A Puzzle to Try

Who Am I?

k	h	t	u

- The product of my digits is not 0.
- $tu = h$
- k is my only odd digit.
- $t + 1 = k$
- t is a square number.
- None of my digits are the same.
- I'm greater than 5000.

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Organizing Curriculum, PD, and Assessment around Mathematical Habits of Mind

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This work supported in part by the National Science Foundation.

MH$_s$oM (mathematical habits of mind)

One idea; challenge of using it; 3 instantiations

- **Curriculum materials**:
  Transition to Algebra
- **Professional development:**
  Implementing Mathematical Practice Standards
- **Assessment research:**
  Assessing Teachers' Algebraic Habits of Mind

$MH_s oM$ (mathematical habits of mind)

- What do we mean?
  Why do we care?

The idea isn't new

- Math'cians and good educators always knew
  - Mathematics is not just a legacy of *content*
  - It also gives us ways of *thinking/seeing* that *generate* those results
  - For proficient practitioners these become "second nature,"
    habits,
    habits of mind.

Old idea, new attention

- Standards for Mathematical *Practice*
  Common Core
- Tests → Professional Development
- Outcome: as yet unknown
- If good: greater fidelity to real mathematics

Why does this matter? Exposing the lie

- Mathematical *facts* not for “Real Life” Mostly, > grade 4
- Mathematical *habits of mind*—
  a puzzling-it-out, sense-making disposition,
  inclination to think quantitatively,
  logical argument,
  precision and clarity,
  attention to structure not just detail, and
  abstraction from examples—

serve *all people*.

Can’t teach *just* MH_soM

- If we ignore the facts, we disable students, rule out choices they might someday love
- Also, to *acquire* MH_soM, we must *do* math!
- So use MH_soM as an *organizer* for content (Al Cuoco, me, June Mark, 1996)

Today's focus: just two MH_soM

- Disposition to seek/use mathematical structure
- Disposition to puzzle through problems
- (Attention to precision in thought and communication)

- Examples *deliberately* from "dreary" content

Structure encoded in precise language

- An example

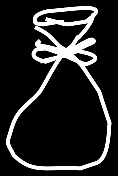
A number trick

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

4th grade, 9th grade
TransitionToAlgebra.com
Samples of the 12 units

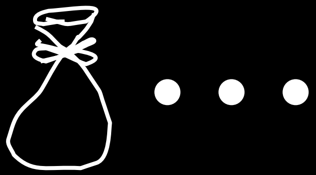
How did it work?

- *Think* of a number.



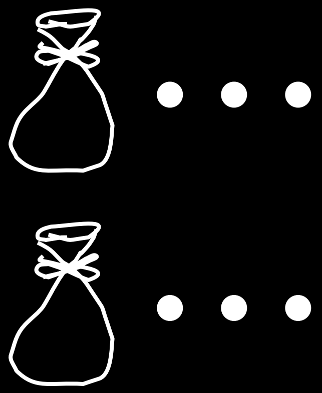
How did it work?

- *Think* of a number.
- Add 3.



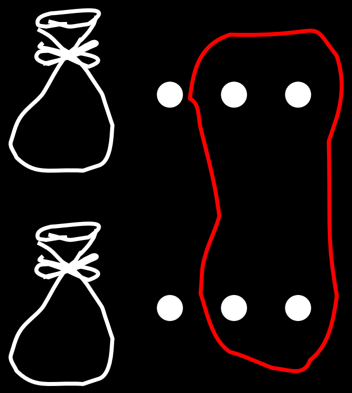
How did it work?

- *Think* of a number.
- Add 3.
- Double the result.



How did it work?

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.



