Elementary Mathematical Writing Task Force Recommendations: Implications for Research and Classroom Implementation

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Problem

"Writing is another important component of the discourse" (NCTM, 1991, p. 34).

- Enhance learning
  (Borasi & Rose, 1989; Rothstein, A., Rothstein, E., & Lauber, 2003; The National Commission on Writing, 2003)
- Become active learners (Kasparek, 1996)
- Foster deeper understanding
  (Emig, 1977, Imscher, 1979, Odell, 1980, and Vygotsky, 1962, as cited in Kasparek, 1996; Rothstein, A., Rothstein, E., & Lauber, 2003)

mathematical WRITING

What is "mathematical writing"?

Students should:

- Use "written communication" (NCTM, 2014, p. 29)
- "Construct viable arguments and critique the reasoning of others" by reading (NGA & CCSSO, 2010, p. 6)
- "Justify and explain ideas in order to make their reasoning clear" (National Research Council, 2001, p. 130)
- "Express themselves increasingly clearly and coherently" (NCTM, 2000, p. 62)

mathematical WRITING

How We Define "Writing"

Must include:

- Words, phrases, and/or sentences

  *May not use correct writing conventions

Might include:

- "Mathematics is so often conveyed in symbols" (NCTM, 2000, p. 60).
- Other representations, such as drawings, tables, and graphs (NCTM, 2000)

mathematical WRITING

Mathematical writing?

How is mathematical writing discussed in the literature?

- Sort these types of writing into categories
- Record your ideas on chart paper
- Be prepared to share

Writing in Math Class Sorting Activity

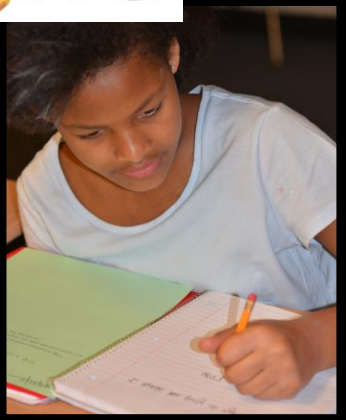
- These are types of mathematical writing described in the literature on mathematical writing (primary and secondary grades).
- Please sort these types of writing into a way you think makes sense.
- Record your group's categories on chart paper, and be prepared to share.

1. Journal writing	2. Stories
3. Creative writing	4. Pose questions
5. Mathematical concepts	6. Composing with key words
7. Reflection	8. Metaphors
9. Word problems	10. Free writing
11. Problem solving	12. Observations
13. Diary writing	14. Process
15. Paraphrasing word problems	16. Learning logs
17. Definitions	18. Elaboration
19. Letter writing	20. Events in history
21. Proof	22. Multiple entry logs
23. Compare and contrast	24. Argumentation
25. Who's who in math	26. Applied use of language
27. Summarizing	28. Predictions
29. Expository	30. Poetry

mathematical WRITING

Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations

mathematical WRITING



Goals

1. Consider various purposes for which students might be asked to write in their mathematics class;

2. Reach a consensus about the types of elementary mathematical writing that are reflective of these multiple purposes and recommend the types that leverage students' mathematical learning; and

3. Account for perspectives from multiple stakeholders, evidence of students' potential for writing productively in mathematics, and multiple sets of curriculum standards.

mathematical WRITING

Task Force Members

Tutita Casa, Janine Firmender, June Cahill, Fabiana Cardetti, Jeffrey Choppin, Jeremy Cohen, Shelbi Cole, Madelyn Colonnese, Juanita Copley, Michael DiCicco , Jack Dieckmann, Jennifer Dorl, M. Katherine Gavin, Michael Hebert, Karen Karp, Erika LaBella, Judit Moschkovich, Karen Moylan, Natalie Olinghouse, Sarah Powell, Erin Price, David Pugalee, Betsy Rupp Fulwiler, Linda Sheffield, Ryley Zawodniak

mathematics and writing education, mathematics, English language learners; regular, special, and gifted education; assessment and curriculum development

Elementary mathematical writing?

