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Exploring iconic interpretation and mathematics teacher development through clinical simulations

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Abstract Field placements serve as the traditional 'clinical' experience for prospective mathematics teachers to immerse themselves in the mathematical challenges of students. This article reports data from a different type of learning experience, that of a clinical simulation with a standardized individual. We begin with a brief background on medical education's long-standing use of standardized patients, and the recent diffusion of clinical simulations to teacher and school leader preparation contexts. Then, we describe a single mathematics simulation and report data from prospective mathematics teachers' interactions with a standardized student on the issue of iconic interpretation. Findings highlight teachers' diagnostic, explanatory, mathematical, and instructional repertoires, as they guide a standardized student through two different graphing problems. Implications focus on the trends in teachers' instructional decisions, contextualized explanations, and the use of clinical simulations to enhance mathematics teacher development.

Keywords Clinical simulation · Iconic interpretation · Mathematics teacher education · Teacher development

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Introduction

Field placements serve as the traditional 'clinical' opportunity for prospective mathematics teachers to engage in the challenges presented in teaching mathematics to students. This clinical standard is not *standard*, though, and results in highly variable experiences. When in secondary classrooms, our prospective teachers (PTs) experience an array of challenges that as teacher educators, we cannot predict, control, or see frequent evidence of how our novice teachers navigate them. In teacher education, our use of varied field placements results in varied learning outcomes, leaving virtually no common denominator for novice teachers and teacher educators to work from.

This article outlines a different form of clinical preparation. We describe a clinical mathematical simulation, the resulting data, and the implications of that experience on mathematics teacher preparation. We begin by reviewing the use of situated learning in medical education, and the diffusion of medical simulations to subject-specific teacher education contexts. We outline our first clinical simulation for mathematics PTs and examine data on how each PT engaged in the same, shared problem of practice. We conclude with implications of clinical simulations on enhancing mathematical teacher preparation.

Theoretical and conceptual foundations

In 1963, Howard Barrows implemented the first medical simulation with his cohort of neurology medical residents at the University of Southern California (Barrows and Abrahmson 1964). The crux of Barrows' work was the standardized patient—a healthy individual carefully trained to present distinct symptoms and communicate questions/concerns to medical professionals in training (Barrows 1987, 2000). From that beginning, the use of standardized patients in medical education diffused regionally and nationally. Today, medical students participate in both formative and summative Observed Structured Clinical Examinations (OSCEs) with multiple standardized patients as part of their professional training (Coplan et al. 2008; Hauer et al. 2005).

Beginning in 2007, Dotger began designing and implementing clinical simulations across teacher and school leader preparation contexts. Early simulations targeted problems of practice that teachers and school leaders commonly encounter. In each simulation, a prospective teacher or leader would interact with a standardized individual—an actor deliberately scripted and carefully trained to simulate a student, paraprofessional, parent, community member, or district leader. Clinical simulations within teacher and school leader preparation contexts are anchored by cognitive development and situated cognition frameworks (Brown et al. 1989; Kohlberg 1969; Korthagen and Kessels 1999; Lave and Wenger 1991; Mead 1934; Piaget 1959; Putnam and Borko 2000; Vygotsky 1978; Wenger 1998). As summarized by Reiman and Peace (2002), these frameworks share (a) the assumption that knowledge/meaning is constructed by individuals through authentic experience, (b) the recognition of cognitive disequilibration, (c) an emphasis on disposition and skill development, as learners' experiences, interpretations, and professional reasoning synthesize over time (von Glasersfeld 1989), (d) the belief that genuine professional growth requires a supportive, yet progressively challenging environment, and (e) the importance of social interactions and the social negotiation of meaning (Lebow 1993) as individuals engage within complex professional environments.

To illustrate these theoretical assumptions, consider the *William Mills* simulation (Dotger 2013). This particular simulation illuminates specific socioeconomic contexts and the common worries of parents/guardians regarding post-secondary opportunities for their children. Reflecting the theoretical assumptions, the *Mills* simulation is intended to be a disequilibrating learning experience, as each PT individually faces Mr. Mills' questions about his son's academic progress, sees him express frustration with his son's performance, and is witness to Mr. Mills hopes and worries for his son's "opportunities in this world." The *Mills* simulation provides PTs opportunities to speak from data in conference with this concerned father, demonstrate empathic response patterns, and practice co-constructing plans of action and communication. In recognition that PTs develop through progressively challenging situations, the *Mills* simulation is one in a deliberately scaffolded series of clinical simulations that increase in challenge. The video-informed debriefings that follow each clinical simulation seek to both support the PT in his/her successes in the simulation, while also challenging the PT to identify a plan for improved action. Finally, all clinical simulations rely on standardized individuals and focus on data and dialogue, recognizing that teacher effectiveness hinges on one's ability to communicate clearly with students, parents, and colleagues across complex scholastic environments.

In 2009, Dotger and colleagues began shifting design attention to how subject-specific challenges come to life through standardized students in clinical simulations. Through a National Science Foundation Discovery Research K-12 (DR-K12) exploratory grant, seven secondary mathematics and science simulations were designed. Interviews with twenty-five senior secondary mathematics and science teachers yielded problems of practice that mathematics and science teachers commonly face in the induction years of teaching. For example, senior teachers suggested that simulations be designed around the secondary mathematics and science themes of graphs and the relationships of variables, genetics, factoring, probability, natural selection, and fractions. Building from those themes, we also consulted with mathematics and science teacher educators, who suggested examining these suggested themes through a lens of student misconceptions. From these building blocks, we constructed seven clinical simulations and began implementing them with small cohorts of secondary mathematics or science PTs.

This study examines how PTs engage in a mathematics simulation focused on iconic interpretation. When a student uses iconic interpretation in understanding a graph, the student is "interpreting a graph as a literal picture" (Monk 2003, p. 257). For example, a student using iconic interpretation might see a graph showing the distance over time that a vehicle traveled as the actual path that the vehicle traveled. This is a common difficulty that students have in understanding graphs. Researchers have found that "visuality is a key source of difficulties" when students try to make sense of graphs (p. 257). Being able to interpret and make sense of graphs is important mathematically. The Common Core State Standards for Mathematics (CCSSM) states that students should be able to "graph data, and search for regularity or trends" (National Governors Association Center for Best Practice 2010, p. 6). While graphing is mentioned in many parts of the CCSSM in relation to measurement and data, algebra, functions, and modeling, its central place in mathematics is illustrated by the above quote, emphasizing that students must make sense of problems and persevere in solving them.

Our study of the iconic interpretation simulation is further supported by mathematics educators' work in teacher noticing (Francisco and Maher 2011; Jacobs et al. 2010), the use of multimedia case studies (Masingila and Doerr 2002), and the use of video tools in mathematics education contexts (e.g., Borko et al. 2008; Santagata and Guarino 2011).

The manner and degree to which mathematics teachers—across the career span—recognize, attend to, and engage in teaching and teacher reflection in mathematics education has more recently been examined as teaching 'noticing' (Jacobs et al. 2010). Sherin et al. (2011) outline several methodologies for examining mathematics teacher noticing. These include studying what and how teachers notice by using videos of other teachers' instructional activities, encouraging teachers to engage in retrospective recall (with and without supporting video) of events in their own mathematics classrooms, and traditional researcher-inferred connections between the visible actions of the mathematics teachers and his/her likely efforts in noticing. Using unique teacher point-of-view cameras, Sherin et al. (2011) extend the work of teacher noticing methodologies through their investigation of high school mathematics teachers ($n = 13$) and the video data that each teacher recorded, marked at the moment of capture, and later reflected on. This study reinforced the range of teaching thinking, documenting that mathematics teachers noticed and reflected on general pedagogical actions, student thinking, and mathematics concepts. Their study highlights the teacher participants' rationales in recording specific video segments. For example, some of the segments captured by teachers document divergence between what the mathematics teacher expected and what he/she actually experienced. Sherin et al. (2011) connect this particular finding back to cognitive development assumptions, where illuminating and at times concerning moments (Reiman and Peace 2002) occur when there are apparent disconnects between what one expects and one actually experiences in the classroom.