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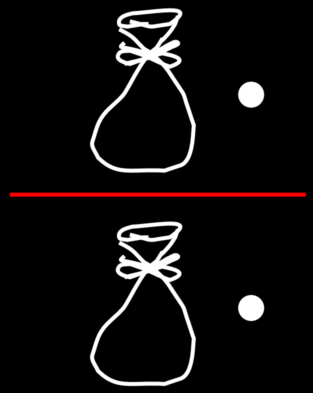

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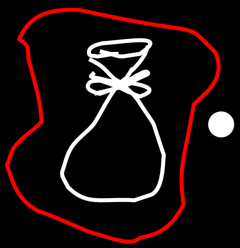
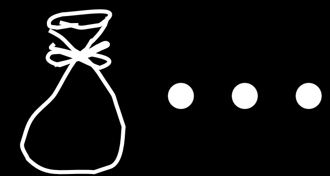
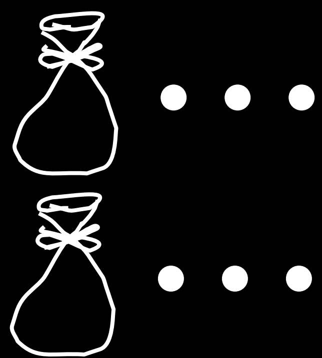


How did it work?

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

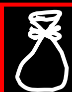
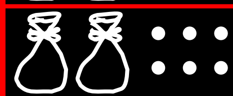
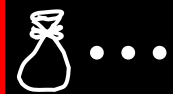
The distributive property before multiplication!

- When you described doubling 
- …to make 
- You were implicitly using the distributive property. *Kids* do that, too, before they learn the property in any formal way.

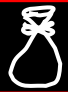
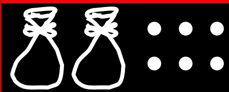
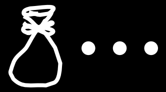
Power in the *notation*

- Transition to algebra growing directly out of the “natural” logic and language of kids
- But to use it, kids need practice.

Using notation: *following* steps

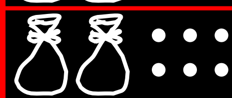
Words	Pictures	Imani	Cory	Amy	Chris
Think of a number.		5			
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20

Using notation: *undoing* steps

Words	Pictures	Imani	Cory	Amy	Chris
Think of a number.		5			
Double it.		10			
Add 6.		16	14		
Divide by 2. What did you get?		8	7	3	20

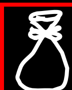
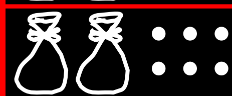
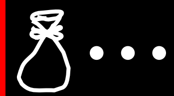
Hard to undo using the words.
Much easier to undo using the notation.

Using notation: *simplifying* steps

Words	Pictures	Imani	Cory	Amy	Chris
Think of a number.		5	4		
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20

Abbreviated *speech*: simplifying *pictures*

- Recording structure

Words	Pictures	Imani	Cory	Amy	Chris	
Think of a number.		5	4			b
Double it.		10				$2b$
Add 6.		16				$2b+6$
Divide by 2. What did you get?		8	7	3	20	$b+3$

In *Transition to Algebra*, this is week 1.
By unit 5, students can encode this sequence of steps as $(2b + 6) / 2$ and see its equivalence to $b + 3$.

Developing attention to structure

- Begin as soon as children learn to count
- Use problems and pedagogy that call attention to structure
- Beware random facts: randomness *hides* structure
- Remember: Even very conventional content can be encountered in ways that exhibit structure and reward *seeing* that structure.

Developing use of structure

- Begin as soon as children learn to count!

Find this video on
thinkmath.edc.org
Professional development

http://thinkmath.edc.org/index.php/Professional_development_topics

Developing use of structure

- This is fact practice. But the drill and *thrill* (and success) was from building a structure.

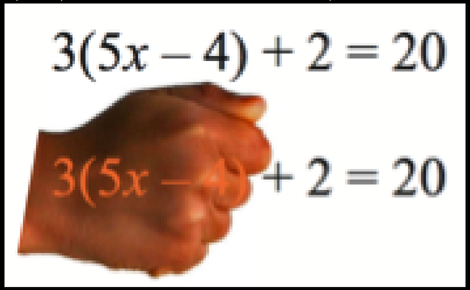
Find this video on
thinkmath.edc.org
Professional development

http://thinkmath.edc.org/index.php/Professional_development_topics

Developing use of structure

- Notations and naming:
  - *orally* “three + four”
  - *orally* 63, 73, 83, 93
- Calculation:
  - adding 8 to any number
  - halving/doubling any number

