NSF Measurement Project

Evaluation of a Developmental Progression for Length Measurement Using the Rasch Model

Douglas H. Clements, Julie Sarama
University at Buffalo, State University of New York
Jeffrey Barrett, Craig Cullen +
Illinois State University
Doug van Dine
University at Buffalo, SUNY

Building Blocks

What Develops?

- Children's Measurement Project (NSF) (childrensmeasurement.org)
- *Learning Trajectories*, Pre-K to 5
- Mainly clinical interviews and teaching experiments (individual and class)
- But pretested in schools, and *conducted Rasch analysis of this pretest data.*

Measurement Learning Trajectory

- ***Length Quantity Recognizer*** Identifies length as attribute.
  - "I'm tall, see?"
- ***Length Direct Comparer*** Physically aligns two objects to determine which is longer or if they are the same length.
  - Stands two sticks up next to each other on a table and says, "This one's bigger."
- ***Indirect Length Comparer*** Compares the length of two objects by representing them with a third object.
  - Compares length of two objects with a piece of string.

Measurement Learning Trajectory

- ***End-to-End Length Measurer*** Lays units end-to-end. May not see the need for equal-length units.
  - Lays 9 inch cubes in a line beside a book to measure how long it is.
- ***Length Unit Relater and Repeater***
  - Relates size and number of units
    - "If you measure with centimeters instead of inches, you'll need more of them, because each one is smaller."
  - Repeats, or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance.
    - Measures a book's length accurately with a ruler.

Measurement Learning Trajectory

- ***Length Measurer*** Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers.
  - Begins to estimate
  - Considers the length of a bent path as the sum of its parts (not the distance between the endpoints).
  - "I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all."

Measurement Learning Trajectory

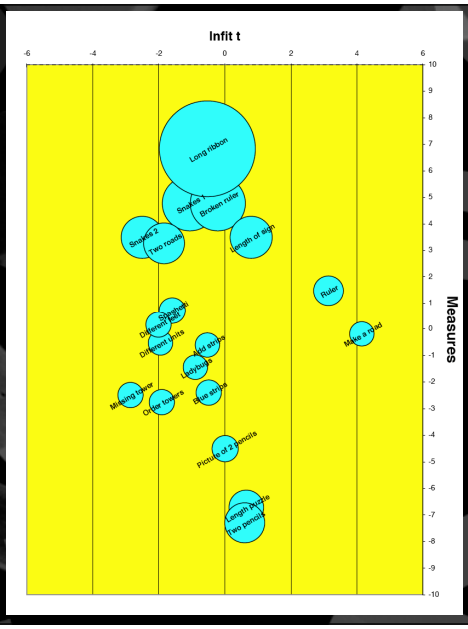
- ***Conceptual Ruler Measurer*** Possesses an "internal" measurement tool. Mentally moves along an object, segmenting it, and counting the segments.
  - Estimates with accuracy.
    - "I imagine one meter stick after another along the edge of the room. That's how I estimated the room's length is 9 meters."
  - Operates arithmetically on measures ("connected lengths").

Foreshadowing

- General confirmation; measure of all dimensions yields a single latent trait
- But some were earlier; children sometimes used less sophisticated strategies
- First, earlier Rasch studies…

Rasch Analyses

- LT was confirmed by Rasch #1 in *Building Blocks* evaluation, but comprehensive test
- Rasch #2: Dissertation
  - Two countries
  - Rasch analyses of only length measure

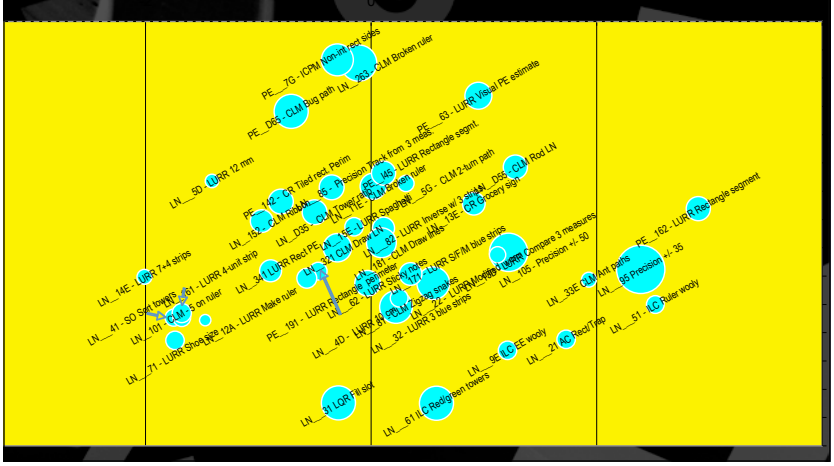


Rasch Analyses

- Fit Rasch model; unidimensional trait, fundamental measurement
- Qualitative supported 'action on objects' of theory
- Alterations: Combined Relator and Repeater (previously separate)
- Path and length measurer—still investigating

