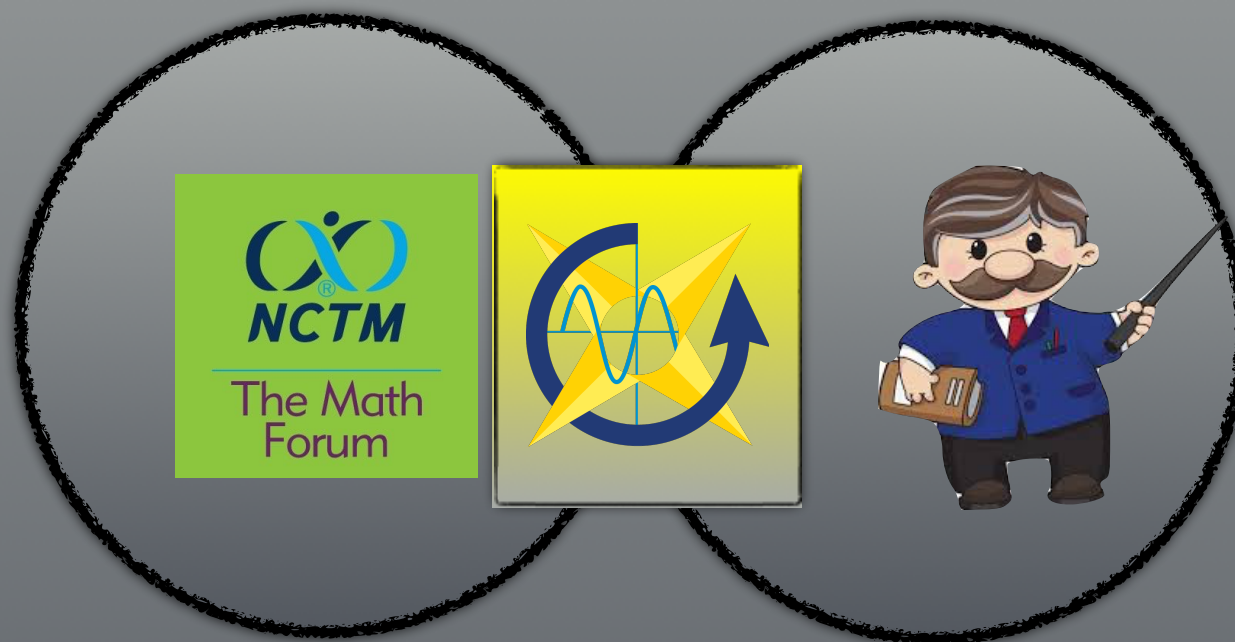
**“I Notice, I wonder”**

# The EnCoMPASS Project

## Goals

- Provide a broader entry-point for engaging with and participating in the Math Forum practices *and effective use of the PoWs*
- Support the emergence of an online community that supports and sustains these practices

# Conjectured PD Model



## EnCoMPASS as a Boundary Object

- Legitimacy in each of the communities

# EnCOMPASS Environment

## Development of this boundary object

- Use cases and features were co-created by project staff and teachers
- To purposefully provide structure and support for assessment (formative & summative) and instructional planning that is centered around productive mathematical and pedagogical practices
- The environment is interactive, personalizable and collaborative

# EnCOMPASS Environment

## Goal directed:

- The goal of this boundary object is to support professional practices. In particular, **noticing** and develop capacity to attend to student thinking, interpreting via **wondering** and use that to **decide** how to respond to students.
- The environment relies on attending to actual elements in students work, or evidence, to make claims and categorize student thinking and responses
- Productive Disruption of traditional teaching practices: answer oriented assessment, procedural focused pedagogy, and associating speed with good thinking

# Demo of Environment

- Selecting (Noticing) and Commenting
- Organizing Noticing
- Feedback to Students
- Collaboration

# EnCOMPASS Tool

Drexel | EnCoMPASS
home
workspaces
responses & reflections
users
logout

## Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4

by Valerie Klein | info

A=(b)(h)
0
0

A=(d1)(d2)/2
3
4

Correct work/incorrect v...
3
6

Illogical reasoning
1
2

incorrect solution
1
1

Properties of Parallelogr...
2
3

Used 30-60-90
2
3

Keslie J.
1 \*

SHOW SELECTIONS
SELECTING
Area of a Rhombus
Respond

Keslie and maria the area is 37.8

First we found the angles of one triangle of the rhombus and got a 60 90 30 triangle. then we used 6 tan 30 = X to find one side of the triangle the 30 is one angle and 6 is half of the diagonal. We got 3.15 then times it by 2 getting 6.3. With that we multiplied it by 12 and got 37.8

60 90 30 triangle
6 tan 30 = X
We got 3.15 then times it by 2 getting 6.3
With that we multiplied it by 12 and got 37.8
60 90 30 triangle
6 is half of the diagonal. We got 3

I NOTICE
I WONDER
FEEDBACK

I would like...