secondary and beyond

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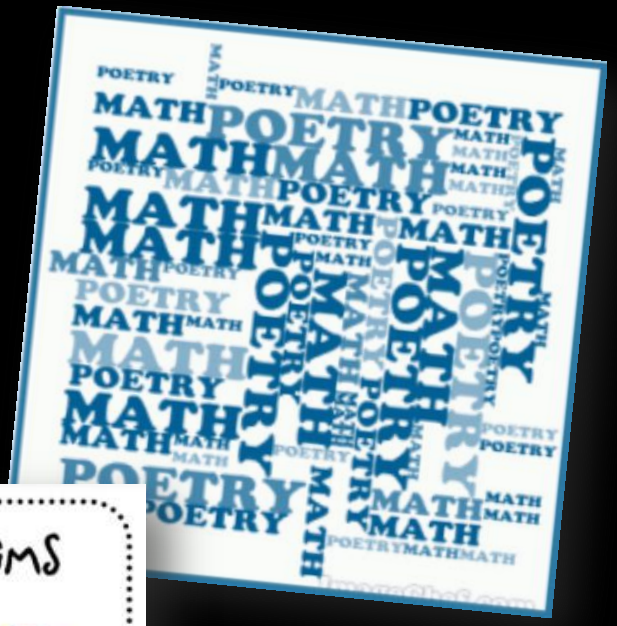
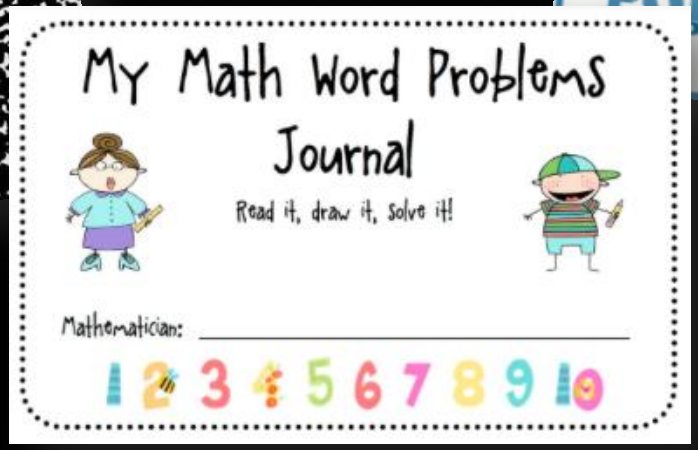
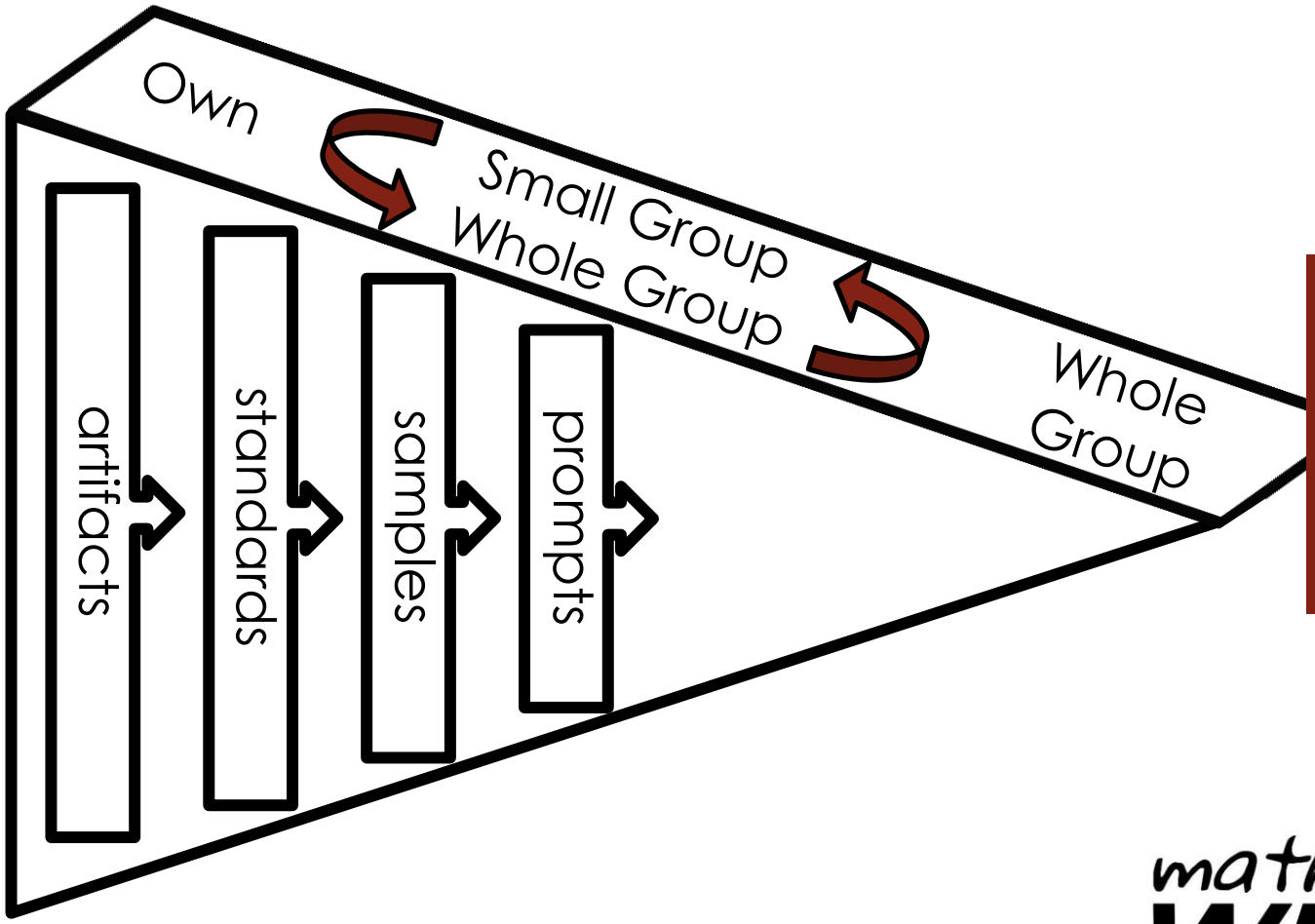