Our study is further informed by scholarship on shared mathematical experiences and the use of video and multimedia tools to support mathematics teacher learning (Borko et al. 2008; Masingila and Doerr 2002). Through a problem-solving cycle (PSC), sixteen mathematics teachers enrolled in a series of workshops on algebraic thinking. Through the workshops, teachers engage with distinct algebraic problems and then later record themselves teaching this same algebraic problem to their respective student groups. The shared practice is the algebraic problem, while the recorded video provides the lens through which teachers view their own instructional approaches—and the approaches of their peers—to the same mathematical concept (Borko et al. 2008). In similar fashion, Masingila and Doerr (2002) report on the use of multimedia materials (i.e., video case studies, teacher journals, lesson plans, and student work) on a four-lesson sequence focused around ranking and weighting data. Participating mathematics PTs ($n = 9$) identified how one current issue within their own instructional practice connects to the representations of practice within the multimedia case study. Thus, the case study becomes the shared, common ground through which each PT further explores his/her specific mathematical instructional challenge(s).

Through a clinical simulation, PTs share a common thread by each engaging with the same questions, challenges, or issues (i.e., "triggers") presented by the standardized individual. In a well-designed simulation, the standardized individual is carefully trained to adhere to specific protocol (Barrows 1987). The teacher is not directed beyond the simple encouragement to utilize his/her professional training to appropriately engage in the simulation. Importantly, though, the teacher educator can tailor this learning experience, placing emphasis as desired on parent–teacher relationships, students' social/emotional needs, or discreet content-specific challenges (Dotger 2013). This helps account for what PTs otherwise experience through hit and miss field settings, where a few PTs have a unique learning opportunity with a parent or student, but the majority of PTs do not (Dotger 2013; Wilson et al. 2001). In both studies above (Borko et al. 2008; Masingila and Doerr 2002), the authors document a similar rationale, seeking a common shared mathematical practice to experience individually and analyze collectively. Clinical simulations build from these studies by presenting a shared problem of practice, that is, (a) live, engaging, and expects professional responses and (b) a limited, professional interaction that is bounded to focus both PT effort and reflection. In traditional field placements, PTs often struggle to untangle and make meaning from the myriad of challenges they face each day. Clinical simulations provide PTs with opportunities to carefully deconstruct professional knowledge, instructional strategies, and dispositional approaches. Multiple-angle video recordings of each PT's simulation provide a data-informed opportunity for the PT to initially self-assess what he/she said and did and then later collectively analyze those practices with peers who faced the same questions, issues, and challenges (Dotger 2013). Again, we see connections between simulations and past efforts to capture teacher point-of-view data (Sherin et al. 2011) or efforts to closely analyze teachers' instructional decision making through more distant third-person perspectives (Borko et al. 2008; Masingila and Doerr 2002).

As PTs engage with a standardized student in the iconic interpretation simulation, we asked the following questions:

- Who assumes responsibility for fostering the instructional dialogue?
- What content or instructional strategies do PT's employ (in simulation) and/or reflect (in debriefing)?
- When and how were instructional decisions implemented within the simulation?

The following methodological review outlines the steps we took to investigate these research questions.

Methodology

In this section, we discuss the design of the iconic interpretation simulation, followed by a description of how we trained the standardized students. Then, we discuss simulation implementation and data collection procedures, followed by our steps in analyzing the resulting video data.

Simulation design

The impetus for this simulation came from our interviews with experienced mathematics teachers, who suggested one simulation focus on basic graph interpretation and the relationships between variables. That repeated suggestion was coupled with our own interest in observing how PTs address iconic interpretation, where a person deciphers a graph by simply looking at the graph as a picture, instead of understanding the connection between the variables represented on the x-axis and y-axis.

To foster a live simulation, two sets of documents—a Teacher Interaction Protocol and a Standardized Individual Protocol—must be carefully designed. The Teacher Interaction Protocol (TIP) provides each mathematics PT with an appropriate amount of background and contextual information. Given 1 week before the actual simulation, the intent of this document is to provide each PT with enough information to realistically situate oneself within the simulated environment, but not so much information that it foretells, scripts, or overly directs the PT on what to do or say when interacting with the standardized student. For this simulation, the TIP was 1¼ pages in length. It indicated the PT—in simulation—is a novice first-year teacher at Pleasantville High School and responsible for five instructional periods, three of which were Algebra I. In these Algebra I sections, the teacher had recently introduced graphs, plotting coordinates on x-axis and y-axis, and interpreting graphed data. The TIP indicates that the teacher gave two homework problems. The first problem was a basic height versus time graph, with the data already plotted, and the prompt for the student to interpret the graphed data. The second homework problem was a basic distance versus time word problem, with the charge for the student to derive and graph the coordinates. The final portion of the TIP indicates that the teacher introduced the homework, and in doing so, noticed that one student (Marcia Taylor) seemed confused. As class concluded, the teacher spoke briefly to Maria, saying, “Stop by tomorrow morning if you want and we’ll work on this homework.” This dialogue and the broader TIP give the basic parameters for the upcoming simulation, where the standardized student, Marcia Taylor, will stop by to consult with each teacher (i.e., each PT) on the homework.

The second set of documents is the Standardized Individual Protocol (SIP), which serves as a training guide for the standardized students and differs dramatically from the TIP given to PTs. While the TIP does not script or direct PTs on what to say or do in the simulation, each standardized student is carefully trained to adhere with high fidelity to the details presented in the SIP. Beginning with extensive character-building information, the SIP describes Marcia Taylor as a ninth grade student at Pleasantville High School, who is well supported by her parents, heavily involved in junior varsity basketball, and working very hard to maintain a general “B” average across her seven high school classes. The purpose of the broad explanation of the *Marcia Taylor* character is to allow each actor to ‘get in character’ so she may realistically and appropriately respond to a wide variety of questions the PTs may ask during the simulations.

The second part of the SIP focuses on the verbal and non-verbal triggers that each standardized student will issue during the simulation. A *trigger* is a question, statement, concern, or response spoken by the standardized student to the PT. Triggers may include non-verbal mannerisms as well, such as sighs, defensive body language and confused facial expressions. For this simulation, the triggers are structured in conjunction with the two homework problems and broadly summarized here:

A. An introductory “I tried the homework, but I don’t know if I got the problems right.”
B. A display of the first homework problem, and a simple “Is this right?”
C. If teacher notes interpretation is problematic, then follow-up questions:
    a. I’m confused, what did I do wrong?
    b. But I thought that when we interpret a graph, we are supposed to interpret what it looks like!
D. Transition to second homework problem, with trigger, “This is my graph. How did I do?”
    a. Student response if teacher asks prompting/guiding questions.
E. Final trigger asking for general suggestions for interpreting graphs.

Standardized individual training

At the request of the first author, the Director of SUNY Upstate Medical University’s (UMU) Clinical Skills Center recruited three actors from local university and community theater organizations who were over the age of 18, but could still authentically present as the 15-year-old high school student, Marcia Taylor. On a dedicated morning, the research team and the Director of the Clinical Skills Center gathered with these actors to train them to become Marcia Taylor during a simulation with each PT. The SIP grounded the training session for this group of standardized individuals. The first author served as the primary facilitator in this 1.5-h training session, guiding the actors through the four-page SIP. Two-thirds of the training session was dedicated to the triggers for this simulation, accompanied by description and explanation by the second author on how the actors should reference the attempted homework problems. During this portion of the training, the authors collectively highlighted for the actors the misconceptions and errors that Marcia has made, noting the mistakes in logic that should be reflected as they enact her in simulation.

Implementation and data collection

On a designated morning, the mathematics PTs gathered at UMU's Clinical Skills Center. These PTs ($n = 8$) were enrolled in a methods course and were participating in this simulation as a non-evaluative course requirement. There were six female students and two male students. Two of the female students were graduate students, and the remainder were undergraduate students. Upon arrival, the PTs were provided a 10-min orientation to the Clinical Skills Center, a facility of 22 medical simulation rooms, modeled to look and function exactly as a general physician's examination room. The only difference between these simulation rooms and a common medical exam room is the presence of two multi-angle cameras and two microphones mounted in the ceiling. These recording technologies feed audio and video data directly to the Clinical Skills Center's control room and servers, resulting in a basic video file that serves as a performance assessment tool for the PTs and reporting researchers.