Developing use of structure

- Deferred evaluation (strategic laziness)
  - $7 + 5 \;\overset{?}{<=>}\; 7 + 4$
  - $1\tfrac{3}{4} - \tfrac{1}{3} + 3 + \tfrac{1}{4} - \tfrac{2}{3} = ?$
  - $f(x) = 5 - (3x - 4)^2$
  - 

- Classification (numbers, shapes, polynomials)
  - Not arbitrary, features of interest

1) *Algebraic Habits of Mind:* **Seeking and using structure**

Challenges to proficiency in algebra

- Seeing the details bu
  not the meanin
- Seeing
  the o
  str
  is

How *Transition to Algebra* bridges the gap

Helps students see the
re and logic;
calculation

7: *Look for and*
structure

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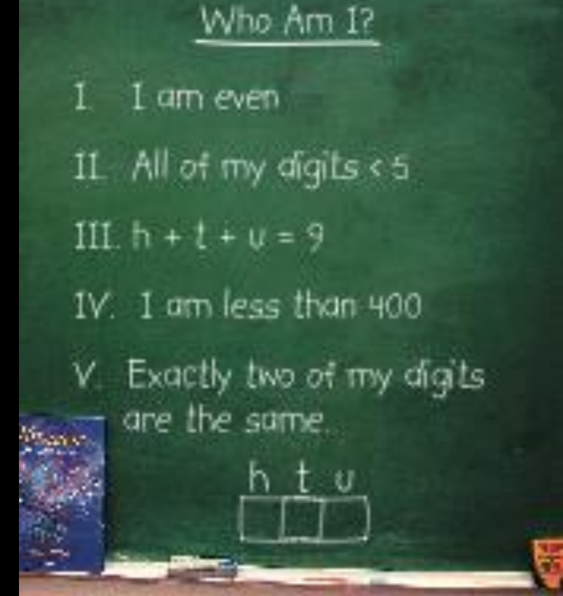
Puzzling things through

- Part of child's world
- Permission to think
- "Pure mathematical thinking" minus content
- But could car

8 year old detectives!



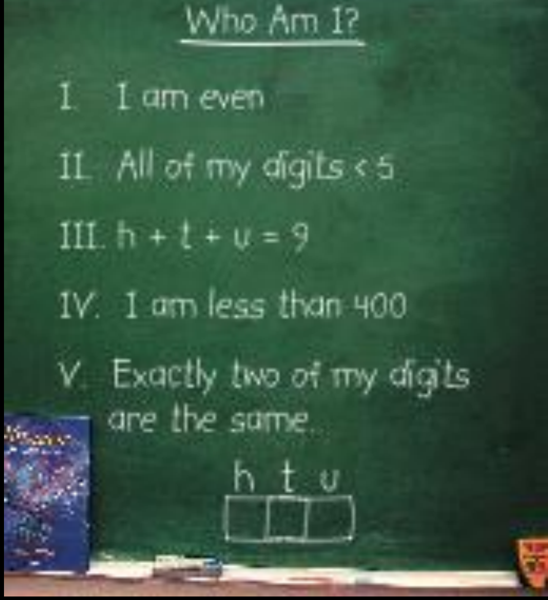
I. I am even.

II. All of my digits < 5

III. $h + t + u = 9$

h	t	u
	0	0
1	1	~~1~~
2	2	2
3	3	~~3~~
4	4	4
~~5~~	~~5~~	~~5~~
~~6~~	~~6~~	~~6~~
~~7~~	~~7~~	~~7~~
~~8~~	~~8~~	~~8~~
~~9~~	~~9~~	~~9~~

8 year old detectives!



I. I am even.

II. All of my digits < 5

III. $h + t + u = 9$

IV. I am less than 400.

V. Exactly two of my digits are the same.

h	t	u
1	4	4

h	t	u
	0	0
1	1	~~1~~
2	2	2
3	3	~~3~~
~~4~~	4	4
~~5~~	~~5~~	~~5~~
~~6~~	~~6~~	~~6~~
~~7~~	~~7~~	~~7~~
~~8~~	~~8~~	~~8~~
~~9~~	~~9~~	~~9~~

~~1,1,7~~

~~2,2,5~~

~~3,3,3~~

4,4,1

Why a puzzle?