CANCEL
SAVE

- Ideas are correct, but final value is not correct.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- calculations are correct, but values are incorrect and formula is not complete.  
by amandareinsburrow in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- What is the formula for the area of a rhombus student is using?  
by amandareinsburrow in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- How they could create a representation to assist them with the relationship of the special right triangle.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- ratios do not fit the 30/60/90 triangle relationship.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- Trigonometric ratios, special right triangles characteristics  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- using properties of parallelogram to create a special right triangle  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4

ADD
EDIT

# EnCOMPASS Tool

Drexel | EnCoMPASS
home
workspaces
responses & reflections
users
logout

Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4 by Valerie Klein | info

A=(b)(h)
0
0
Keslie J.
1\*
9 / 18

A=(d1)(d2)/2
3
4
SHOW SELECTIONS
SELECTING
Area of a Rhombus
Respond

Correct work
Illogical reasoning
incorrect solutions
Properties
Used 30-60-90

Keslie and maria the area is 37.8

First we found the angles of one triangle of the rhombus and got a 60 90 30 triangle. then we used  $6 \tan 30 = X$  to find one side of the triangle the 30 is one angle and 6 is half of the diagonal. We got 3.15 then times it by 2 getting 6.3. With that we multiplied it by 12 and got 37.8

60 90 30 triangle
6 tan 30 = X
We got 3.15 then times it by 2 getting 6.3
With that we multiplied it by 12 and got 37.8
60 90 30 triangle
6 is half of the diagonal. We got 3

ADD
EDIT

I NOTICE
I WONDER
FEEDBACK

I would like...

CANCEL
SAVE

it.  
metry
ect and  
burrow -  
s student  
burrow -  
sist them  
with the relationship of the special right triangle.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4  
+ ratios do not fit the 30/60/90 triangle relationship.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4  
+ Trigonometric ratios, special right triangles characteristics  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4  
+ using properties of parallelogram to create a special right triangle  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4

# EnCOMPASS Tool

Drexel | EnCoMPASS
home
workspaces
responses & reflections
users
logout

## Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4

by Valerie Klein | info

A=(b)(h)
0
0

A=(d1)(d2)/2
3
4

Correct work/incorrect v...
3
6

Illogical reasoning
1
2

incorrect solution
1
1

Properties of Parallelogr...
2
3

Used 30-60-90
2
3

Keslie J.
1\*

SHOW SELECTIONS
SELECTING
Area of a Rhombus
Respond

Keslie and maria the area is 37.8

First we found the angles of one triangle of the rhombus and got a 60 90 30 triangle. then we used 6 tan 30 = X to find one side of the triangle the 30 is one angle and 6 is half of the diagonal. We got 3.15 then times it by 2 getting 6.3. With that we multiplied it by 12 and got 37.8

60 90 30 triangle
6 tan 30 = X
We got 3.15 then times it by 2 getting 6.3
With that we multiplied it by 12 and got 37.8
60 90 30 triangle
6 is half of the diagonal. We got 3

I NOTICE
I WONDER
FEEDBACK

I would like...

CANCEL
SAVE

- Ideas are correct, but final value is not correct.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- calculations are correct, but values are incorrect and formula is not complete.  
by amandareinsburrow in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- What is the formula for the area of a rhombus student is using?  
by amandareinsburrow in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- How they could create a representation to assist them with the relationship of the special right triangle.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- ratios do not fit the 30/60/90 triangle relationship.  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- Trigonometric ratios, special right triangles characteristics  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4
- using properties of parallelogram to create a special right triangle  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4

ADD
EDIT

# EnCOMPASS Tool

Drexel | EnCoMP
Correct work/incorrect values  
3 submission(s), 6 selection(s)

Area of a Rhombus
Browse: ☒ By Student ☐ By Evidence (selection) ☒ Include Sub-Folders

