Supporting children's problem solving

Teachers demonstrated four categories of reasoning when deciding how to respond to students.

By Victoria R. Jacobs and Randolph A. Philipp

"Help! I'm stuck!" How many times have you heard a student make a similar plea? What do you consider when deciding how to respond? What reasoning is most productive?

As part of a large research project, we explored these questions with 131 prospective and practicing teachers and found patterns in their reasoning. Consider how teachers and professional developers can use these patterns to better support children during problem solving.

The video

We began our project by sharing a video of a one-on-one conversation that took place in June between a teacher and Rex, a kindergartner. (See **fig. 1** for the three problems discussed. View the three-minute video at **www.nctm.org/tcm/**. We encourage you to view the video before reading the rest of this article, but if that is not possible at the moment, please read the video transcript in the **appendix** on p. 105 before continuing.) The video ends when the teacher poses the tadpole problem and Rex comments, "I don't even know that one. That's hard."

How might you respond to Rex, and why would you choose that response?

Four responses

After we showed this video, we posed the same question to the teachers in our study. When you read the four samples of written responses

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FIGURE 1

Three problems were posed to Rex.

1. Rex had thirteen cookies. He ate six of them. How many cookies does Rex have left?
2. Today is June 5, and your birthday is June 19. How many days away is your birthday?
3. Rex had fifteen tadpoles. He put three tadpoles in each jar. How many jars did Rex put tadpoles in?

below, think about how your own proposed response compares with theirs. In particular, if you had to choose the sample response most similar to yours, which would it be?

Response 1: I would help him draw a picture and guide him through the problem. I would ask him to draw fifteen dots or lines to represent the fifteen tadpoles. Then I would tell him that there will be three in each jar, so to represent each jar, he could circle tadpoles in groups of three. I would then ask him how many circles he has.

Another method I would guide him through would be to use the cubes that were on the table. I would ask him to count out fifteen cubes and then make them into sticks of three (stick them together). I would then ask him to count how many sticks he has.

Response 2: I might say something like, "Yes, that does seem a little bit harder than our last problems, but you're a smart boy. I'm sure if we work together, we could solve it." I'd agree with him that it's a difficult problem—to let him know I understand how he feels. I'd use positive reinforcement by telling him I think he's smart to boost his confidence. I'd offer to work on the problem with him because he obviously needs help. I believe after solving the problem together, Rex would feel very proud of himself.

Response 3: Rex really prefers to use his fingers as a tool to solve problems. In the first problem he used them to count down from thirteen, keeping track of when he'd counted down six times. In the second problem, he counted on from June 5 to June 19, but was thrown—ever so slightly—when his counting on continued beyond his ten fingers. Considering this, I think the third problem caused some difficulty because he couldn't represent fifteen tadpoles with his fingers. Also, since his other strategies involved counting on and counting back, he might think he could use that here.