- Fun (intellectual surprise/reward)
- Feels smart (intellectual <u>effort</u>, boredom is torture)
- *Because it's puzzling*
  - "Problems" are *problems*
  - Puzzles give us permission to think
- Puzzles can easily be adjusted to fit students
- And, because we're not cats

transition to algebra

make algebra make sense

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Researching the Problem

Grant-funded by the National Science Foundation

Developed by Education Development Center, Inc.

Authors:
June Mark, E. Paul Goldenberg, Mary Fries, Jane M. Kang, and Tracy Cordner

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Algebra . . .

- opens doors to more advanced math
- is a predictor of eventual graduation
- is a gateway to a bachelor's degree
- is linked to job readiness/higher workforce earning potential

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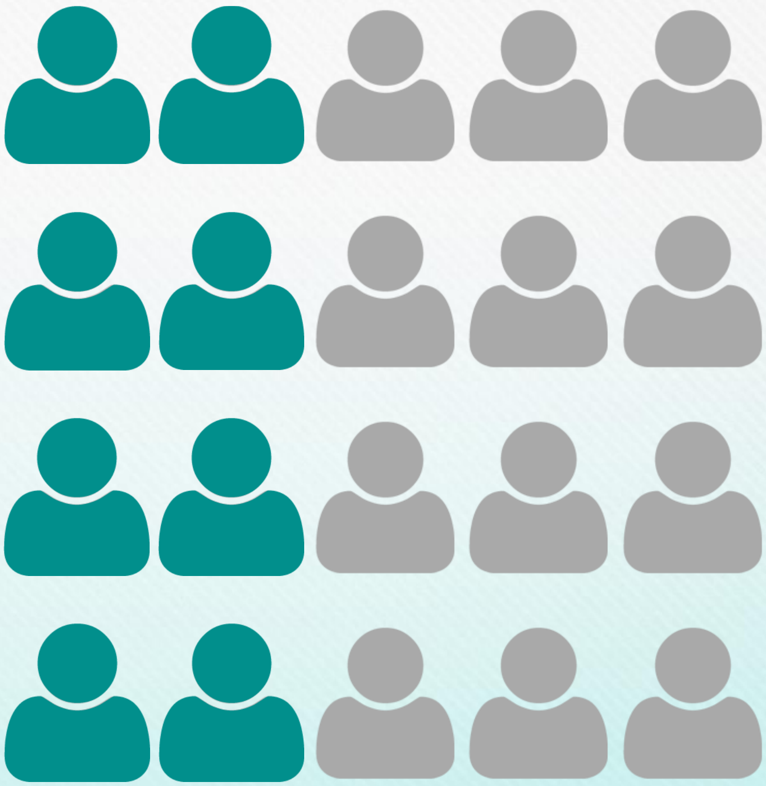
Facts About Algebra

As many as 40% of students need to retake algebra



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Common responses. . .

- Review, double-period algebra: “slower and louder”
- Remediation
- Test preparation

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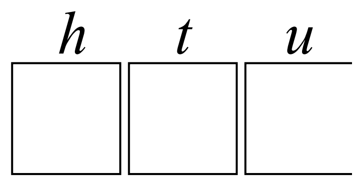
A different approach. . .

- As smart and intrepid as when they were 6!
- Build on students' own logic & common sense
- Focus on key algebraic habits of mind
- Connect arithmetic to algebra
- Make algebra make sense
- Make students producers, not just consumers, of mathematics

Invent your own! Who Am I?

- I'm a three digit number. Who am I?