After orientation, three simulations were run simultaneously. Outside of each simulation room is a computer. Before beginning their simulations, PTs logged into their respective computers and responded to three pre-simulation questions. These questions inquired as to the PTs' goals/objectives, expectations, and any questions/concerns. After the PTs submitted their responses, a UMU staff member in the control room turned on the cameras and microphones in each simulation room. Each PT entered his/her simulation room, where a table and two chairs had been placed. On the table, a small dry-erase white board and markers were present. PTs were given approximately 30 s before a knock at the door signaled that Marcia Taylor had arrived for help with her math homework. From that point, the PTs were entirely responsible for what occurred as they engaged with Marcia Taylor. As PTs concluded their respective interactions, they exited the simulation rooms and turned off their cameras by logging out of their computers. Of note, while the PTs engaged in their simulations, the second author viewed, from the monitor room, live portions of each PT's simulation.

Each PT was escorted to a computer laboratory to watch video of his/her respective interaction with Marcia. After reviewing their data, PTs had approximately 15 min of unstructured time, but were specifically asked not to discuss the simulation with each other until the debriefing session. Once all had completed the simulation and data review processes, the second author guided the PTs through a semi-structured debriefing. Focusing on each homework problem that Marcia presented, the researcher asked the cohort to describe and reflect on their approaches and strategies toward Marcia's questions and misconceptions. During the debrief, a whiteboard and markers were used to recreate mathematical approaches and examples given earlier in simulation.

Data analysis

This study resulted in a significant amount of qualitative data, including PTs' pre-simulation responses, videos of their interactions with Marcia Taylor, and the whole-group debrief session. Our analysis of these data used the primary code constructs of a PTs' diagnoses, explanations, mathematics-specific, and general teaching repertoires (Clermont et al. 1993; Magnusson et al. 1999). To begin, the third author transcribed the PTs' individual simulation videos and the whole-group debriefing video. Then, the authors independently open coded three of the simulation transcripts. This broad listing of sub-codes was then categorized under the appropriate primary codes. For example, for the first homework problem, Marcia provides a written interpretation of the graph. We began with the *diagnosis* code construct, knowing that we would code whether or not each PT recognized Marcia's iconic interpretation. There were two sub-codes under the *diagnosis* construct, applied when each PT either agreed with Marcia's written interpretation or, alternatively, recognized the error in her interpretation. As a second example, though, consider the more complex *explanation* code construct. As we initially open coded a subset of transcripts, six different explanations emerged as PTs worked with Marcia on the first homework problem. PTs provided explanations that focused on height, point of origin, and alternative interpretations and were 'within the context' of the explanation provided by Marcia. Thus, our *explanation* code construct lists all different approaches PTs used when explaining Problem 1 to Marcia. Table 1 shows the broader simulation code list.

We applied the complete list of primary and secondary codes to the three predominant triggers in this simulation: where Marcia presents her attempt at the first homework problem; she questions her attempt on the second homework problem; and concludes by asking for general suggestions when interpreting graphs. After each author independently coded all transcripts, he/she then constructed a data summary for each PT. These summaries provided the research team with a format to examine PTs' responses across trigger, instead of looking at a single PT's responses to all the triggers. Data are reported below in consideration of each trigger.

Findings

After the knock at the door, Marcia entered and immediately issued her first trigger, indicating she's unsure of how she did on her homework. As expected, each PT welcomed Marcia into the room with greetings similar to, "Okay, come in" or "No problem." Each "Marcia" sat down and presented her homework. The first significant trigger focuses on Marcia's presentation of her first homework problem, with the question, "Is it right?" Figure 1 shows Problem 1, and Marcia's written interpretation.

Recall that simulation data were coded across four primary constructs: *diagnosis, explanation, mathematics repertoire*, and *teaching/instructional repertoire*. In consideration of this first homework problem, we present the data associated with PTs' diagnoses, explanations, and mathematical repertoires. More often, a PT's diagnosis of Marcia's interpretation was closely associated with his/her follow-up explanation, and both included mathematical terms, strategies, and representations. Thus, for clarity, we present those data in association with each other.

Table 1 Codes for simulation triggers

Trigger #1

[T1D] Diagnosis

- T1D1: Agrees with student's interpretation of graph and does not indicate or prompt student to think of potential flaw with this interpretation
- T1D2: Recognizes potential flaw in student's interpretation

[T1E] Explanation

- T1E1: Explains why graph reaches maximum and then decreases
- T1E2: Focuses on height of graph at the start ($T = 0$)
- T1E3: Confuses two meters with being short (agrees with student)
- T1E4: Interprets graph within context given by student
- T1E5: Explains thinking process
- T1E6: Explains why there could be another interpretation of the graph
- T1E7: Explains graph within teacher's context
- T1E8: Explains why the graph does not represent a person walking uphill
- T1E9: Confuses constant rate with zero slope
- T1E10: Confuses meaning of starting point

[T1MR] Mathematics Repertoire

T1MR1: Graph	T1MR13: Compare
T1MR2: Unit	T1MR14: Motion
T1MR3: Height	T1MR15: Projection
T1MR4: Time	T1MR16: Representation
T1MR5: Meters	T1MR17: Point
T1MR6: Curve	T1MR18: Interpretation
T1MR7: Slope	T1MR19: Path
T1MR8: Positive	T1MR20: Plot
T1MR9: Negative	T1MR21: Constant/straight line
T1MR10: Increasing/decreasing	T1MR22: Direction
T1MR11: Distance/far	T1MR23: Meters per second (rate)
T1MR12: Axis/axes	T1MR24: Vertex/maximum/minimum

[TR] Teaching Repertoire

- TR1: Asks a question
- TR2: Uses repetition
- TR3: Uses textbook/outside materials
- TR4: Corrects student's incorrect response
- TR5: Gives new/revised language to student
- TR6: Uses a visual/diagram/drawing
- TR7: Interprets student's response
- TR8: Reads problem
- TR9: Reads student's solution
- TR10: Scaffolds student's thinking
- TR11: Confirms student's response
- TR12: Questions student's response
- TR13: Prompts student's action

Table 1 continued

TR14: Offers student a learning strategy (e.g., mnemonic, starting point, steps to consider/use, underlining/highlighting, draw a picture)	
TR15: Reduces content down to essential components	
TR16: Checks for student's understanding	
TR17: Praise or acknowledgment	
Trigger #2	
[T2D] Diagnosis	
T2D1: Agrees with student's graph	
T2D2: Recognizes error in student's graph	
[T2E] Explanation	
T2E1: Explains that graph starts at (0,0)	
T2E2: Explains why section of graph where person is stopped should be horizontal	
T2E3: Explains other parts of the graph	
T2E4: Corrects student's incorrect response	
T2E5: Clarifies which coordinates correspond to time and distance	
T2E6: Explains strategy for creating graph	
T2E7: Explains thinking process	
T2E8: Adds contextual information	
T2E9: Demonstrates some confusion about constant speed	
[T2MR] Mathematics repertoire	
T2MR1: Far	T2MR18: Constraints/factors
T2MR2: Distance	T2MR19: Direction
T2MR3: Coordinate(s)	T2MR20: Line
T2MR4: x	T2MR21: Displacement
T2MR5: y	T2MR22: Visualize
T2MR6: Graph	T2MR23: Negative/positive
T2MR7: Point/starting point	T2MR24: Independent variable
T2MR8: Add/plus	T2MR25: Dependent variable
T2MR9: Miles	T2MR26: Axis/axes
T2MR10: Seconds/minutes/time	T2MR27: Path
T2MR11: Position	T2MR28: Closer/farther
T2MR12: Straight line/horizontal	T2MR29: Interpret/interpretation
T2MR13: Calculate	T2MR30: Plot
T2MR14: Fast	T2MR31: Slope
T2MR15: Constant speed	T2MR32: Equation
T2MR16: Miles an (per) hour (rate)	T2MR33: Increasing/decreasing
T2MR17: Representation	T2MR34: Vertex/maximum/minimum