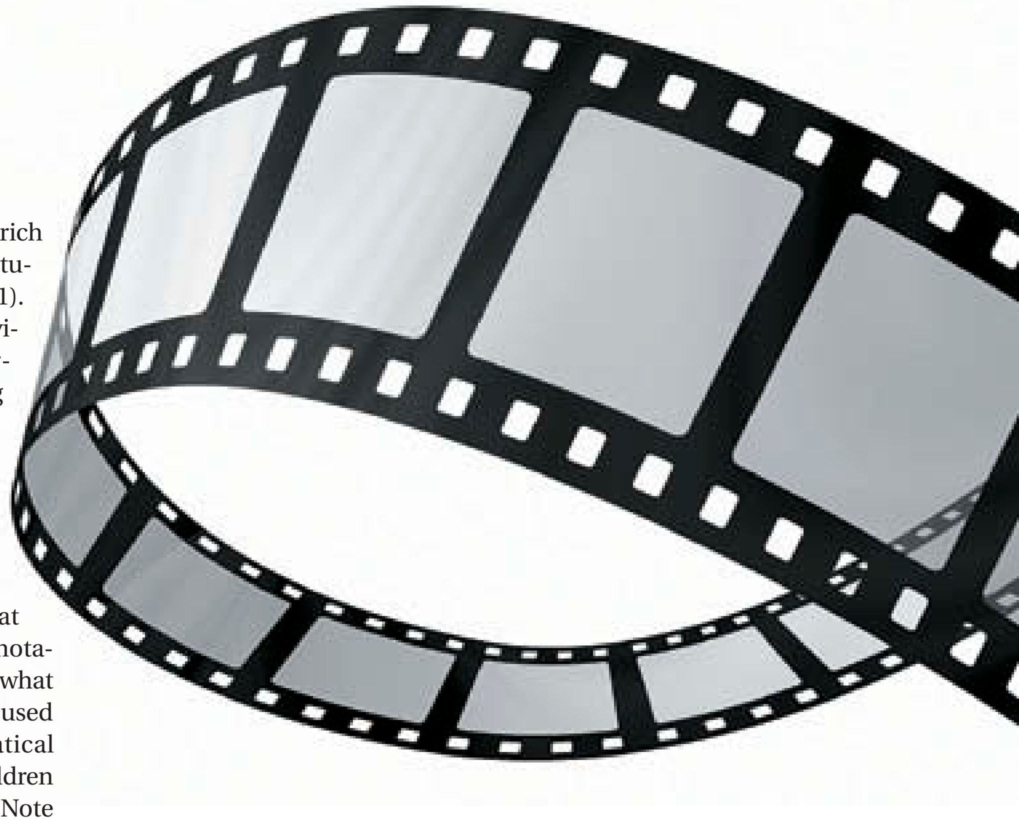
OK, the original question, what to do from here: I'd start by asking him why that problem was hard. Is it because of the language and context of tadpoles? Is it because he can't use a counting on or back strategy? Does he recognize that his *previous* counting strategies won't work? Where I'd go from there would really depend on his response. I'm going to assume that he understands what the problem is asking. I might adjust the numbers to sixteen and two to see if he'd skip count by twos up to sixteen and keep track on his fingers. If Rex explained that it was hard to use his fingers for this one, I might ask if there's another tool that would help him.

Response 4: I would ask him what he knows about the problem, or what the story tells us, and what we're trying to find out. Then I would have him start with what he knows and build from there. I would ask questions along the way as a guide to get him started. I think questioning is a way to guide students in the process of how to start and where to go next.

Children's mathematical thinking

The range of goals and teacher moves proposed in these responses highlights the inherent ambiguity in teaching—a teacher must always choose among multiple paths when supporting a child during problem solving. We were not expecting teachers to describe any particular path for working with Rex, and all four sample responses contain pieces to appreciate. However, because research has shown the power of paying attention to children's mathematical thinking, we were particularly interested in the role that children's mathematical thinking played in teachers' decision making: Did teachers use what they learned about Rex's mathematical thinking on the first two problems when deciding how to respond? Did their instructional suggestions leave space for Rex's future thinking?

Children often have ways of thinking about mathematics that differ from adult ways, and research has shown that instruction that builds on children's ways of thinking can lead to rich instructional environments and gains in student achievement (NCTM 2000; NRC 2001). However, creating these instructional environments has proven challenging, particularly because this vision of instructing requires that teachers keep children's mathematical thinking central when making in-the-moment decisions that occur hundreds of times a day. Specifically, to use children's mathematical thinking when deciding how to respond, teachers must not only detect children's ideas that are embedded in comments, questions, notations, and actions but also make sense of what they observe in meaningful ways. We focused on teachers' use of children's mathematical thinking in deciding how to respond to children who need support during problem solving. Note that equally challenging is the decision making required to extend children's mathematical thinking after they have successfully solved a problem (Jacobs and Ambrose 2008–2009).