Leverage mathematical learning?
Encourage reasoning?

Overview of Our Process



Types of and purposes for mathematical writing

mathematical WRITING

Group Work Process

Writing that Takes Place in Math Class

mathematical WRITING

Writing that Takes Place in Math Class

Writing about math	Mathematical writing

mathematical WRITING

Writing that Takes Place in Math Class

Writing about math

Forefronts literacy

Can "substitute" other content area

Mathematical writing

mathematical WRITING

Writing that Takes Place in Math Class

Writing about math	Mathematical writing
Forefronts literacy	Furthers learning of mathematics
Can "substitute" other content area	Distinct to the mathematics discipline

mathematical WRITING

Writing that Takes Place in Math Class

Writing about math

Mathematical writing

Depends:

mathematical WRITING

Elementary mathematical WRITING Task Force

Considerations

- All elementary students should write mathematically, with any necessary accommodations
- Recommendations start in kindergarten
- Writing develops across multiple continua, including within and across years
- The audience influences students' mathematical writing
- Mathematical writing may take multiple forms

mathematical WRITING

Type	Purpose	Form
Method of conveying thinking in written form	The intention or desired result of the writing	Presentation of the writing
Persuasive	Convince someone of your position	Letter

mathematical WRITING

To Reason and Communicate Mathematically

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Exploratory

- To personally make sense of a problem, situation, or one's own ideas

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- To document original ideas, problems, and/or solutions
- To convey fluency and flexibility in thinking
- To elaborate on ideas

Student Writing Samples

- Review the 10 samples
- Decide what type of writing each depicts
- Be prepared to share your reasons for categorizing each sample

mathematical WRITING

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To Reason and Communicate Mathematically

Exploratory

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1. Sasha's challenge card read "Multiply to include all the members in your family." She has 4 people in her family and her trip was 6,997. So she rounded the number to 7,000 and multiplied by 4 and got 28,000. She thought, "Now, all I have to do is subtract 3 to get my answer."

   a) Do you agree or disagree with her reasoning? Why?