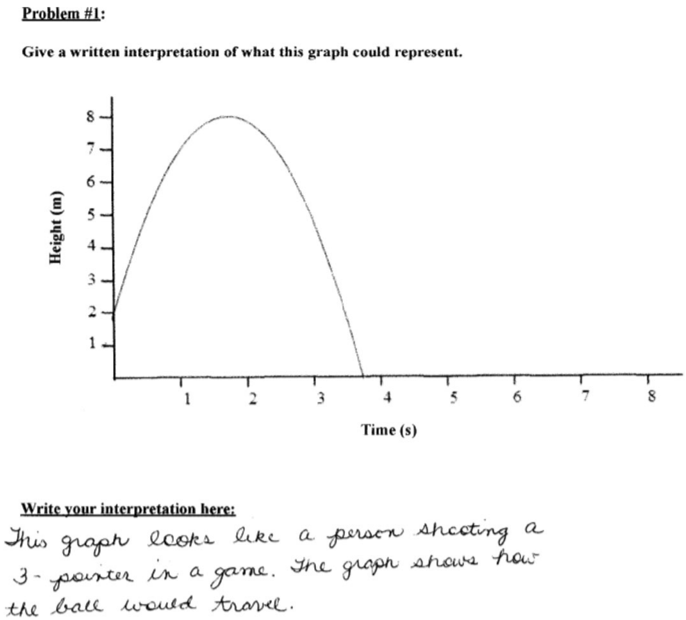

Fig. 1 Problem 1

Problem 1: "…how the ball would travel"

Five of the eight PTs offered an affirmative diagnosis of Marcia's interpretation, essentially indicating that she was correct. As Marcia read aloud her graph interpretation, Springer responded with an initial "*Mmmhmm, definitely*," indicating that Marcia's interpretation was correct. Just a few moments later, Marcia clearly stated the challenge of iconic interpretation and students' common misconception, saying, "… *when we're interpreting graphs, we're supposed to look at what it looks like right (motioning to the arc on the graph)?*" Springer confirmed Marcia's literal interpretation:

> S: mmmhmm, yeah … so we have a height (pointing to y-axis) so the ball travels up, up, up (pointing to arc with pencil) and then all the way back down over the time (motioning to x-axis). So it starts right when you shoot the ball, so as soon as the ball leaves your hand, it goes up, up, up, then goes through the basket ….

Similar to Springer's response, two other PTs used visualization in their response to Marcia's interpretation. Focusing on the starting point of the graph, Pastle explained, "… *so we're starting at two meters, so its obvious we're not starting at zero. So what are we kind of thinking as like the visual interpretation of this going … what does the two meters represent?*" Marcia replied that two meters might represent a middle school student shooting a basketball: "*um maybe someone could be really short ….*" Pastle agreed with Marcia, confusing two meters with 'short' by saying, "*Yeah, maybe we could consider it like maybe like a middle school student.*" [Note that using metric units was perhaps an added layer of complexity that was not helpful. We did not have a goal of PTs addressing student understanding of metric units. We assumed the metric unit would be a non-issue, which it was except for this instance.] Aligning with Pastle's visualization response, Batista also agreed with Marcia's interpretation and similarly cued into the starting point of the graph: "*So I think that this is a good example, 'cause you noticed that it was two meters off the ground and it didn't start at the ground. 'Cause if it was off the ground, then you'd have to think of a different example.*"

Two other PTs offered affirmative diagnostic comments, but qualified their diagnoses. Their qualifications served as extensions to contexts beyond that provided in Marcia's interpretation. Such extensions then provided room for a closer examination of the data points in the graph itself. In response to Marcia's interpretation, Henley noted, "*Okay, that's a good start.*" As she continued, though, Henley did not counter Marcia's interpretation, but instead qualified her affirmative '*good start*' with an extension comment that called for an interpretation more closely associated with data points. Henley indicated,

> H: So, I think you got the main point that it could be like a ball … how it travels … and um, I was expecting or hoping a little bit more … that you would tell me a little bit more saying time and when does the ball fall or when does it hit the vertex or highest point … um, a little bit more I had hoped. But okay, since you have written this, let's go from here.

Another PT, Jordan, technically offered a confirmation of Marcia's interpretation with his initial "*Okay, yeah*" when she read aloud her interpretation. When Marcia asked, "*Is this right?*," Jordan offered a significant qualification by saying, "*Um, I mean there's no really right answer … just your own interpretation.*" From there, Jordan engaged in an explanation initially grounded in the three-point basketball context proposed by Marcia. While still operating in that context, Jordan explained another way to visualize this graph:

> J: If you want to stick with basketball … pretend I have a ball (stands up and motions as if he's throwing the ball with force to the ground) and I just throw it at the ground. It's gonna bounce up and its gonna bounce down. So, if I throw the ball and as soon as it hits the ground, I (start) a timer, and as time ticks on, it goes up (motioning with hand), and then it comes back down, what is that graph gonna look like do you think?

One PT offered a contrast to Marcia's interpretation of the first homework problem, indicating that her interpretation was not quite 'right.' Ford began by directing Marcia's attention to the axes and their labels. As Marcia confirmed that time and height are the respective x-axis and y-axis, Ford reminded Marcia that measurements of height focus on objects that move up and down. Ford followed this reminder with an important question, by asking, "*… but when you're looking at a person shooting a 3-pointer, is there anything else that's changing … like are there any other aspects of the basketball's position that's (sic) changing besides just the height?*" Marcia was unsure how to respond, so Ford presented a clarifying question that challenged Marcia to visualize, "*So, when you shoot a 3-pointer, are you like standing right underneath the hoop?*" Marcia answered with the obvious, 'No,' and Ford followed up with an important, directing question, "*So, it (the basketball) was thrown, and then which direction would it be thrown in?*" Here, Ford has asked Marcia to find the data on direction or distance, which are not represented in the graph. Marcia realized this and indicated that the direction the ball would be thrown in would be 'up.' Ford confirmed, "*It would just be thrown up because the height is going up and its coming back down and hitting the ground.*" In accordance with the SIP, Marcia presented the misconception clearly, "*Oh, cause I thought when we were supposed to interpret graphs … we were supposed to interpret what it looked like.*" Ford's response was conclusive: "*… if this (graph) was like the path of an object, then that (interpretation)* *would be completely right, but this is just … you have to look at what the axes are labeled … 'cause its height vs. time.*"

Problem 2: Determine coordinates; construct graph

The second significant trigger in this simulation centered on the second homework problem. Figure 2 shows this problem, along with the coordinates and graph that Marcia constructed.

Note that the coordinates and graph are incorrect. Marcia has made an error by assigning a value of "0" in the third set of coordinates (60, 0), when the coordinates should actually be (60, 10) in accordance with the word problem. In the problem, Marcia read that the driver stops traveling and is talking on his cell phone for 30 min. She interpreted this as "0" on the y-axis representing distance. During those 30 min, it is true that the driver did not travel any additional distance on his trip. However, the driver did not regress, which is what Marcia mistakenly plotted on her graph with the "0." Like the first trigger, the data for this trigger were coded within the four primary code constructs. We now turn attention to the PTs' diagnostic, explanatory, and mathematical repertoires when working with Marcia on this second problem.

All eight PTs identified Marcia's error early in the discussion of this homework problem. Their diagnoses typically occurred through a sentence-by-sentence evaluation of the word problem and its graphic representation. Batista's (B) interaction with Marcia (M) serves as an exemplar of this diagnostic interaction:

B: "Okay. (reading). A car leaves a parking lot and travels 10 miles in 30 min. Okay, so let's just underline that. The driver stops to talk for 30 min. So, when the driver stops, what is his distance?
M: Uh, he's stopped.