MARCELA BARSSE

Rex's thinking

What did we learn about Rex's mathematical thinking? At first glance, we learned that this five-year-old successfully solved two problems by counting on his fingers before deciding that the third problem was too difficult. We wondered what else we could have learned. Research on children's mathematical thinking has shown that paying attention to the *details* of children's strategies matters because these details provide a window into children's understandings—information that teachers can use to decide their next instructional steps (Carpenter et al. 1999). By attending closely to the details of Rex's problem solving on the cookie and birthday problems, we could learn the following, for example, about his thinking:

- **Willingness** to try to solve both problems—When Rex had difficulty, he was willing to continue working. For instance, Rex immediately began engaging with both problems and, on the birthday problem, after declaring, "I can't figure that one out," he was willing and able to proceed after the teacher offered only minimal assistance. In short, Rex displayed a productive disposition (NRC 2001) toward solving problems.
- **Ability** to successfully solve two problem types—Rex answered a subtraction problem and a missing-addend problem.
- **Range** of counting strategies—Rex counted up on the birthday problem and counted down on the cookie problem.
- **Emerging** understanding of tens—On the birthday problem, he was able to think of ten as a group: After he had counted to June 15 and had ten fingers extended, he paused and said, "That's ten," before continuing his counting to June 19. He was then able to conserve ten in his head and count on to the answer of fourteen by recounting the four extended fingers.
- **Preference** for using his fingers as a tool—Although other problem-solving tools (e.g., cubes) were available, Rex chose to use his fingers on both problems that he solved.

The next steps

How might these details inform instructional next steps? The tadpole problem is a measurement-division problem in which the total number and size of each group is provided, but the number of groups is unknown. Research has shown that measurement-division problems are accessible to young children and not substantially more difficult than problems with the mathematical structures of the first two problems. Because Rex correctly solved the first two problems using counting strategies with his fingers, we can reasonably assume that he should be able to solve the tadpole problem with either a counting strategy or a less sophisticated strategy in which all the tadpoles would be represented and distributed into jars. If Rex chose this less sophisticated strategy, he might need a tool other than his fingers (e.g., cubes) so that he could represent all fifteen tadpoles and place them in groups of three (Carpenter et al. 1999).

