Elementary Mathematical Writing
Task Force Recommendations: Implications for Research and Classroom Implementation

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“Writing is another important component of the discourse” (NCTM, 1991, p. 34).

- Enhance learning  
  (Borasi & Rose, 1989; Rothstein, A., Rothstein, E., & Lauber, 2003; The National Commission on Writing, 2003)

- Become active learners  (Kasparek, 1996)

- Foster deeper understanding  
What is “mathematical writing”?

Students should:

- Use “written communication” (NCTM, 2014, p. 29)

- “Construct viable arguments and critique the reasoning of others” by reading (NGA & CCSSO, 2010, p. 6)

- “Justify and explain ideas in order to make their reasoning clear” (National Research Council, 2001, p. 130)

- “Express themselves increasingly clearly and coherently” (NCTM, 2000, p. 62)
How We Define “Writing”

Must include:
- Words, phrases, and/or sentences
  * May not use correct writing conventions

Might include:
- “Mathematics is so often conveyed in symbols” (NCTM, 2000, p. 60).
- Other representations, such as drawings, tables, and graphs (NCTM, 2000)
Mathematical writing?
How is mathematical writing discussed in the literature?

- Sort these types of writing into categories
- Record your ideas on chart paper
- Be prepared to share
Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations
Goals

1. Consider various purposes for which students might be asked to write in their mathematics class;

2. Reach a consensus about the types of elementary mathematical writing that are reflective of these multiple purposes and recommend the types that leverage students’ mathematical learning; and

3. Account for perspectives from multiple stakeholders, evidence of students’ potential for writing productively in mathematics, and multiple sets of curriculum standards.
Task Force Members


mathematics and writing education, mathematics, English language learners; regular, special, and gifted education; assessment and curriculum development
Elementary mathematical writing?

secondary and beyond
Leverage mathematical learning?
Encourage reasoning?
Overview of Our Process

Types of and purposes for mathematical writing

Own

artifacts

Small Group

standards

Whole Group

samples

Whole Group

prompts

Types of and purposes for mathematical writing
Group Work Process

- Small groups revise working definitions
- Small groups share working definitions with the Task Force
- Small groups engage with writing items
- Members identify types of and purposes for mathematical writing
Writing that Takes Place in Math Class

Writing about math

Mathematical writing Forefronts literacy Furthers learning of mathematics

Can “substitute” other content area Distinct to the mathematics discipline
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Writing about math
Mathematical writing

Depends:
Elementary Mathematical Writing Task Force
Considerations

- All elementary students should write mathematically, with any necessary accommodations
- Recommendations start in kindergarten
- Writing develops across multiple continua, including within and across years
- The audience influences students’ mathematical writing
- Mathematical writing may take multiple forms
<table>
<thead>
<tr>
<th>Type</th>
<th>Purpose</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method of conveying thinking in written form</td>
<td>The intention or desired result of the writing</td>
<td>Presentation of the writing</td>
</tr>
<tr>
<td>Persuasive</td>
<td>Convince someone of your position</td>
<td>Letter</td>
</tr>
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</table>
Exploratory

- To personally make sense of a problem, situation, or one's own ideas
To Reason and Communicate Mathematically

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**Mathematically Creative**
- To document original ideas, problems, and/or solutions
- To convey fluency and flexibility in thinking
- To elaborate on ideas
Student Writing Samples

- Review the 10 samples
- Decide what type of writing each depicts
- Be prepared to share your reasons for categorizing each sample
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1. Sasha’s challenge card read “Multiply to include all the members in your family.” She has 4 people in her family and her trip was 6,997. So she rounded the number to 7,000 and multiplied by 4 and got 28,000. She thought, “Now, all I have to do is subtract 3 to get my answer.”

a) Do you agree or disagree with her reasoning? Why?

b) Find the total mileage.

Disagree. Why subtract 3?
Not have rounded
Mr. Pack asks his students to draw the fraction $\frac{3}{5}$. Here are the drawings of four students.

Alex

Bo

Cole

Deb

Some students made mistakes in their drawings. First, write down the names of all the students who made mistakes. Then, choose one student you would like to help. Write about their mistake and how you would help them solve the problem correctly.

Alex, Bo and Cole all made mistakes.

Alex
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How Many Ways Can You Measure a Cup?

Using the measuring tools show a teacher different ways you can measure the paper cup. Then choose 1 way. Draw and tell about your findings.

It is six blocks to the top of the cup.

The cup is six blocks tall.
The difference between a cube is that a square has 4 sides and a cube has 12 edges. And that you can put something in a cube and nothing in a square. A cube has 6 faces and a square has 1 face. A square is 2D and a cube is 3D. A square is flat and a cube is not. A cube is held in your hands and a square can be pinched.
2. This graph represents one team's results of the Orange Nose Push experiment. Explain what the horizontal line is telling you about the relationship between the variables.

**Graph B** shows a Orange Pusher starting some distance from the start and staying at this same distance as time increases. The variables are time and distance from the start. The distance stays the same while time increases.
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Play Weight Challenge!

Put our bags in order.

coconut  pennies  papaya

heaviest   lightest

The coconut bag is heavier than the papaya bag.

How do you know? Because the papaya bag is the lightest of all the rest. The pennies are heavier than the papaya and lightest than the coconut.
a. Which measuring tool was best to measure the circumference of the eggs?

The best tool to use was the tape measure.

b. Why?

I think that because it can bend and it has numbers on it. The ruler has numbers on it but it can't bend. The pipe cleaner can bend but it doesn't have numbers on it.
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Awesome Algebra
The Name Game

Is it easier to find the 137th letter for Sara, Alexander, or Christiana? Why?

Answer: Christiana because her name has 10 letters. The 140th is n, 139 is n, 138 is a, so 137 is i.
Sample 7

I have made up a formula that may help to figure this problem. The formula is shown below:

\[ N = (P-1) \times N \]

\[ N \div 2 = \text{your answer} \]

I know this doesn't really make sense so I'll explain myself a little clearer. The 'P' stands for people and the 'N' stands for the number you come up with. Here is an example:

Take the number 76:

76 \times 76 = 5,776

5,776 \div 2 = 2,888

That would be your answer. All of my charts came out to be 325. Let's see what happens we use the formula:

26 \times (26+1) = 660

660 \div 2 = 330

We got the same answer!
Lack of Guidance

- Reviewed over 1,900 prompts
- Grade 3 student books from 9 common curriculum series (e.g., Everyday Math)
- Expectations re: what to write about and how much were unclear
- Frequency 59 (Saxon) to 486 (My Math)
- 36.2% explain what; 27.4% explain why
  - “What” procedural 42.7%, conceptual 17.2%
  - “Why” procedural: 30.7%, conceptual 27.3%
Teaching Considerations

- What do teachers need to attend to when implementing:
  1. Exploratory writing?
  2. Informative/explanatory writing?
  3. Argumentative writing?
  4. Mathematically creative writing?

- How would you support preservice and inservice teachers’ learning of these?
Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations

Download the task force recommendations

http://mathwriting.education.uconn.edu
Teaching Considerations
Research Implications

- What implications are there for researchers studying:
  1. Exploratory writing?
  2. Informative/explanatory writing?
  3. Argumentative writing?
  4. Mathematically creative writing?
- What questions are important to ask?
Research Implications
Thank you!

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