Rasch Analyses #3

Length and Perimeter



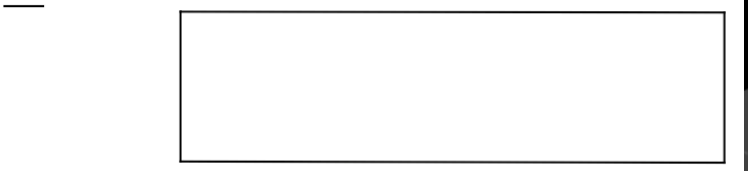
Confirmation

- *Direct Compare*, lowest
- *Indirect Compare* (with objects)
- *End-to-End* and *Length Unit Relater and Repeater* (we need more *E-to-E*)
- *Length Measurer* and *Conceptual Ruler*
- *LM*/perimeter items are mostly more difficult (some *CR* mixed), but single latent trait.

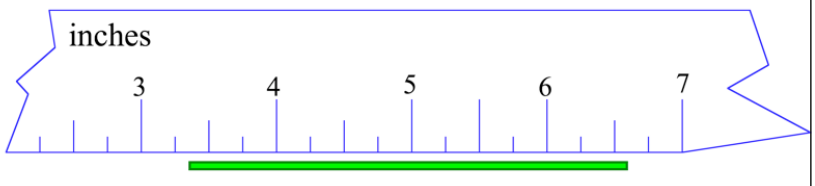
Highest Conceptual Ruler

ITEM 4. [6] Estimate the number of tape pieces needed to wrap around the rectangle. _____

(Rectangle is on overhead and not on the actual sheet)



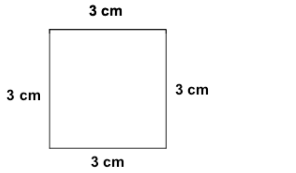
Highest *Length Measurer* (not perimeter)



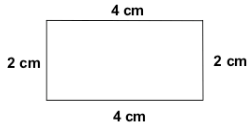
Exactly **how many inches** long is this green rod?

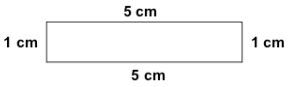
Perimeter

12 cm long string



- Draw one more
- Note: both last 2 *fractions*

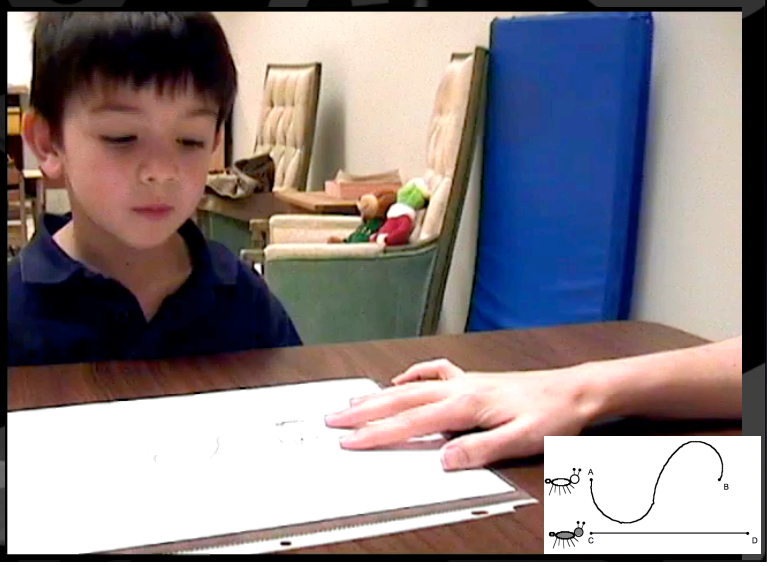




Direct Length Compare

- Ant Paths—original thought to be measurement, but difficulty -1

Ant Paths



Direct Length Compare

- Direct measurement tasks were -3 and less
- So, postulate new development, Direct Compare (Mental)

Conceptual Ruler?

Point to the line segment on the child's left in the image on the screen.