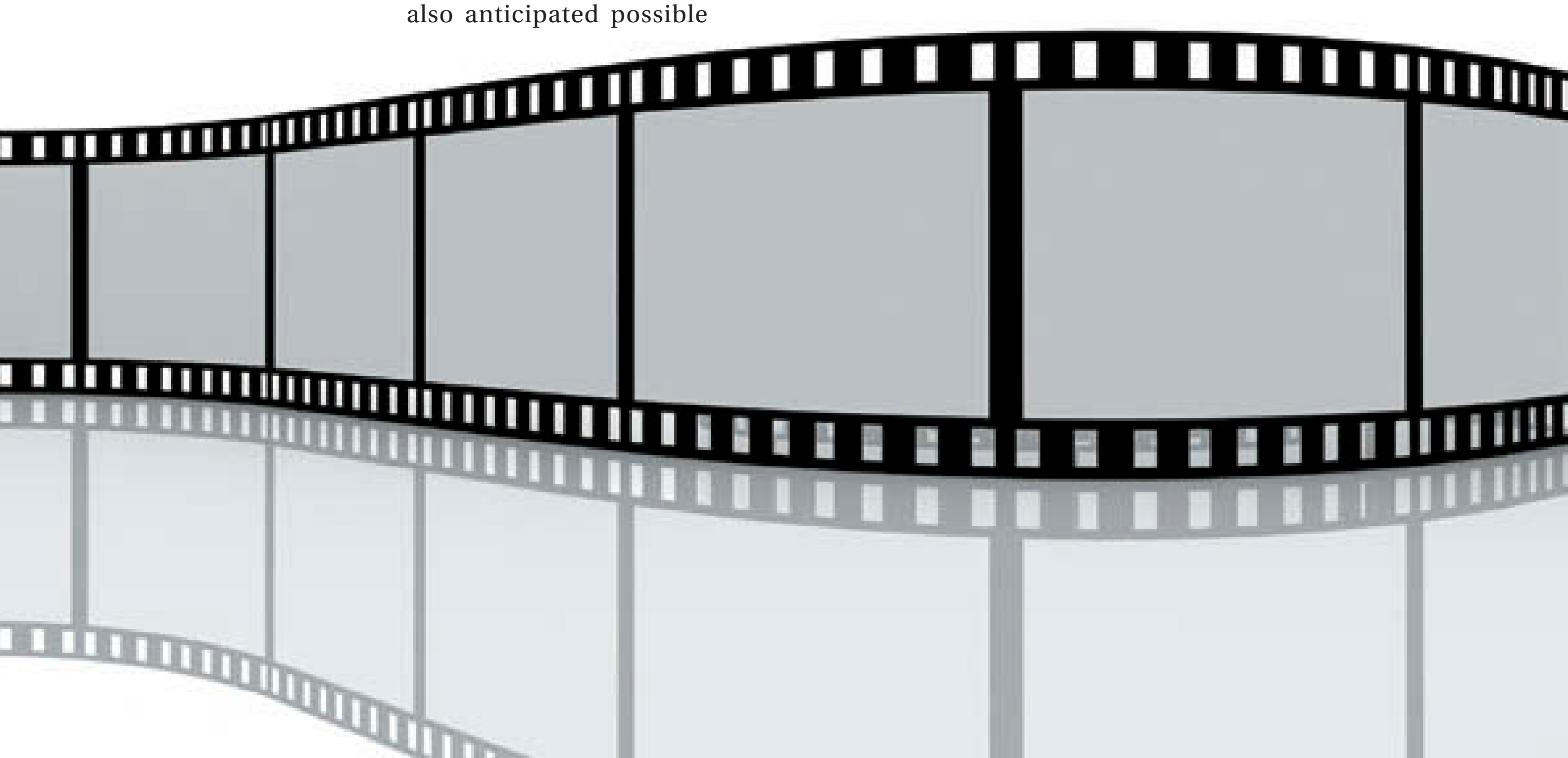
We recognize that attending to and reasoning about the details of Rex's mathematical thinking does not prescribe a specific response, nor do we believe that there is a single best response. However, we do believe that teachers can use the types of details described above to inform their instructional next steps so that they are likely to make the mathematics accessible to children and ensure that the children (not the teachers) do the mathematical thinking. Thus, when reading the teachers' responses, we looked for two characteristics: First, did the teacher attend to the details of Rex's mathematical thinking on the first two problems? Second, did the teacher's instructional suggestions build on Rex's thinking on the first two problems *and* leave space for Rex's future thinking?

A focus on Rex's thinking

We found that only one sample, response 3, focused on Rex's mathematical thinking. The teacher who gave this response not only considered what she had learned about Rex's mathematical thinking on the first two problems but also anticipated possible strategies for the tadpole problem—strategies that were consistent with Rex's existing strategies and the research on children's mathematical thinking.

In the first paragraph of her response, this teacher showed that she had carefully attended to how Rex had solved the first two problems. Details she highlighted included Rex's facility in and preference for using his fingers, his counting-up and counting-down strategies, and his emerging base-ten understanding. She then used her observation that Rex was thrown "ever so slightly" when the numbers went beyond ten in the birthday problem to hypothesize why Rex might be struggling with the tadpole problem ("he couldn't represent fifteen tadpoles with his fingers"). Note that the teacher's reasoning is not generic reasoning about a division problem but, instead, is particular to how she thinks Rex might engage with the tadpole problem on the basis of what she learned from his mathematical thinking on the previous two problems.