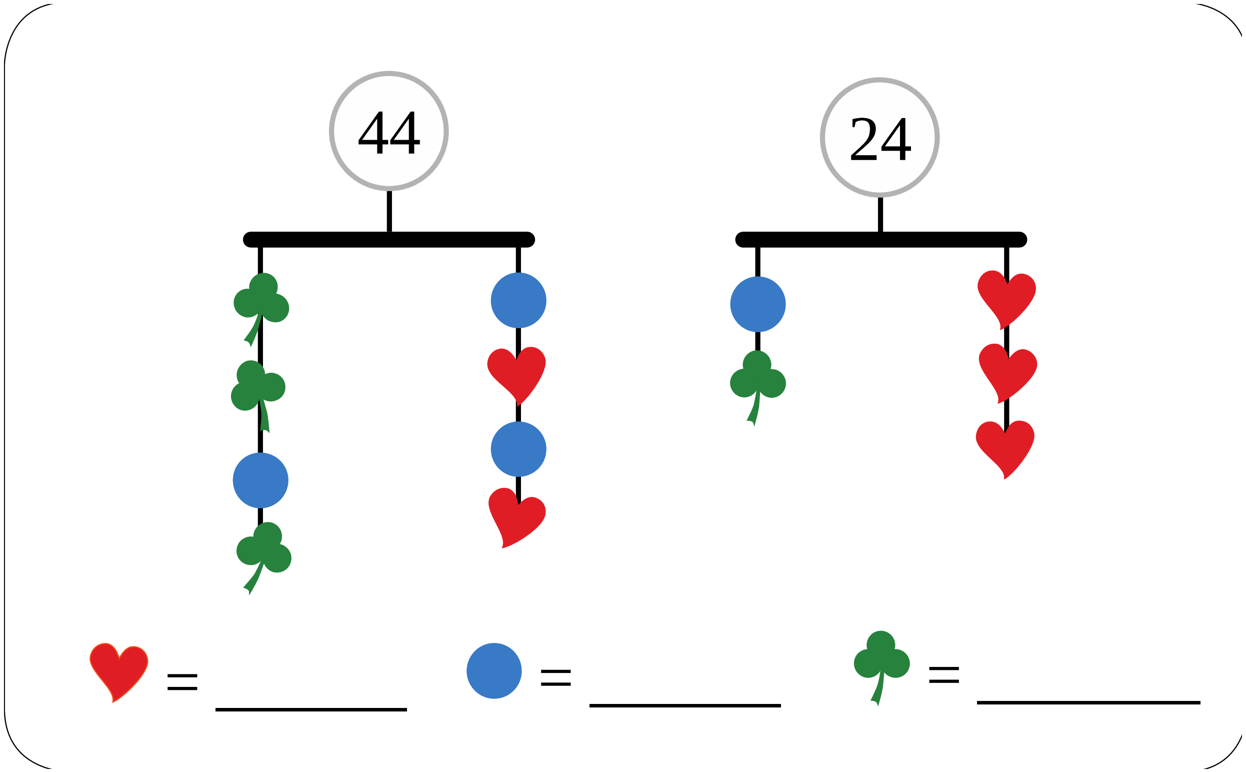


- Clues:
  - Blah
  - Blah
  - Blah
  - …

When kids invent their own puzzles,
they invent clues *at their level*.

Good puzzles are ones that are
easy enough to solve,
hard enough to be fun, and
the hard part is the puzzling through,
not the arithmetic.