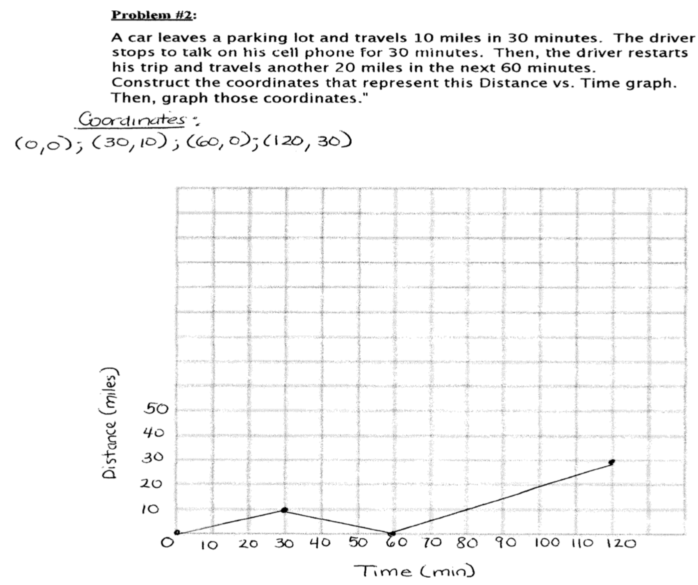

Fig. 2 Problem 2

B: Right. So, how far can you travel when you stop?
M: You can't.
B: So it's …
M: Uh … so its zero, right?
B: Yeah.
M: 'Cause you wouldn't travel.
B: Right. So, it's zero miles when he stops (reading). Then, the driver restarts and travels for another 20 miles in 60 min.
M: mmmhmm.
B: (Reading). Construct the coordinates. Okay, we have zero, 'cause that's the starting point.
M: mmmhmm.
B: Good, and then you traveled for 30 min and you went 10 miles.
M: mmmhmm.
B: The driver stops to talk on his phone for 30 min. Okay, so you got 30 min, but you said he went negative 10 distance.
M: Like backwards?
B: Yeah.
M: Oh.
B: So what should this be? What should that point be instead?
M: It should stay the same?
B: Right, so it should be … yeah 10 …."

While each PT correctly identified Marcia's mistake, their explanations took different forms. Most PTs followed a diagnosis of the mistake by explaining why that portion of the graph would be horizontal, rather than going back to a y-value of 0. Consider Sidley's remarks: "*He's not moving any closer or farther away from the parking lot so for those 30 min, his distance is just going to stay fixed. So it's just gonna be a straight line across (gesturing on graph with hand to represent a horizontal line).*"

As their explanations continued, PTs encouraged Marcia to identify variables and clarify the coordinates as two strategies for creating graphs. In making these suggestions, Jordan referenced both homework problems:

> J: *Yeah, the biggest thing when it comes to all graphs … not just the parabola which is what this is called (pointing to Problem #1) … is I always want to identify what the variables were …. So in this case (Problem #2), I would look at this and say, I know my two variables are …*

Batista took this strategy one step further, by gently interrogating Marcia on the coordinates and how they align to the axes. Note, though, that in this interaction, Batista (B) briefly confirmed Marcia's incorrect "fix" of the x-coordinate, before addressing this error:

B: Okay, so which coordinates do you have to fix now in your coordinates?
M: Uh, 10 would stay the same, right? So, it would be the 'x' that I would have to fix?
B: mmmhmm
M: Okay.
B: So which one? Well …
M: This one.
B: No, the 'x' is the same.
M: Oh, 'y'…

B: Yeah, 'cause this is 'x' and this is 'y', right, if this is our coordinate?

Ford mirrored Batista's emphasis on coordinates and axes, indicating, "*… just always remember that pretty much for the majority of the graphs that we're gonna look at, be really careful to read the axes, 'cause it's a pretty rare occasion that the graph is gonna be the path ….*" Interestingly, Ford was the only PT to cue into the iconic interpretation in the first homework problem, and we see her reference it again when coaching Marcia on the second homework problem.

As their explanations for the second homework problem continued, PTs provided Marcia with suggestions for graphing. Readers will recall that each "Marcia" is trained to present a third trigger and ask for general examples on how to interpret graphs. At times, though, PTs began making general suggestions, thereby negating the need for this specific third trigger. Through interactions with each PT, each "Marcia" could decide whether or not to present this trigger independent of, or aligned with, discussion of the second homework problem. In response to this third trigger, Batista gave an encouraging, concrete example and structure for Marcia to follow. Referencing the second homework problem and future word problems like it, Batista noted, "*So, I like to underline. So, that's my technique of keeping track, but maybe you want a list up here (referencing top of paper above graph). So, you could just list miles and minutes in like a little graph(sic).*" Batista's reference to underlining key information and then constructing a table of values stands alone as an outlying suggestion.

Most other PTs suggested visualization as a strategy. For example, Ford encouraged Marcia to visualize by "*just imagine(ing) the real situation, and that might help …*" Jordan makes an identical remark, suggesting, "*… try to come up with a picture or a situation where that could happen.*" To bring the visualization suggestion to life, Springer (S) uses a manipulative when working with Marcia on the second homework problem:

S: So if we started at zero (grabs an object to represent a car driving on the table between them) …. so, our car started here, zero miles, we travel ten miles to here (moves object along table) … another 30 min …. then another 30 min, are we going anywhere?

To follow this manipulative, Springer engaged in noteworthy dialogue about visualization. In response to Maria's request for any suggestions, Springer referred back to the first homework problem, and actually encouraged visualization, confirming he did not recognize how iconic interpretation and visualization had contributed to Marcia's error on that first problem:

S: It definitely helps to visualize. So, take like this one is perfect (pointing to Problem #1). The basketball, so you can visualize the ball leave the hand, it went up and down … But the visualizing can cause a little bit of problems (sic) when you have something like this (flipping back over to look at Problem #2).

Mathematical and instructional/teaching repertoires

As shown earlier in Table 1, our mathematical repertoire coding scheme for the first and second homework problems generally consisted of the same codes. For example, the code 'path' was used to identify data across the entire simulation. Some additional codes were initially proposed for the second trigger (i.e., Problem #2), like 'independent' and 'dependent variable,' but these codes were rarely assigned to data. The above data excerpts provide the reader with examples of how PTs used mathematics concepts and vocabulary within their diagnoses and explanations to Marcia. To report these data differently—outside of the diagnostic and explanatory contexts in which they were used—does not yield any new insight into the PTs' mathematical reasoning. Reporting the frequency with which the PTs used the terms 'axes,' 'distance,' or 'height' does not yield deeper understandings of their decisions beyond those revealed above. We are better served to know that PTs used 'visualization' and 'representation' often in their explanations to Marcia, particularly as they tried to help her more fully interpret Problem 1 or attend to mistakes in Problem 2.

While the PTs' mathematical repertoires are noteworthy in context, their instructional repertoires are best represented independently. Doing so allows us to better understand the instructional practices PTs employed when crafting their diagnostic and explanatory comments.

PTs frequently used questions when engaging with Marcia. Often, these questions were used to either check for understanding or to interrogate Marcia's procedures. At times, checking for understanding came from very brief, predictable question stems, like when Springer turned to Marcia on several occasions, made a statement and then checked with Marcia by simply asking, "Okay?" Referencing more of the context of the second problem, Henley gave us another example of checking for understanding: "*Right, if it's a red light, you stop and then continue. Does that make sense?*" Importantly, we do see PTs checking for understanding by asking Marcia to explain her thinking, and not simply responding to classic 'yes, I understand' or 'no, I don't get it' types of questions. For example, Ford asked Marcia, "*How'd you get that answer? What'd you base that on?*" in reference to the first homework problem. For the same problem, Batista asked, "*Shooting a three-pointer, okay. So, how did you interpret this?*" Later, Ford questioned Marcia on her process/approach for the second homework problem: "*Okay, so did you write down your coordinates first and then make your graph?*" Questions such as these—where Marcia must explain her thinking—were less frequent, but clear in purpose.

Noted earlier, several PTs used sketches and diagrams, manipulatives in referencing distance traveled, and prompts that centered on visualization and representation. In addition, concrete instructional directives emerged in some simulations. Two PTs—Henley and Jordan—used clear examples of instructional directives, meant to place the responsibility for action on Marcia so that she is active and engaged. Referencing the first problem, Jordan prompted Marcia to "*… make a little dot at where you think it would start.*" Later, Jordan again directed, "*So, let's work with that example (erasing the white board)…Why don't you take a pen and make a sketch of this graph? So, it's gonna be distance and time.*" Henley gives similar, action-oriented directives to Marcia: "*I would write that too … kinda explaining that the ball would reach the highest height at that time … so I would write that also.*" Of note, Henley also directed Marcia to read her interpretation of the first homework problem. This serves in contrast to several other PTs, who read aloud either the homework problems or Marcia's interpretation of Problem 1.