This is an inch long.

Point to the rectangle.

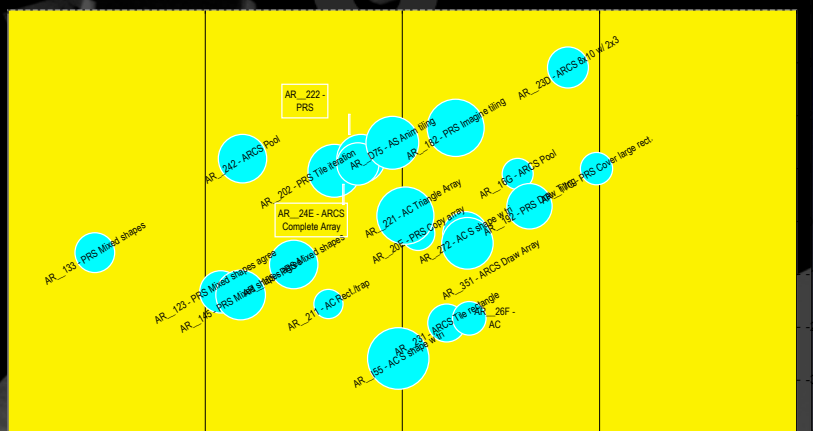
Estimate how many inches would wrap around this rectangle.

If the child asks, "What does 'estimate' mean?" or appears confused, say, **An estimate is a good gue**

Correct: **Estimate from 15 to 21**

	Code 2 0 = incorrect 1 = correct (any estimate from 15 to 21) 9 = no response

Area



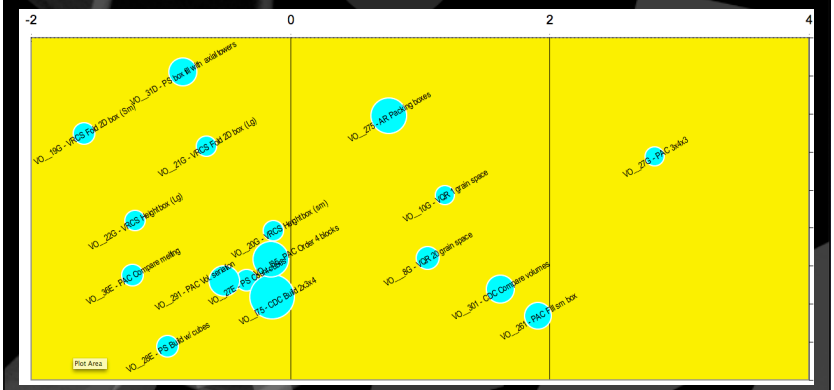
Area Summary

- Area items are measuring what we intended them to measure.
- None of the area items assessed levels lower than *Area Unit Relater and Repeater* (AURR).
- Children progress from *AURR* to *Area Row and Column Structurer*.

Area Summary

- Area Conserver (AC) isn't a level in and of itself, and instead may develop alongside or concurrent with different levels.
- Spatial structuring doesn't tell the whole story. More research needs to be done to determine how spatial structuring develops concurrent with Length and Volume understandings.

Volume



Volume Items' Levels

Copying a Building

Easy—e.g., Direct Volume Compare? Or parallel development.

How many cubes?

With no internal cubes students at the *Primitive Array Counter* level can deal with this task.

Grains of Sand

These items will be left in the analysis, however, we will use the results of the Rasch analysis to suggest level placements.

Overall story:

1. most items fell within acceptable infit/outfit parameters
2. A few items (See above notes) fell out differently than expected; these were analyzed individually and their levels reconsidered
3. We need to give further thought to how students move from a concrete, but intuitive understanding of volume to a more mental–conceptual understanding of volume.
4. There are still gaps in the item

Rasch

- Strengths
  - Fundamental measurement
- Weaknesses
  - One path or multiple
  - Profiles -- 2 students same IRT score, different knowledge
  - Many items

Q-Matrix and RSM

- Classify these students according to profiles of attribute mastery
- Transforms unobservable attributes involved in test items into observable attribute mastery probabilities that are defined as the probability of using each attribute correctly to get the correct answer for given problems.
- RSM converts students' item response patterns into the attribute mastery probabilities.

Q-Matrix and RSM

- Johnny's diagnostic report will be:

Scale Score of 500

Bring material together from 2 passages	Excell.	95 %-tile
Processing grammatically complex texts	Good	80 %-tile
Understand main idea when not explicitly stated	Good	75 %-tile