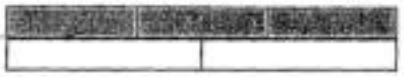
   b) Find the total mileage.

Disagree, Why subtract 3?
Not have rounded

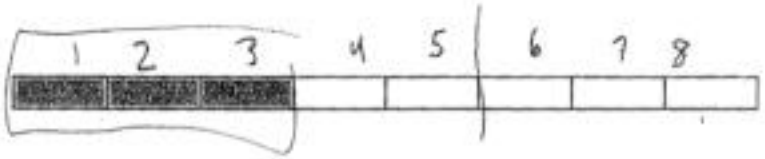
Sample 1

Mr. Pack asks his students to draw the fraction $\frac{3}{5}$. Here are the drawings of four students.

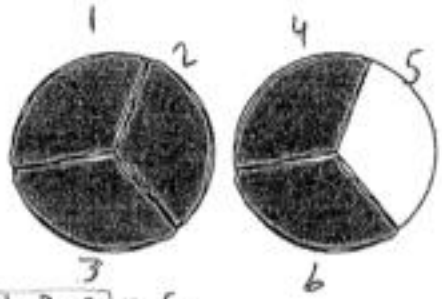
Alex



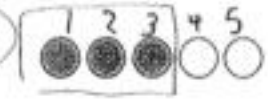
Bo



Cole



Deb



Some students made mistakes in their drawings. First, write down the names of all the students who made mistakes. Then, choose one student you would like to help. Write about their mistake and how you would help them solve the problem correctly.

Alex, Bo and Cole all made mistakes.

Alex

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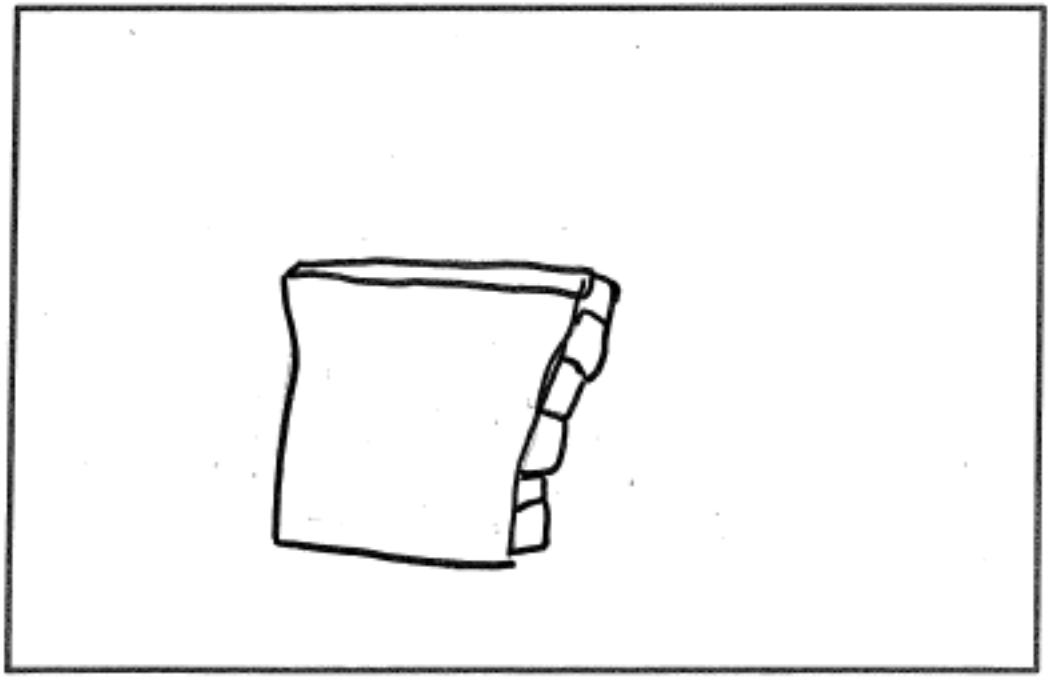
Sample 5

How Many Ways Can You Measure a Cup?

Using the measuring tools show a teacher different ways you can measure the paper cup. Then choose 1 way. Draw and tell about your findings.