Show:  
☒ Evidence  
☒ Comments  
☒ Folders

Student	Evidence		
	Selection	Comments	Folders
Xavier G.	area=(d1)(d2) divided by 2	<ul style="list-style-type: none"> <li>Correct formula. good place to start</li> <li>Correct Formula for area of a rhombus</li> <li>Was this given? Memorized? Referenced?</li> </ul>	<ul style="list-style-type: none"> <li>A=(d1)(d2)/2</li> </ul>
	For d1 we put in 6 and for d2 we put 6	<ul style="list-style-type: none"> <li>Why the values of 6 were used when the problem says one of the diagonals was 12?</li> <li>What they were using to have the given 12cm divided by 2?</li> <li>Property of parallelogram, diagonals bisect each other</li> </ul>	<ul style="list-style-type: none"> <li>A=(d1)(d2)/2</li> </ul>
	answer 18.	<ul style="list-style-type: none"> <li>Mathematics for finding this value are correct, values used are incorrect.</li> </ul>	<ul style="list-style-type: none"> <li>Correct work/incorrect values</li> </ul>
	The answer we got was 36	<ul style="list-style-type: none"> <li>What the 36 could represent in this scenario</li> </ul>	
Kesslie J.	60 90 30 triangle	<ul style="list-style-type: none"> <li>you understood that 30-60-90 triangles are created</li> </ul>	<ul style="list-style-type: none"> <li>Used 30-60-90</li> </ul>
	6 tan 30 = X	<ul style="list-style-type: none"> <li>is this the correct trig function to use?</li> <li>Trigonometric ratios, special right triangles characteristics</li> </ul>	<ul style="list-style-type: none"> <li>Correct work/incorrect values</li> </ul>
	We got 3.15 then times it by 2 getting 6.3	<ul style="list-style-type: none"> <li>this would give you the second diagonal if the 3.15 was correct.</li> </ul>	<ul style="list-style-type: none"> <li>Correct work/incorrect values</li> </ul>
	With that we multiplied it by 12 and got 37.8	<ul style="list-style-type: none"> <li>What is the formula for the area of a rhombus student is using?</li> <li>calculations are correct, but values are incorrect and formula is not complete.</li> <li>Ideas are correct, but final value is not correct.</li> </ul>	<ul style="list-style-type: none"> <li>Correct work/incorrect values</li> </ul>
	60 90 30 triangle	<ul style="list-style-type: none"> <li>using properties of parallelogram to create a special right triangle</li> </ul>	
	6 is half of the diagonal. We got 3	<ul style="list-style-type: none"> <li>ratios do not fit the 30/60/90 triangle relationship.</li> <li>How they could create a representation to assist them with the relationship of the special right triangle.</li> </ul>	

ADD +
EDIT ✖

right triangle  
by cmj86 in Area of a Rhombus / Reinsburrow - Geometry  
AmandaPoW4

logout

FEEDBACK

CANCEL | SAVE

## EnCOMPASS Tool

Drexel | EnCoMPASS

homeworkspacesresponses & reflectionsuserslogout

Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4 by Valerie Klein | info

A=(b)(h)00Keslie J.1\*

9 / 18

I NOTICEI WONDERFEEDBACK

SHOW SELECTIONSSELECTINGArea of a RhombusRespond

I would like...

## New Response

To: Keslie J.

Hello Keslie,

You wrote:

60 90 30 triangle

...and I noticed that...

you understood that 30-60-90 triangles are created

You wrote:

6 is half of the diagonal. We got 3

...and I noticed that...

ratios do not fit the 30/60/90 triangle relationship.

...and I wondered about...

How they could create a representation to assist them with the relationship of the special right triangle.

Edit

Save

Send a copy to the PoW (Keslie J.)

?

This response was generated from Valerie Klein's workspace Area of a Rhombus / Reinsburrow - Geometry AmandaPoW4. In particular, it came from Keslie J.'s [submission](#) ([more details](#))

# EnCoMPASS Use Cases

## **Planning for Instruction Orchestrating Discussion**

- Looking at and organizing archived student work to plan for instruction planning
- Identify misconceptions, misunderstandings and unique interpretations and solution strategies
- Front-load the five practices

## **Formative Assessment**

- Quickly explore and organize student homework or classwork

## **Facilitating Student-led learning**

- Sharing comments/folders/categories/comments/feedback

## **Crafting and Sharing “Good” Feedback**

- Workspaces and folders can be public and have direct links

## **Professional Development**

# Look at Data

## The Task: Area of a Rhombus (#3627)



A rhombus has an angle measure of 120 degrees, and its longer diagonal has a length of 12 centimeters. Find the area of the rhombus.

Extra: A rhombus has an angle measure of 60 degrees and an area of 32. What is its edgelen<sup>g</sup>th?