Finally, questions arose often as PTs worked to scaffold Marcia's thinking and support her reasoning. The focus here is not on the interrogative nature of the PTs' dialogue, but instead on how they used questions to build student understanding. To begin, consider how Sidley coached Marcia on her interpretation. Moving from what the graph in Problem 1 actually resembles, she encouraged Marcia to be more intentional about citing the coordinates in her interpretation:

> S: … so you're interpreting this curve (point to graph) as the path of the ball, right? So, like maybe in keeping with this … it doesn't start at zero, right?… Maybe we

could along, like more mathematically, say stuff about this point (top of arc) like what happens right here.

In a similar effort to scaffold Marcia toward constructing an interpretation more closely oriented to the data, Ford asked her a series of questions:

F: "So, when you look at what axes are labeled as, what are the two labels that we're looking at?"
M: Height and time?
F: Height and time. So height…what is (sic) height really measure, like if you're changing the height of something, like with your hand?
M: How tall it is.
F: Yeah, how far up and down it is, but when you're looking at a person shooting a 3-pointer, is there anything else that's changing…like any other aspects of the basketball's position that's (sic) changing besides just the height?
Jordan presented a final illustration. In response to a prompt from Marcia, Jordan gave her a separate driving example that is similar to Problem 2. In unscripted fashion, note how Marcia was struggling to follow Jordan's representation, and therefore, could not build from his distinct questions:
J: "If we leave our house, 5 min later we'll be a certain distance away. Another 5 min, we'll be a certain distance away. At any point, do you see that coming back to our house if we're just heading downtown?
M: What do you mean?
J: Um, I mean, if we're just driving, just driving straight, uh, should we end up…notice how we come back to our starting point at zero (referring to graph)
M: Yeah, yeah
J: Should we ever come back to our starting point if we're just driving straight?
M: Yeah
J: We should?
M: Yeah
J: If you start at your house and you just drive straight to (city), when are you gonna end up being back home?"

Post-simulation debriefing: approach and deltas

After the PTs engaged in their individual simulations, they used their same login/password combinations, accessed their simulation videos, watched them, and then gathered together as a group in a simulation to debrief. Once again, audio and video were captured as the second author guided the PTs through a semi-structured debriefing. The first author open coded the entire debriefing transcript once, resulting in twenty-one different codes. This code list was provided to the other authors, who used it to independently code the debriefing transcript. After this coding process, each author constructed a data summary for the debriefing.

PTs reflected extensively on their approaches in simulation. To start, consider Pastle's frank reflection,

P: …when (the video) was going on, I was like a lot more critical of what I was saying…I'm like 'think of a way to…think of a different way to help (Marcia) understand'…and I was just blanking. So, I walked out thinking I almost did a horrible job. But then I guess rewatching it and seeing what she had to say, I guess it was just a lot more of me being critical of myself.

As the group debrief progressed, Pastle elaborated on her approach, noting the level of questions that she asked. In her reflection, she is both proud of her use of questions, but cognizant of how she could have structured the questions to probe more for student thinking: "*I felt like I was asking good engaging questions, but I was so quick to be like, 'yeah!', 'right!', and not like 'why?', 'why do you think?'… I forgot to ask 'why do you think that?' and leading up to have her support it with mathematical evidence?*" Aligned with Pastle's reflection on her use of questions, Batista also commented on how often she questioned Marcia's thinking and the unintended consequence:

B: I think I did good focusing questions but I also think that in certain times I like to question a student's thinking even if she's right…and I think I do that too much, 'cause then they come to expect they're wrong when they're right, but I just want them to justify why they're right. So, maybe I should start with 'yeah, this (is) right', but instead I'm just kind of like, 'So, why is this right?' and they're like, 'Is it?'…

As Pastle and Batista's comments demonstrate, PTs' organization, choices, and frequency of language surfaced in the debriefing process. To further illustrate, consider Kline's reflection on her decision making in real-time:

K: I just start saying one thing and then say another and I just wasn't organized with what I was saying and I just felt like…I should have been more organized with my thoughts and my words, because I was just like starting saying one example, then be like 'okay, well this one would be better, so let's go back to that example'… I felt like I wasn't clear enough.

While Kline indicated a need to improve how she organized and presented her efforts to Marcia, Springer commented on the directional nature of his comments to Marcia, and the need to use more open-ended cues in future interactions:

S: …I asked good questions when talking about the visual representation…but when we had to physically put the new coordinate points in the graph, I probably funneled her a little bit. Like I said, 'Okay, now where would we put this point?' you know, instead of saying like, 'which point would we change?'

Perhaps, the most straightforward reflections came from Sidley and Springer, again referencing their use of language as a placeholder. Sidley reflected, "*I think I talked a lot. Like I kind of really wanted to help … I mean I asked her questions and then she would answer them, and then I would elaborate on them for her.*" Sidley's remarks reflect how uncomfortable silence can be for novice teachers, where wait time of three seconds is difficult because it feels like an eternity to the teacher who feels the need to fill that gap. Springer's comments are similar and quite poignant, as he focused on both the silence and the struggle that it represents:

S: I think it's hard sometimes to watch them struggle. You know what I mean? They ask you a question and then you ask them a follow up question and they're really struggling with the answer. Its hard to sit there and watch them go, 'uh, uh, uh' It's almost our nature to want to help them.

The post-simulation debriefing fostered discussion of PTs' approaches, but also allowed room to discuss their mistakes and changes they would make in the future. Early in the debriefing session, the researcher guided PTs back through their approach to the first homework problem presented by Marcia. Batista's remarks in the debrief reinforce the instructional decision she made with the first homework problem. Batista noted, "*I think I anticipated her to get it wrong, so like I went in thinking the first problem could be wrong, but she had it correctly (sic) so I didn't spend as much time on it as I like should have I guess.*" When asked by the debriefing facilitator why she thought the first homework problem was right, Batista's response is striking, "*Because I thought it was right.*" This provided the facilitator an opportunity to discuss the problem with the group of PTs. Ford explained why she thought Marcia had made a mistake—"Well, she was looking at it as if it was the path of the object as opposed to position versus time or height versus time graph"—and the facilitator asked if the graph could represent someone throwing a ball. The group discussed under what conditions this could be an accurate representation.

To her credit, as she listened to the facilitator and the other PTs, Kline realized she maybe should have approached the first homework problem differently. "*I gave her an example of a person walking and then changing direction and going in the opposite direction…well, like, was I wrong? Like if you're walking and then changing direction and going down…okay, well, I did, I did that wrong.*" Of equal importance, the follow-up dialogue between the debriefing facilitator and Kline indicates the origin of her mistake. Earlier in the semester, the PTs had engaged in a series of completely separate activities, over several class periods, using a motion detector and a graphing calculator to examine graphs created by walking. These activities occurred in a methods course taught by the debriefing facilitator. Kline had thought about the graph on Homework Problem 1 as a graph that could be formed by walking away from the motion detector and back toward it. This example could be correct if one is only looking at the graph without the units on the axes because the increasing part of the graph could represent walking away from the motion detector, the vertex of the parabola could represent the turning point, and the decreasing part of the graph could represent walking back toward the motion detector. In fact, Kline and the other PTs had created graphs that looked similar to Homework Problem 1 using motion detectors; however, this scenario does not match the label on the y-axis of "Height." Additionally, this example would likely be confusing to a student who had not used a motion detector to explore graphs formed by walking. Moreover, Kline did not attempt to understand Marcia's interpretation of the problem, but rather moved quickly to explaining the way she thought about it to Marcia.

Later in the debrief, Springer reflected on the vocabulary he used with Marcia, and how he would refine it when engaging with future students:

> S: I picked up on speech mistakes…I say 'okay' a lot…so I'm going to try and work on (that). And sometimes the preciseness of my language, like I said constant 'speed' instead of constant 'distance'…which isn't technically wrong, but could confuse a student…like I said when there's a horizontal line that's a constant speed, when I really meant to say a constant distance…the distance isn't changing, but I said the speed isn't changing…it stayed zero speed, but that could've been confusing.