It is sixtl ox to the top
blocks
of the KUP
the KUP is 5 9/ bL ox tall

The difference between a cube is that a square has 4 sides and a cube has 12 edges. And that you can put something in a cube and nothing in a square. A cube has 6 faces and a square has 1 face. A square is 2D and a cube is 3D. A square is flat and a cube is not. A cube is held in your hands and a square can be pinched.

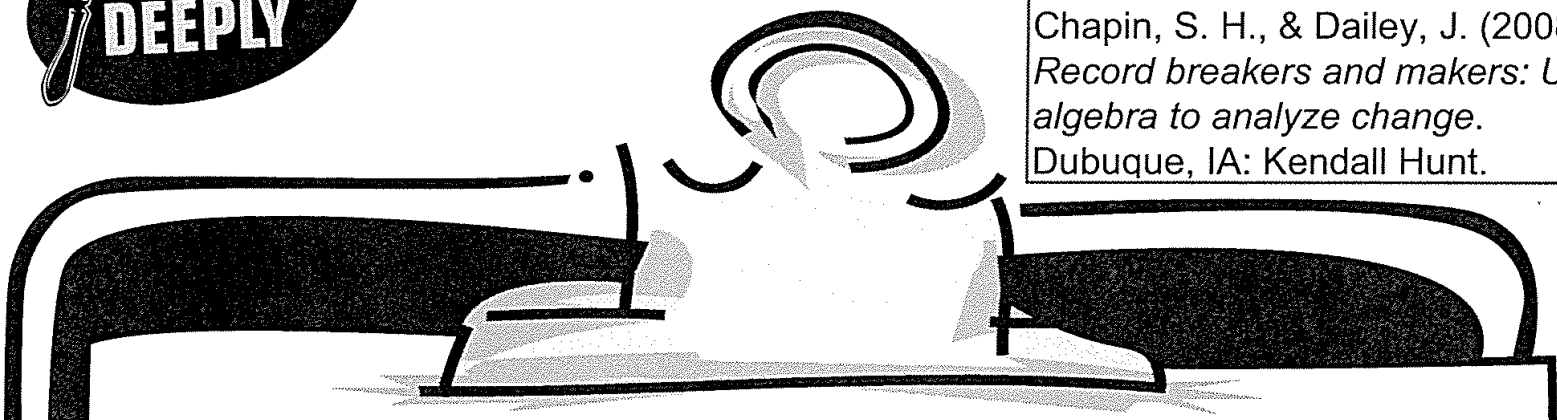
Sample 10

THINK DEEPLY

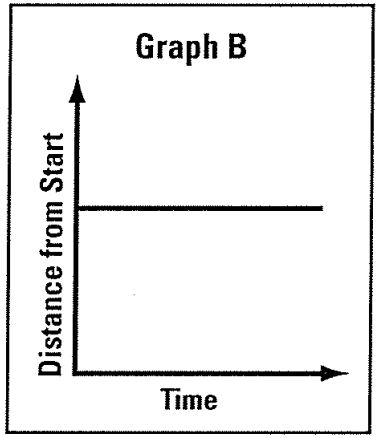
Student Mathematician: Date:

Mathematician's Journal

Gavin, M. K., Sheffield, L. J., Chapin, S. H., & Dailey, J. (2008). *Record breakers and makers: Usin algebra to analyze change.* Dubuque, IA: Kendall Hunt.



2. **This graph represents one team's results of the Orange Nose Push experiment. Explain what the horizontal line is telling you about the relationship between the variables.**



Graph B shows a Orange Pusher starting some distance from the start and staying at this some distance as time moves on. The variables are time and distance from the start. The distance stays the same while time increases.