# Look at Data

## What do you Notice? What do you Wonder? For Discussion...

1. What are the Professional Development benefits and challenges for
  - taking learner's work and thinking seriously thinking
  - the features of EnCoMPASS
  - the online nature of the environment(s)
2. What are the benefits and challenges of professional development that has these activities and practices at the core
  - Iterative; Symmetric PD

# Discussion

## EnCoMPASS Conjectures and Questions

- Is the EnCoMPASS tool a boundary object?
- Emerging Results:
  - a focus on how students' written work helps teachers focus on what students are thinking and doing
  - collaboration around students' work, in various forms, allows teachers reflect on their own thinking of mathematics and mathematics instruction
  - ability for student thinking not to be “binary” is significant
  - teachers explicit about how this work affected practice
  - more efficient feedback was “hook” but ultimately not driver

# For more information

- <http://mathforum.org/encompass>



**EnCOMPASS**  
*Emerging Communities  
for Mathematical Practices  
and Assessment*

Home Project Summary Research Faculty/Staff Summer Institutes Fellows Teacher Participation



● ● ● ● ● ● ●

## Project Overview

The EnCoMPASS Project is developing an online professional teaching community of mathematics educators focused on understanding and improving mathematical thinking through work with formative assessment rubrics and feedback to student problem solving. In this community, members extend their content knowledge for teaching and seek to become more effective at supporting the mathematical

# Computer-Supported Math Discourse Among Teachers and Students

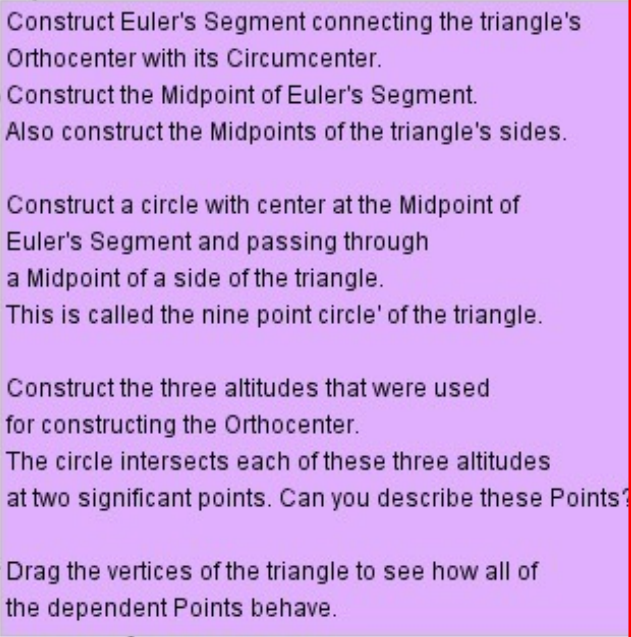
Arthur B. Powell, Rutgers University  
Stephen Weimar, The Math Forum @ NCTM