Mobile Puzzles

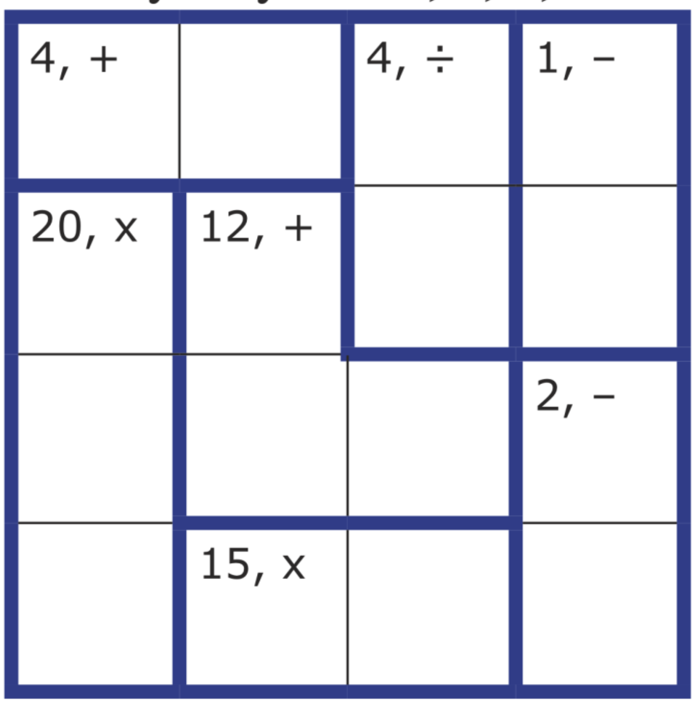


TransitionToAlgebra.com video

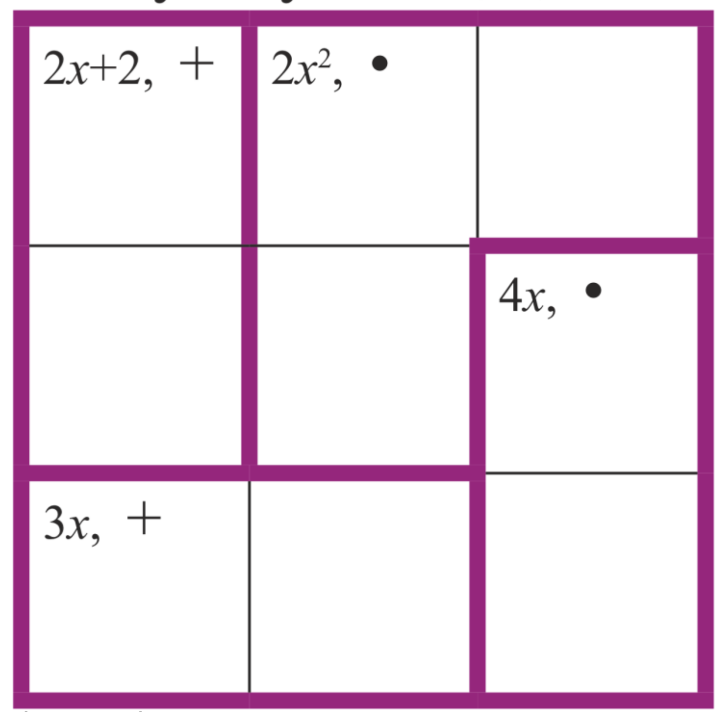
MysteryGrid Puzzles

Find puzzles like these on kenken.com

MysteryGrid **1, 3, 4, 5**



MysteryGrid **2, *x*, 2*x***



TransitionToAlgebra.com

2) *Algebraic Habits of Mind:* Puzzling and Persevering

Challenges to proficiency in algebra

- Students think math is rules to know and follow; formulas to apply
- Real problems aren't formulaic
- Even word problems aren't formulaic

How *Transition to Algebra* bridges the gap

Builds skill at figuring out where to start, looking for clues, puzzling through problems, all skills essential in algebra

CCSS, SMP#1: *Make sense of problems and persevere in solving them*

Program Components

Skip ahead

Professional Support Resources

Series Overview

- Research and guiding principles
- Pathways for embedding instruction into the curriculum

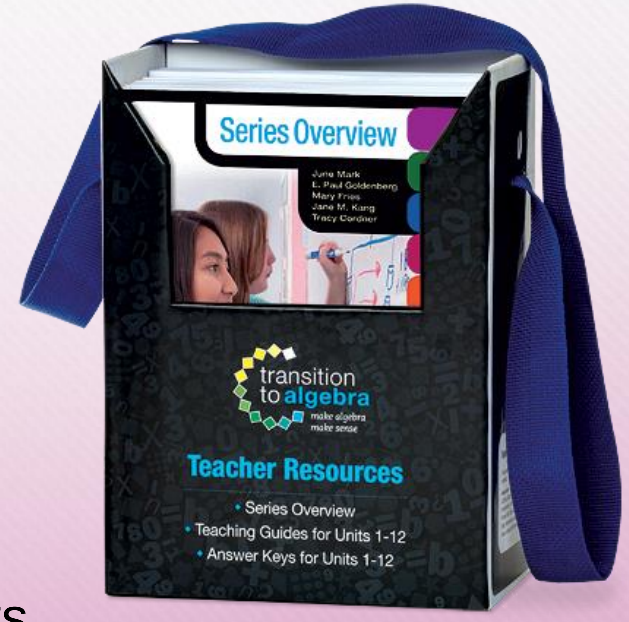
Teaching Guides (one per Unit)