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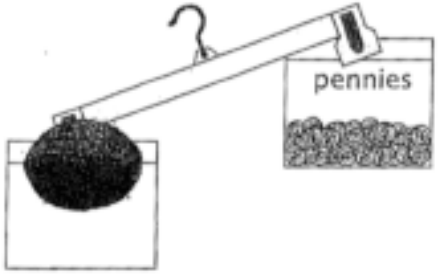
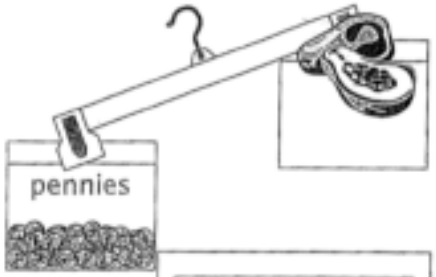
Sample 9

THINK DEEPLY

Play Weight Challenge!





Put our bags in order.

coconut pennies papaya

heaviest lightest

Word Wall
heavier than
lighter than
the same as

bag is heavier than the

bag.

How do you know? because the papaya bag is the lightest of all the rest. The pennies are heavier the the papaya and lightest than the coconut!

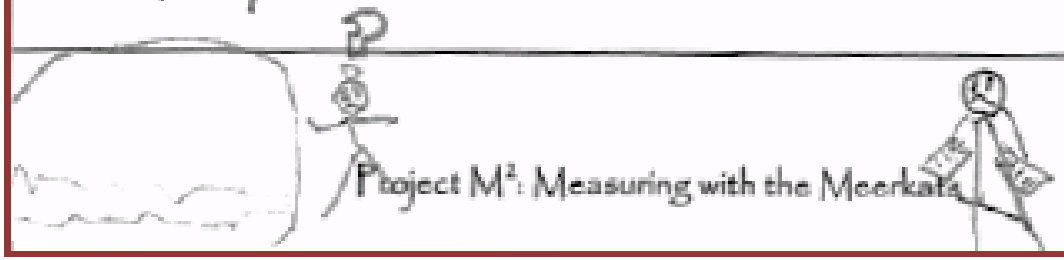
Sample 6

a. Which measuring tool was best to measure the circumference of the eggs?

the best tool to use was the tape measure

b. Why?

I think that because it can bend and it has numbers on it. The ruler has numbers on it but it can't bend. The pipe cleaner can bend but it doesn't have numbers on it.



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Sample 3

Awesome Algebra

The Name Game

Is it easier to find the 137th letter for Sara, Alexander, or Christiann? Why?

Answer: Christiann because her name has 10 letters. The 140th is n, 139 is n, 138 is a, so 137 is i.

Sample 7

People	Handshakes
1	25
2	24
3	23
4	22
5	21
6	20
7	19
8	18
9	17
10	16
11	15
12	14
13	13
14	12
15	11
16	10
17	9
18	8
19	7
20	6
21	5
22	4
23	3

People	Handshakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66
13	78
14	91
15	105
16	120
17	136
18	153
19	171
20	190
21	210
22	231
23	253

I have made up a formula that may help to figure this problem. The formula is shown below:

$\#p \times \#(P-1) \div N$

$N \div 2 =$ your answer.

I know this doesn't really make sense, so I'll explain myself a little more. The "P" stand for people, and the "N" stands for the number you come up with. Here is an example:

(Take the number 76.)

$76(76-1=75)$

$75 \times 76 = 5,700$

$5,700 \div 2 = 2,850$

That would be your answer! All of my charts came out to be 325. Let's see what happens we use the formula:

$26 \times (26-1)\ 25 = 650$

$650 \div 2 = 325$ We got the same answer!

Lack of Guidance

- Reviewed over 1,900 prompts
- Grade 3 student books from 9 common curriculum series (e.g., *Everyday Math*)
- Expectations re: what to write about and how much were unclear
- Frequency 59 (*Saxon*) to 486 (*My Math*)
- 36.2% explain what; 27.4% explain why
  - "What" procedural 42.7%, conceptual 17.2%
  - "Why" procedural: 30.7%, conceptual 27.3%

Teaching Considerations

- What do teachers need to attend to when implementing:
  1. Exploratory writing?
  2. Informative/explanatory writing?
  3. Argumentative writing?
  4. Mathematically creative writing?
- How would you support preservice and inservice teachers' learning of these?