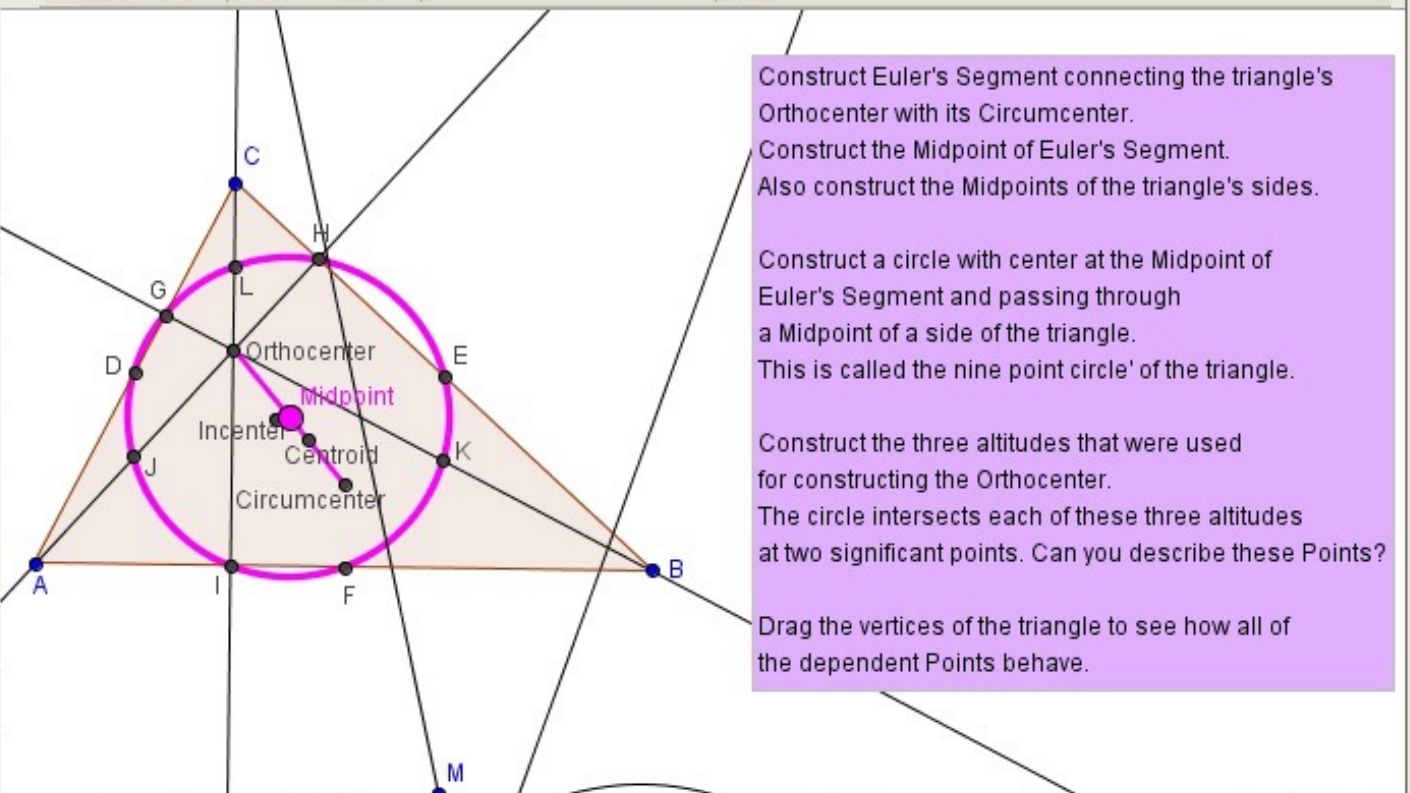
# Context for Virtual Math Teams

- Research situated in an online math education community, The Math Forum
- A focus on learners' mathematical thinking:
  - The development of mathematical practices
  - Understanding the development of group cognition
  - Facilitating the role of mathematical practices in learning math
- What do computer-supported and networked environments bring to this focus?
- What supports help teachers facilitate productive mathematical interaction in an online multi-user dynamic mathematics environment?









Construct Euler's Segment connecting the triangle's Orthocenter with its Circumcenter.

Construct the Midpoint of Euler's Segment.

Also construct the Midpoints of the triangle's sides.

Construct a circle with center at the Midpoint of Euler's Segment and passing through a Midpoint of a side of the triangle.

This is called the 'nine point circle' of the triangle.

Construct the three altitudes that were used for constructing the Orthocenter.

The circle intersects each of these three altitudes at two significant points. Can you describe these Points?

Drag the vertices of the triangle to see how all of the dependent Points behave.

Muteb

vvvvvvvvvv

**vkрупник** 6/12/13 6:38:35 PM EDT:  
cool

**vkрупник** 6/12/13 6:38:40 PM EDT:  
everyone see it

**allowry** 6/12/13 6:38:41 PM EDT: I  
think we're ready to move on?

**ira** 6/12/13 6:38:43 PM EDT: very  
cool

**vkрупник** 6/12/13 6:38:44 PM EDT:  
yes

**allowry** 6/12/13 6:38:45 PM EDT:  
to the next topic

**Alice\_L** 6/12/13 6:38:47 PM EDT:  
yes

**Alice\_L** 6/12/13 6:38:52 PM EDT:  
next topic?

**allowry** 6/12/13 6:38:52 PM EDT:  
topic 8?

■ ■

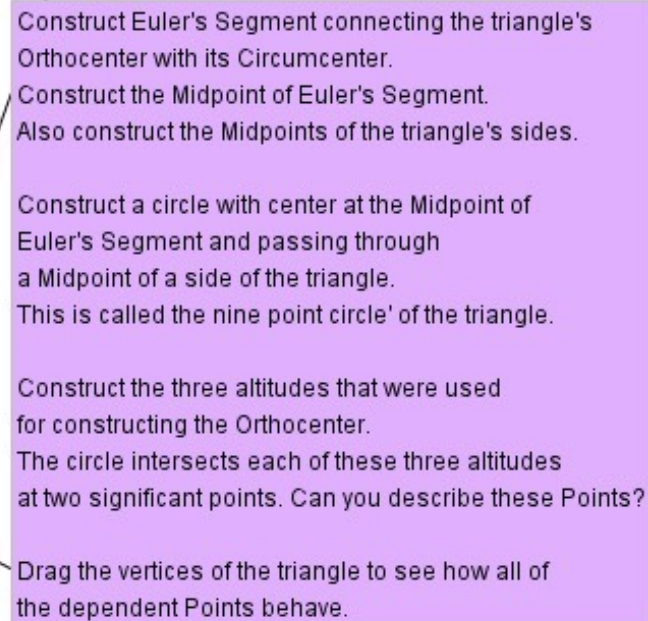






Drag the vertices of the triangle to see how all of the dependent Points behave.