In the second paragraph, she focused on problem difficulty ("asking him why that problem was hard"), leaving space for Rex's thinking while considering connections to his past work ("Is it because he can't use a counting-on or [counting]-back strategy? Does he recognize that his *previous* counting strategies won't work?"). She then explicitly stated that her next steps "would really depend on his response," indicating that Rex's thinking would play a prominent role



in the proposed interaction. She acknowledged the importance of ensuring that Rex understood the problem and then continued by proposing a variety of possible supporting moves, all of which were consistent with what the video showed about Rex's mathematical thinking. For example, she suggested changing the problem numbers (to sixteen tadpoles distributed into jars of two tadpoles each) making the skip counting easier (twos instead of threes) to facilitate Rex's use of a familiar counting strategy while still enabling the use of a familiar tool (i.e., Rex could use each finger to represent two tadpoles and thus count by twos to sixteen without having to count beyond his two hands).

Although we found this teacher's suggestions to be interesting moves for supporting Rex, we recognize that there are many other helpful moves that a teacher could have made in response to Rex. Thus, the expertise in this teacher's response depends not on a specific move she suggested but instead depends on her consistent and extensive consideration of Rex's mathematical thinking on the previous problems as well as her attention to the importance of his future thinking in solving the tadpole problem.

The next section explores the other three sample responses, in which teachers did not focus on Rex's mathematical thinking.

Alternatives

We identified three categories of responses that did not focus on Rex's mathematical thinking. Each has important kernels that teachers can use as starting points for incorporating a focus on children's mathematical thinking into their decision making.

The best instructional next steps build on students' strategies and leave room for their future thinking.

Reasoning that teachers use

Distinguishing among these four categories of reasoning that teachers use when deciding how to support a student during problem solving can serve as a self-reflection tool for teachers and a reflection tool for professional developers:

1. The child's mathematical thinking
2. The teacher's mathematical thinking
3. The child's affect
4. General teaching moves

The teacher's thinking

Response 1 illustrates a focus on the *teacher's* mathematical thinking. This teacher suggested two specific and effective strategies for solving the tadpole problem, and these strategies are ones that children are likely to use. However, in this case, the strategies are the teacher's strategies, and whether any attention has been (or would be) paid to Rex's understandings of these strategies is unclear. In general, teachers who focused on their own mathematical thinking did not build on Rex's past thinking and, in particular, did not create space for his future thinking. Instead, they generally emphasized reaching a correct answer and suggested guiding Rex—step by step—through the solving of the tadpole problem.

Teachers with responses in this category did not build on children's mathematical thinking, but they *did* provide explicit details about strategies. As illustrated in responses in the next two categories, not all teachers provided such detail. Therefore, this attention to detail is a strength and can provide a starting point for teachers who want to learn to redirect their attention to the details of children's (rather than their own) mathematical thinking.

Rex's affect

Response 2 illustrates a focus on Rex's affect and lacks the specificity about strategies found in the previous two categories of responses. This teacher emphasized nurturing Rex's confidence and positive feelings but made no reference to his past or future mathematical thinking. Research has connected lack of confidence or dislike of mathematics with low achievement (Ma 1999), and thus these affective goals are important, but they are insufficient for offering instruction that builds on children's mathematical thinking. Teachers with responses in this category can work to augment this affective focus so that they also consider the details of children's mathematical thinking when deciding how to respond.

General teaching moves

Response 4 illustrates a focus on general teaching moves, again with a lack of specificity about strategies. For example, this teacher mentioned the importance of asking questions without articulating specific questions or even types of questions to be posed ("I would ask questions along the way as a guide to get him started"). A defining characteristic of this category was that the responses were general enough to be applied to any problem and any child—nothing in this teacher's suggestions was customized to the tadpole problem or Rex's thinking. Nonetheless, teachers with responses in this category often expressed an intention to use Rex's mathematical thinking ("I would have him start with what he knows and then build from there"). Research has shown how challenging attending to and building on children's thinking is. Thus having these general goals is an important starting point for teachers who can then work to incorporate the *details* of children's mathematical thinking into their decision making.