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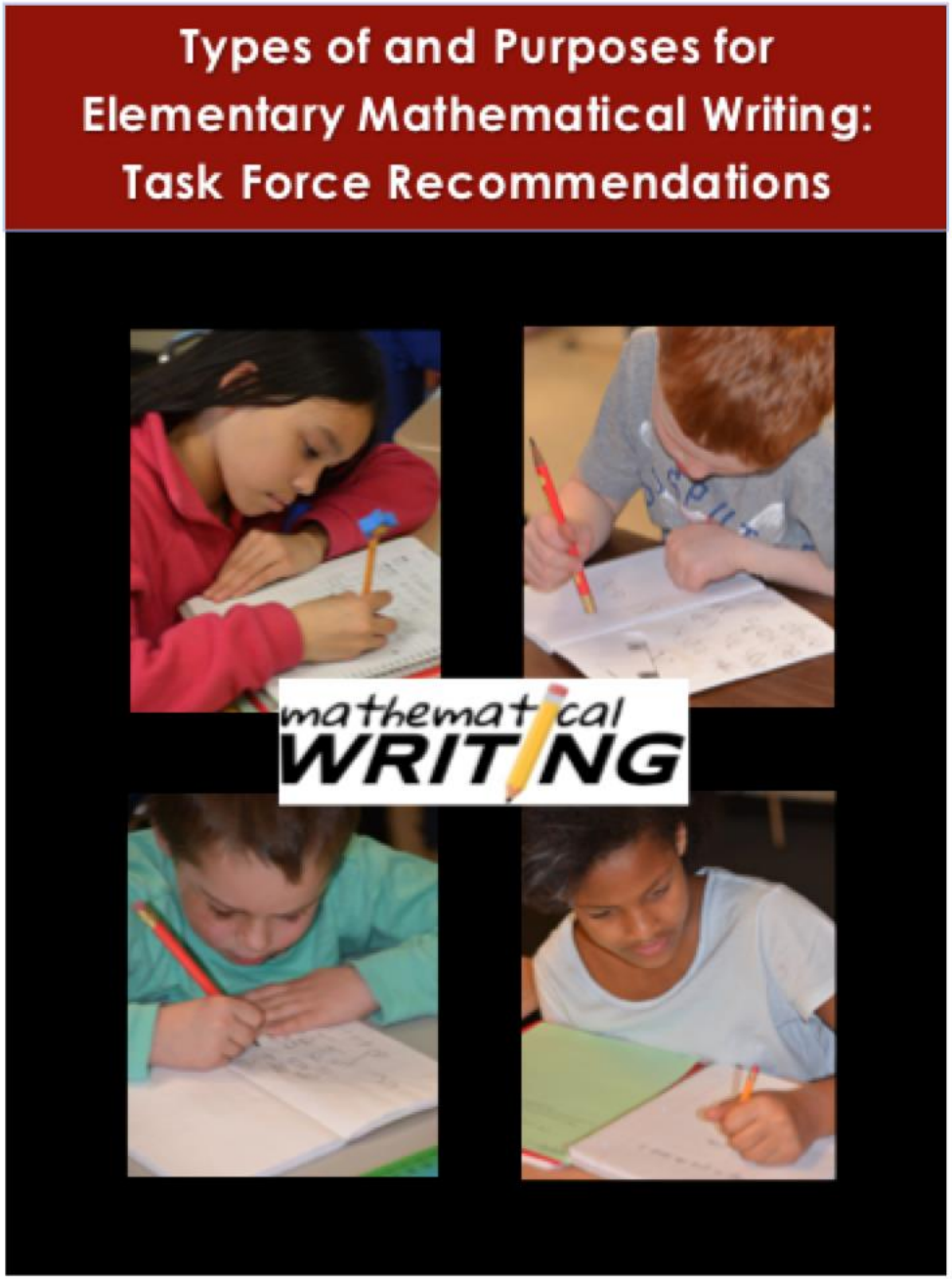

Download the task force recommendations

http://mathwriting.education.uconn.edu

Teaching Considerations

mathematical WRITING

Research Implications

- What implications are there for researchers studying:
  1. Exploratory writing?
  2. Informative/explanatory writing?
  3. Argumentative writing?
  4. Mathematically creative writing?
- What questions are important to ask?

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Research Implications

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Thank you!

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