Similar to Springer's emphasis on precision of language, other PTs noted non-verbal and verbal changes they would make in the future. Reflecting on her impatient disposition, Ford noted that, "*I look like I drank 10 cups of coffee and ran around the block a couple of times before I was ready to chill out.*" After reviewing her video, Sidley also noted a need to slow down and be more deliberate: "*I should try to maybe slow down and be more authoritative in what I say.*" The debriefing researcher clarified, asking if she wanted to speak more "confidently," to which Sidley responded, "*Yeah, well, rather than just a* *regular conversation like I'm having right now, I guess just plan out my sentence before and then just say it with authority ….*" Citing the non-verbal components of working with students, Springer reflected,

> S: I feel like in that situation at that table sitting across from her, I felt like I almost kind of dominated the table…like I moved really close to the table, my hand was right on top of the (homework) paper, and the student was moving back a little…like (my) body language could be a little better.

Later, Springer reinforced the exact reason why each PT's simulation is video recorded and reviewed by participants. He stated, "*It (video) lets you pick up on subtleties … looking back (through video), I go like, 'Oh, that was a mistake I made, make sure to correct that for next time.'*" Of note, the PTs were asked to review their videos, but were not directed on what to look for. The self-direction present in Springer's comment reinforces one purpose of a clinical simulation.

Discussion and implications

Our discussion is best presented through a classic *who, what, when*, and *how*. We begin by examining *who* holds responsibility, *what* content or instructional strategies were considered, and *when* and *how* instructional decisions should be implemented. Our implications center on the *where,* through our emphasis on the clinical experience as a location and context that is distinctly different from traditional field placements.

First, we consider *who* holds responsibility in a one-to-one learning environment. This clinical simulation is designed to closely approximate an interaction that a novice mathematics teacher would have with a high school student. Thus, we discuss the instructional and learning roles that these PTs hold in simulation as a representation of the same roles each would maintain within a high school classroom environment. Our data suggest that the PTs recognized their instructional responsibility. At first glance, all worked to guide Marcia through her questions and struggles with the two homework problems. Closer inspection of the data, though, indicate nuanced instructional styles and perspectives on responsibility. For example, we see two PTs who are comfortable with and versed in providing instructional directives. They not only saw themselves as the instructional authority, but also felt comfortable exercising that authority through appropriate instructional directives. In direct contrast, the data indicate that other PTs read aloud the homework problems and Marcia's written interpretation of Problem 1. It is our argument that the student should hold—at the very least—this minor responsibility and that the PT holds responsibility for cueing Marcia to 'reread the problem' and 'read aloud how you interpreted this first problem.' In debrief, PTs referenced instructional responsibilities they hold and the degree to which they met those responsibilities in the earlier simulation. For example, Pastle reflected on her use of surface-level questions, and her desire to structure future questions that investigated students' mathematical reasoning. This suggests her recognition of the instructional responsibility she holds, how her performance did not fully meet that metric, and her desire to improve. Pastle's interrogation of her own questions, coupled with her peers analyses on the degree to which they were authoritative and directive, are trends we see from other studies. Borko et al. (2008) report very similar data, where the video-informed intervention more naturally drew out teachers' self-assessments of instructional styles.

Second, we briefly discuss *what* strategies and content were selected and *how* they were implemented within—and reflected on after—the clinical simulation. Each PT used visualization prompts to either support Marcia's understanding of the homework, or as a suggestion she might use when working with graphs in the future. Although drawn from a small sample, this trend suggests visualization is either a pervasive self-developed strategy used by these PTs as students of mathematics, or it stems from effective teaching practices they encountered in prior mathematics coursework at the secondary or collegiate level. The manner in which visualizations were implemented showed through the use of manipulatives and representations. Springer's physical demonstration with a manipulative—moving a car from a defined point to a second point to convey it does not regress in distance traveled—remains the clear example, with other PTs crafting other written representations to help Marcia organize and visualize data. By marking variables, axes, or moving toy cars forward, these PTs embodied Arcavi's (2003) reference to the use of any object to transition Marcia from the cognitively abstract concept to a more concrete, spatial representation, where she can "see" the mathematical concepts.

As an additional example of *what* strategies were selected and *how* they were implemented, these data suggest PTs' considerations of the types of questions they posed. Through their debriefing comments, we see emerging attention to the quality as well as quantity of questions. Each PT worked to scaffold Marcia's understanding of the homework problems through questions, but there is emerging cognizance of how questions should be structured to help Marcia engage more actively. These findings approach those from Masingila and Doerr (2002), where the multimedia case study resulted in PTs' pinpoint critiques of the types and levels of questions used to interrogate mathematical reasoning.

Additionally, the degree to which PTs operated within or extended beyond Marcia's mathematical contexts also represents a point of discussion. For the first homework problem, some PTs operated within the 'basketball three-pointer' context that Marcia presented, while others started their explanations there, and later extended or qualified those contexts. In the debriefing, Kline provided two important realizations that speak to the operational context decisions that PTs made. Kline commented on wanting to be more deliberate in her explanations and examples she provides to students. Such a comment suggests a willingness to more critically examine her adoption of a student's mathematical context, or her willingness to introduce alternative or potentially contrary mathematical contexts that would better illuminate a particular concept or refute a misconception. Importantly, Kline admitted her poor choice when working with Marcia on the first homework problem. Later in the debriefing, she and the facilitator briefly explored her thinking, locating it within a similar mathematical context explored in a recent 'methods' course. This is an especially promising moment for two reasons. Not only did Kline admit her poor choice for her interaction with Marcia, but she also identified the outside context from which her reasoning originated. Thus, we see Kline acknowledge the error in not interrogating the context that Marcia presented, and we see this teacher began to distinguish applications and misapplications of one mathematics context to another.

Santagata and Guarino (2011) describe the benefits of using video to help teachers analyze their practices. Specifically, they note how video of practice informs: (a) teachers' reasoning about instructional decisions, (b) considerations of how such decisions impact student learning, and (c) alternative instructional moves. We see similar trends, as our data provide evidence of mathematics PT reasoning on *when* instructional decisions should be enacted. Sidley's, Batista's, and Pastle's comments on the frequency with which they spoke, interrogated, and intervened suggest the challenge of *when* teachers should engage to foster student growth. Experienced teachers understand the struggle that comes with wait time, and how effective teachers have learned to accept slowing down with silence to prompt student learning. With a larger group of students, wait time is commonly awkward for the novice teacher, as he/she pauses and waits for someone in the group to respond and engage. In a one-to-one instructional environment, though, wait time and other pacing decisions are even more challenging. The pace seems much quicker and the issue of when to question or pause for understanding is magnified. The teacher is not relying on *students* from the group to engage, but is instead relying on *the only student* to respond. Springer's words bring this concept to life, as he emphasized the difficulty in watching a student struggle with content. His words exemplify the internal pull a teacher feels in deciding when to engage, prompt, question, or just sit quietly and wait for the student. Interestingly, Springer's critique of his 'funneling' of Marcia mirrors data Borko et al. (2008) recorded, where one of their mathematics teacher participants also wrestled with the 'mistake' (p. 433) of forcing a student toward a particular mathematics solution.

Limitations, strengths, and directions for future research

Our discussion of *who* holds responsibility, *what* and *how* content or strategies were utilized, and *when* instructional decisions were implemented suggests final attention to *where* this study took place. The *where* is a clinic, a location and approach that extends the traditional views of 'clinical preparation' within teacher education. This laboratory setting could be considered a limitation in that fewer teacher preparations institutions have access to such a facility. However, simulations hinge on carefully trained standardized individuals, not on elaborate clinical settings or expensive recording equipment. Dotger (2013) discusses the benefits and financial costs of partnering with a similar simulation facility, but outlines clear steps to implementing simulations, regardless of one's proximity or association with a medical school. One clear limitation is the PTs' predominant reflection on instructional moves, with much less attention to Marcia's mathematical reasoning. In future studies, more elaborate investigations of the PTs' recognition of and reflection on (standardized) student thinking are necessary. A second limitation is this study's sample size ($n = 8$), essentially constricting analysis to the qualitative approaches seen herein or descriptive and nonparametric statistics. It is possible to implement clinical simulations with large cohorts of teachers, mirroring the much larger numbers of prospective physicians engaging in medical simulations.