32




allowry 6/12/13 6:38:52 PM EDT:  
topic 82

--	--



**Message:**





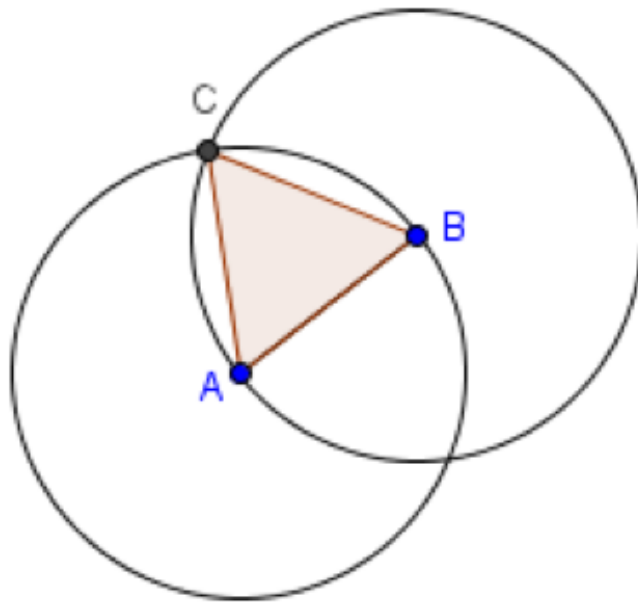
# The Teacher PD Courses

- Epistemic tools: noticing, meaning, wondering
- Productive mathematical discourse (Mercer and Sam)
- Collaborative norms (one reading followed by a discussion about norms)
- Review their work (every other week)
- Mathematical practices
- Accountable talk
- Pedagogical issues: Technology in classroom
- Implementation
  - Five phases to develop a plan
- Support in the spring
  - Discussions with other teachers
  - Sharing experiences

# GeoGebra Construction

## Equilateral Triangle Task

An example of what you will construct.



Make sure that everyone in your team can construct and drag this equilateral triangle and understand why it is equilateral dynamically.

You will construct an equilateral triangle the way that Euclid did in his first proposition, but yours will be a dynamic equilateral triangle.

1. Construct a segment for the base of the triangle.
2. Construct a circle with center at one endpoint, passing through the other endpoint.
3. Construct a circle with center at the second endpoint, passing through the first.
4. Use the Intersection tool to construct a third point at an intersection of the two circles.
5. Drag to make sure the point is on both circles.
6. Use the polygon tool to construct a triangle.
7. Chat about how the third point is dependent on the distance between the first two points.
8. Do you think the triangle is equilateral? Always?

Figure 1: Equilateral triangle task statement

# Student Data

- Constructed figures
- Dragged vigorously
- Teacher monitors and intervenes (lines 16, 23 - 24, and 32, 46, and 49)
- Lens
  - Teacher facilitation of group learning
  - Looking closely at mathematical thinking
  - Epistemic tools (noticing, meaning, wondering )

# Look at Data

**What do you Notice? What do you Wonder?**

# Look at Data

1. What are the Professional Development benefits and challenges for

- taking learner's work and thinking seriously thinking
- the features of VMT
- the online nature of the environment(s)

2. What are the benefits and challenges of professional development that has these activities and practices at the core

- Iterative; Symmetric PD

Excerpt of Team 6's chat posting:

Line	Time	User	Message
6	7:52:02	<u>kim_bchs</u>	would you like to start first?
7	7:52:12	<u>kar_bchs</u>	Sure
8	7:53:08	<u>kim_bchs</u>	may i try next
9	7:53:15	<u>kar_bchs</u>	of course
10	7:55:00	<u>kar_bchs</u>	looks like we both got it [both successfully construct and drag the figures vigorously]
11	7:55:47	<u>kim_bchs</u>	yay, it <u>seem</u> like for a second one of the circles appeared much larger. but that was my imagination.
13	7:56:44	<u>kar_bchs</u>	oh. lol. why is the third point <u>dependent on</u> the distance between the first two points? (number 7)
14	7:57:25	<u>kar_bchs</u>	it just connects the points and the circles. making them all one piece
15	7:58:37	<u>kim_bchs</u>	as the segments change sides so does the radius of the circle. However, the triangle remains an equilateral <u>traiangle</u>
16	7:58:38	<u>bsingh</u>	<i>be sure to read directions, ALL, and make the pledge</i>
17	7:58:42	<u>kim_bchs</u>	triangle
18	8:01:06	<u>kar_bchs</u>	yea. even though the sizes of the sides change, the fact