Final thoughts

In-the-moment decision making is a hidden, but critical, skill of teaching that needs to be discussed and developed. We want to underscore the complexity of this skill and the challenge in developing this expertise. To support teachers' growth, we identified four categories of reasoning that teachers use when deciding how to support a child during problem solving (see **sidebar** on p. 103). Although we recognize that these foci are not mutually exclusive, we think that distinguishing among them can serve as a self-reflection tool for teachers and a reflection tool for professional developers. Teachers may recognize themselves in *each* of these categories, perhaps in different situations or at different times in their own development. We hope that these categories can also indicate paths for future growth toward instruction in which children's mathematical thinking is central. To that end, we encourage teachers to enhance their own decision making about instructional next steps by continually asking themselves the following questions when a child needs support:

- **Which details** provide evidence for my conclusions about what I know of this particular child's strategies and understandings?
- **How can I build** on this child's existing strategies and understandings to give him or her an entry point to engage with the problem?
- **Have I left space** for this child's mathematical thinking? In what ways? Or did I solve the problem for the child?

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Victoria R. Jacobs, vjacobs@mail.sdsu.edu, and **Randolph A. Philipp**, **rphilipp@mail.sdsu.edu**, are mathematics educators at San Diego State University in California. They collaborate with teachers to explore children's mathematical thinking and how that thinking can inform instruction.

View the three-minute video by accessing this article at **www.nctm.org/tcm/**.

This is a transcript of the three-minute video available online at **www.nctm.org/tcm/** with the Jacobs and Philipp article. Rex, a kindergartner, is working individually with a teacher. Unifix® cubes, as well as paper and pencil, are available for his use.

Rex had thirteen cookies. He ate six of them. How many cookies does Rex have left?
[*Quietly counting back six from thirteen, putting up a finger with each count*] Seven.

And how did you figure that out, Rex?
I counted down with my fingers.

OK, tell me how you did that.
I went like, umm, thirteen, and then I went, twelve, eleven, ten, nine, eight, seven [*demonstrating how he counted back six from thirteen, putting up a finger with each count*].

Good. Now, is that how old you are? Are you seven?
No.

Well, how old are you?
Five.

You're five? And when is your birthday?
June 19.

It's coming up pretty soon, isn't it?
And then I'm going to be six.

And how many days away is your birthday? If today is June 5, how many days away is your birthday?
[*Quietly counting on his fingers, beyond ten, but after some counting (and re-counting), stopping*] I can't figure that one out.

Well, let's see. Today is June 5 and your birthday is June 19, so what do you think we could do to figure that out?
Use our fingers or something.

OK, how could we use our fingers? What should we do?
Like this: June 5, June 6—No [*raising one finger for June 6 and then hesitating*].

OK, June 6.
June 7 [*continuing to count, raising a second finger for June 7*].

OK, June 6, June 7 [mirroring what Rex has done, putting up two fingers—one for June 6 and one for June 7].
June 8, June 9, June 10, June 11, June 12, June 13, June 14, June 15 [*continuing to count up, putting up a finger with each count and stopping when all ten fingers are raised. The teacher continues to mirror what Rex has done by putting up her fingers with each of his counts*]. That's ten.

Uh huh.
June 16, June 17, June 18, June 19 [*continuing to count up, putting up a finger with each count until four fingers are raised*]. It must be [*pausing and quietly recounting the fingers above ten by counting on: eleven, twelve, thirteen, fourteen*] fourteen days away.

Wow! Now, Rex, do you know what guppies are?
No.

Do you know what goldfish are?
Yes.

Or would you rather do tadpoles?
Tadpoles!

OK. Rex had fifteen tadpoles. He put three tadpoles in each jar. How many jars did Rex put tadpoles in?
I don't even know that one; that's hard.