One intentional limitation of a clinical simulation is the lack of context surrounding the interaction between each PT and standardized individual. There are no ringing bells, interrupting intercoms, or a group of students that fragments the PT's attentiveness to the standardized student asking questions. Clinical simulations are intentionally bounded, limited experiences, intended to approximate (Grossman et al. 2009) classroom practices. Clinical simulations do not—nor are they designed to—supplant fully contextualized work in schools with students.

Simulations hold pedagogical strength as opportunities for (limited) engagement with the practices of teaching (Dotger 2014; Shulman 2005). Fully contextualized teaching is complex, consequential, and fast-paced. In leading up to and supporting that work, though, clinical simulations provide PTs with opportunities to enact professional knowledge and skills in a consequence-free environment. We know that PTs need opportunities to act, engage, decide, instruct, and make mistakes. As seen in the data, this clinical simulation gives Kline the opportunity to make a mistake and provides Springer with the opportunity to use a physical manipulative in demonstrating distance traveled. Ford had the chance to ask questions and structure follow-up responses, while Jordan could enact instructional directives to move learning forward.

Another strength is the use of simulations to help PTs use video data to reflect on their actual practices, and not on what they *think* they said or did with a student (Sherin et al. 2011). In this mathematics simulation, each PT engaged with the same problem of practice, reflected on his/her individual approaches, and was also able to build from peers' understandings later in a group debriefing. PTs' reflections were grounded in data, and they had the time to deconstruct those data. A video-supported simulation gives Pastle the opportunity to closely examine the pace and style of questions she asked, while also giving Ford the opportunity to critique her ten-cups-of-coffee presentation early in the simulation. Simulations give Springer the chance to critique his consumption of workspace, while providing Henley the opportunity to examine her diction when issuing instructional directives.

An additional strength of simulations is how they result in a shared practice. In many teacher preparation environments, a single PT describes a professional dilemma, while twenty others listen with only mild interest to a problem that they cannot immediately see or embed themselves in. Clinical simulations eliminate the 'that hasn't happened to me' approach of PTs, by literally situating each PT within the professional dilemma. Simulations do not allow for bystanders, idle inaction, and they are not role-plays with a predetermined outcome (Dotger 2011). The only role that Springer portrayed in this simulation is that of his teaching 'self.' He had sole control over what he said and did within the simulation, but was able to later share with his peers how he navigated the simulated challenges. Other researchers use video and multimedia to construct shared practices around specific video segments (e.g., Masingila and Doerr 2002) or shared lesson plans (e.g., Borko et al. 2008), allowing each teacher to envision or immerse, respectively, himself/herself in that broader practice. Beyond teachers' shared experiences across an extended lesson or across different mediums of practice, clinical simulations narrow the lens even more through the use of very specific triggers.

The data reported herein show how a clinical simulation illuminates PTs' mathematical knowledge, instructional abilities, and practices in need of refinement. Moving forward, we will continue to investigate how clinical simulations can potentially enhance the instructional capacity of PTs in very specific subject areas. For example, as PTs engage in multiple algebraic and geometric simulations, how do their experiences translate to the practices in student teaching contexts, where they are teaching the same content they encountered in past simulations? In cooperation with other colleagues, we also intend to examine how clinical simulations serve as a 'signature pedagogy' (Shulman 2005, np), moving beyond a single, episodic intervention. We want to investigate whether and how simulations might serve as a pedagogy that enhances and extends current approaches to teacher preparation across subject-specific and general instructional metrics. Simulations allow us to see PTs practicing, making mistakes, and using data to build from within and from each other. For educators vested in the development of future educators, the simulation concept and resulting data sets are extending our views of 'clinical preparation.'

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References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational studies in mathematics, 52*, 215–241.

Barrows, H. S. (1987). *Simulated (standardized) patients and other human simulations: A comprehensive guide to their training and use in teaching and evaluation*. Chapel Hill, NC: Health Sciences Consortium.

Barrows, H. S. (2000). *Problem-based learning applied to medical education*. Springfield: Southern Illinois University Press.

Barrows, H. S., & Abrahmson, S. (1964). The programmed patient: A technique for appraising student performance in clinical neurology. *Journal of Medical Education, 39*, 802–805.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 28*, 417–436.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.

Clermont, C., Krajcik, J., & Borko, H. (1993). The influence of an intensive in-service workshop on pedagogical content knowledge growth among novice chemical demonstrators. *Journal of Research in Science Teaching, 30*(1), 21–43.

Coplan, B., Essary, A. C., Lohenry, K., & Stoehr, J. D. (2008). An update on the utilization of standardized patients in physician assistant education. *The Journal of Physician Assistant Education, 19*(4), 14–19.

Dotger, B. (2011). From know how to do now: Instructional applications of simulated interactions within teacher education. *Teacher Education and Practice, 24*(2), 132–148.

Dotger, B. (2013). *"I had no idea!": Clinical simulations for teacher development*. Charlotte, NC: Information Age Publishing.

Dotger, B. (2014). *Core pedagogy: Individual uncertainty, shared practice, formative ethos*. Article submitted for publication.

Francisco, J. M., & Maher, C. A. (2011). Teachers attending to students' mathematical reasoning: Lessons from an after-school research program. *Journal of Mathematics Teacher Education, 14*, 49–66.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055–2100.

Hauer, K. E., Hodgson, C. S., Kerr, K. M., Teherani, A., & Irby, D. M. (2005). A national study of medical student clinical skills assessment. *Academic Medicine, 80*(10), S25–S29.

Jacobs, V. R., Lamb, L. L. C., & Phillip, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal of Research in Mathematics Education, 41*(2), 169–202.

Kohlberg, L. (1969). Stage and sequence: The cognitive-developmental approach to socialization. In D. Geslin (Ed.), *Handbook of socialization theory and research* (pp. 347–380). New York: Rand McNally.

Korthagen, F. A., & Kessels, J. P. (1999). Linking theory and practice: Changing the pedagogy of teacher education. *Educational Researcher, 28*(4), 4–17.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lebow, D. (1993). Constructivist values for systems design: Five principles toward a new mindset. *Educational Technology Research and Development, 41*, 4–16.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *PCK and science education* (pp. 95–132). Dordrecht: Klewer Academic Publishers.

Masingila, J. O., & Doerr, H. M. (2002). Understanding pre-service teachers' emerging practices through their analyses of a multimedia case study of practice. *Journal of Mathematics Teacher Education, 5*, 235–263.

Mead, G. H. (1934). *Mind, self, and society*. Chicago: University of Chicago Press.

Monk, S. (2003). Representation in school mathematics: Learning to graph and graphing to learn. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 250–262). Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Piaget, J. (1959). *Logic and psychology*. Manchester: Manchester University Press.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4–15.

Reiman, A. J., & Peace, S. D. (2002). Promoting teachers' moral reasoning and collaborative inquiry performance: A developmental role-taking and guided inquiry study. *Journal of Moral Education, 31*(1), 51–66.

Santagata, R., & Guarino, J. (2011). Using video to teach future teachers to learn from teaching. *ZDM: The International Journal of Mathematics Education, 43*, 133–145.

Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011). Accessing mathematics teachers' in-the-moment noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 79–94). New York: Taylor and Francis.

Shulman, L. S. (2005). *The signature pedagogies of the professions of law, medicine, engineering, and the clergy: Potential lessons for the education of teachers*. Speech presented at the Math Science Partnership (MSP) Workshop: "Teacher Education for Effective Teaching and Learning". Irvine, CA.

von Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese, 80*(1), 121–140.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge, England: Cambridge University Press.

Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations*. Report prepared for the U.S. Department of Education and the Office of Educational Research and Improvement. Retrieved from http://depts.washington.edu/ctpmail/PDFs/TeacherPrep-WFFM-02-2001.pdf.