Line	Time	User	Message
			that it is an equilateral triangle doesn't
19	8:02:27	<u>kar_bchs</u>	each side has the same distance in between it. even when you move the points
20	8:03:52	<u>kim_bchs</u>	i notice that point d and e are on the circumference of one circle. while point f is an <u>intercetion</u> of both circle. making it dependent on both points.
21	8:05:05	<u>kar_bchs</u>	if you try and move the intersected point (F and I), it wont move. but yea you're right, the intersecting point depends on the segment that was made
22	8:05:06	<u>kim_bchs</u>	*point f is an intersect of both circle
23	8:05:44	<u>bsingh</u>	<i>there is something missing, are you reading the directions</i>
24	8:07:04	<u>bsingh</u>	<i>we are only doing tab 1 today</i>
27	8:08:12	<u>kar_bchs</u>	i didnt use the polygon tool, <u>thats</u> missing in mine
28	8:08:37	<u>kim_bchs</u>	i just notice that.
29	8:08:55	<u>kar_bchs</u>	can i try?
30	8:09:58	<u>kar_bchs</u>	okay. i got it now
31	8:10:35	<u>kim_bchs</u>	do you think the triangle will always be an equilateral triangle.
32	8:12:09	<u>bsingh</u>	<i>triangle GHI did not use polygon tool</i>

33	8:12:39	<u>kar_bchs</u>	the sides stay <u>equal</u> . the two circles were formed using one segment, so those circles were even with each other. so any points connecting them <u>will</u> become the same length as the original segment
34	8:12:52	<u>kar_bchs</u>	oh <u>thats</u> right. we <u>didnt</u> fix that one
35	8:13:07	<u>kim_bchs</u>	triangle GHI was our first try. we fix our mistake.
36	8:14:09	<u>kar_bchs</u>	<u>i</u> just used the polygon tool on top. <u>i</u> guess it still worked.
37	8:14:13	<u>kim_bchs</u>	should we delete GHI or use it as a non-example.
38	8:14:22	<u>kar_bchs</u>	we can delete it
39	8:15:29	<u>kim_bchs</u>	<u>i</u> agree with what you said earlier.
40	8:15:44	<u>kar_bchs</u>	about the sides?
41	8:15:49	<u>kim_bchs</u>	yes
42	8:16:06	<u>kar_bchs</u>	you think its anything else?
43	8:16:17	<u>kar_bchs</u>	like other ideas?
44	8:18:22	<u>kim_bchs</u>	<u>i</u> concur with Euclid's argument
45	8:19:19	<u>kar_bchs</u>	<u>i</u> agree too.
46	8:19:32	<u>bsingh</u>	<i>[The teacher] put a textbox with your name, next to your construction</i>
47	8:20:01	<u>kar_bchs</u>	okay
48	8:20:32	<u>kar_bchs</u>	want me to put your name, <u>kim</u> ?
49	8:21:47	<u>bsingh</u>	<i>even with each other?</i>
50	8:22:36	<u>kim_bchs</u>	the radius of a circle is the same distance. segment AB is Sure. the radii of both circles and Segment AC and BC are also radii of both <u>cicles</u> . <u>hense</u> , the triangle should be

Line	Time	User	Message
			<u>euilateral.</u>
51	8:22:46	<u>kar_bchs</u>	the circles are equal. making the <u>circumfrence</u> of each, equal to one another
52	8:22:56	<u>kim_bchs</u>	You can put my name too.
53	8:22:59	<u>kar_bchs</u>	okay

Material: Equilateral Triangle Perpendiculars Add a tab +

File Edit View Options Tools Window Help

Move Graphics View  
Drag graphics view or one axis (Shift + Drag)

An example of what you will construct:

© 2010 Drexel University

Current users:

Chat: (0)

kar\_bchs 3/20/14 8:09:58 AM EDT: okay. i got it now

kim\_bchs 3/20/14 8:10:35 AM EDT: do you think the triangle will always be an equilateral triangle.

bsingh 3/20/14 8:12:08 AM EDT: triangle GHI did not use polygon tool

kar\_bchs 3/20/14 8:12:39 AM EDT: the sides stay equal.. the two circles were formed using one segment, so those circles were even with each other. so any points connecting them will become the same length as the original segment

kar\_bchs 3/20/14 8:12:51 AM EDT: oh thats right. we didnt fix that one

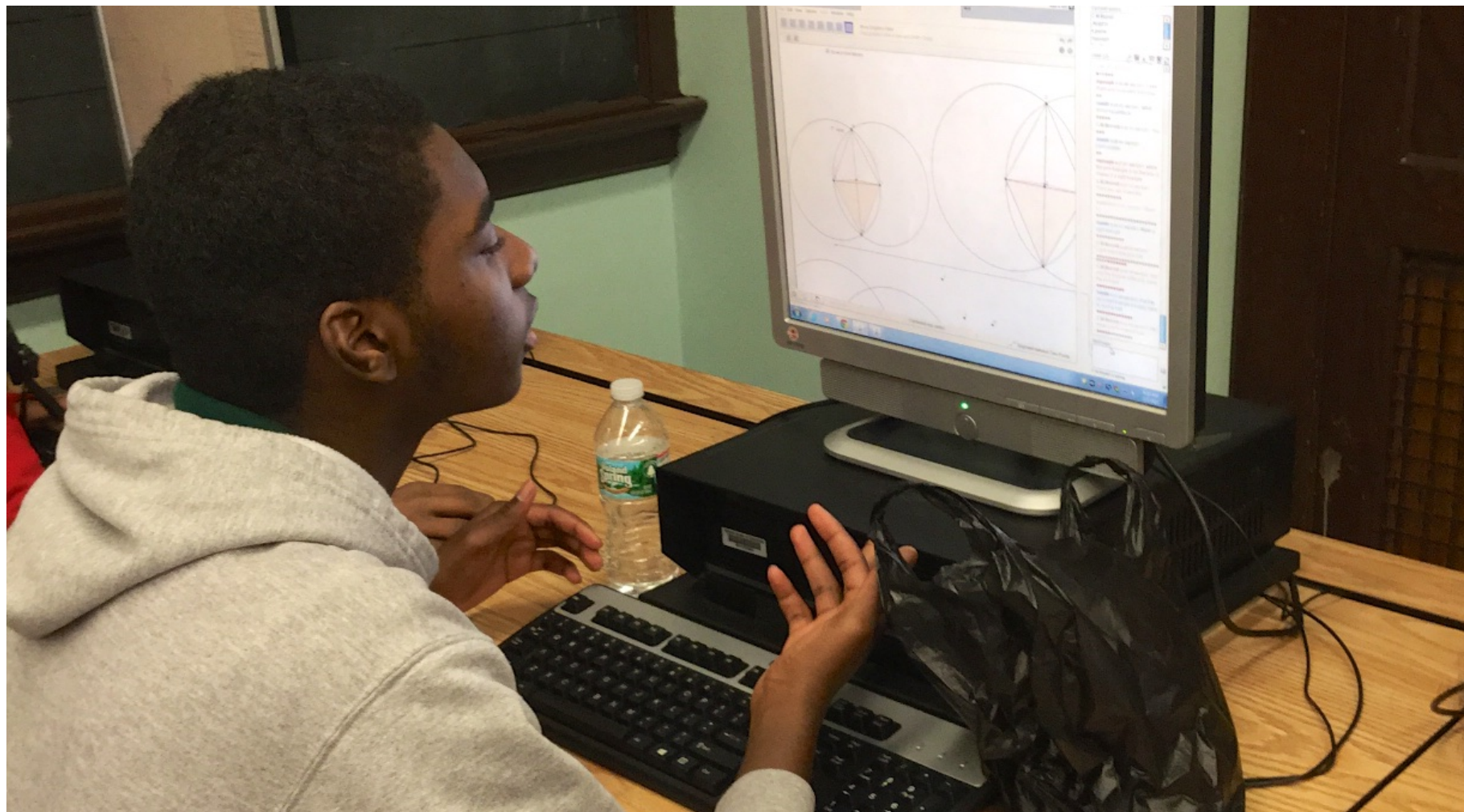
Message:

Figure 2: Team 6's solution in the Virtual Math Teams with GeoGebra application

# Connections

- What did you notice across those two experiences?
- What commonalities or differences?
- Next steps for similar research projects/questions
  - Taking Student Thinking Seriously
  - Relationship of Research to Data Generation
  - Affordances/Challenges of Online/Distributed
  - Collaboration/Community
  - Symmetric PD

# Thank You



# Discussion

- Thoughts about the use of online technologies for facilitating the development of mathematical thinking.
- Thoughts about focusing on collaborative group learning and the group as the unit of analysis.
- How to enable and support teachers to work well in these sorts of contexts.
- Connections between these two projects
- Future research directions and collaborations