- **Lesson Support -** Explorations, practice problems, Activities
- *Snapshot Check-ins*
- *Unit Assessments*

Answer Keys (one per Unit)

Downloadable Teaching Resources

- Templates, activities, assessments



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Program Components

12 UNITS

80 LESSONS

22 EXPLORATIONS

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Curriculum Units

Unit 1: Language of Algebra
Unit 2: Geography of the Number Line
Unit 3: Micro-Geography of the Number Line
Unit 4: Area and Multiplication
Unit 5: Logic of Algebra
Unit 6: Geography of the Coordinate Plane

Unit 7: Thinking Things Through Thoroughly
Unit 8: Logic of Fractions
Unit 9: Points, Slopes, and Lines
Unit 10: Area Model Factoring
Unit 11: Exponents
Unit 12: Algebraic Habits of Mind

Context for *Transition to Algebra*

- Students
- Teachers
- Administrators

- …and use in variety of contexts

Research on *Transition to Algebra*

- Analysis of score gains using a pre-post test design
  - 119 students, test of 48 items worth 100 points
  - Average gain of 37.4 points,
    statistically significant and equivalent to 1.9 standard deviations
  - Average gains statistically significant in every tested area, with largest gains in polynomials and factoring, expressions and equations, and graphs and tables

Research on *Transition to Algebra*

- Analysis of teacher fidelity of implementation
  - Sources: Classroom observation, teacher report
  - Average scores by teacher comparable at start of year; diverging by end of year. Suggests that stronger fidelity may be related to student outcomes
  - Finding is only suggestive due to very small teacher sample and exploratory nature of measures.

Research on *Transition to Algebra*

- Impact on algebra achievement and student attitudes using regression discontinuity design
  - 183 students in 11 classrooms in 2 schools, using a modified MCAS instrument
  - Students assigned to treatment or control classrooms using a cutoff score
  - Results were inconclusive, due to small sample size and limited variation in assignment scores
  - Lesson: regression discontinuity is very hard to do right

Research on *Transition to Algebra*

- Mixed methods study of district algebra support practices
  - 235 survey respondents, most math directors and curriculum coordinators
  - 92% said districts provide Algebra 1 supports for students at risk
  - Most common type of support (79%): additional intervention class; some tutoring or computer or web-based programs
  - Content varies: 86% said same content as Algebra 1;
64% pre-algebra;
52% preparation for state test;
20% study skills;
8% conceptual approaches, or build social and emotional skills
  - Most use teacher-developed materials

PD: an idea from student materials

The Student Worktext: *Lesson Structure*

Lesson 3: Balancing Mobile Puzzles

IMPORTANT STUFF

Thinking Out Loud

Michael, Lena, and Jay are working on this problem.

If = , then what number(s) can the represent?

Lena: I never thought about it before, but that statement uses an equals sign even though the two sides don't look the same!

Jay: They aren't the same, but they do have the same value. That's what it means to be equal.

Michael: So the question is, *when* do they have the same value? When will it be true?

Lena: We need to figure out what has to be in a bucket for the two sides to be the same. Since each bucket holds the same amount, I can remove the matching buckets because that won't affect the balance. *(Lena crosses out two buckets:* = *and Jay writes the new equation* = *.)*

Jay: That leaves us with = and we remove the matching ones. *(Lena draws* = *.)*

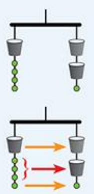
Michael: So = 3. That makes sense! If = 3, then = 7 and = 7. So they're equal!

Discuss & Write What You Think

① If = 2, would the statement " = ." be true? Why or why not?

Thinking Out Loud

Jay: I started in a completely different way. I knew that 1 bucket plus 4 is the same number as 2 buckets plus 1. So, I pictured them balancing on a mobile. *(Jay draws a mobile.)*



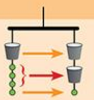
Michael: Oh. Because they're equal, the two sides have to balance!

Jay: Yeah! We can imagine the buckets and ones hanging from the strings. Just like before, the bucket holds my original number, but we can't see inside. Anyway, I saw that the top of each side is a bucket and the bottoms both have ones, so the middles have to match up, too. *(Jay draws arrows.)*

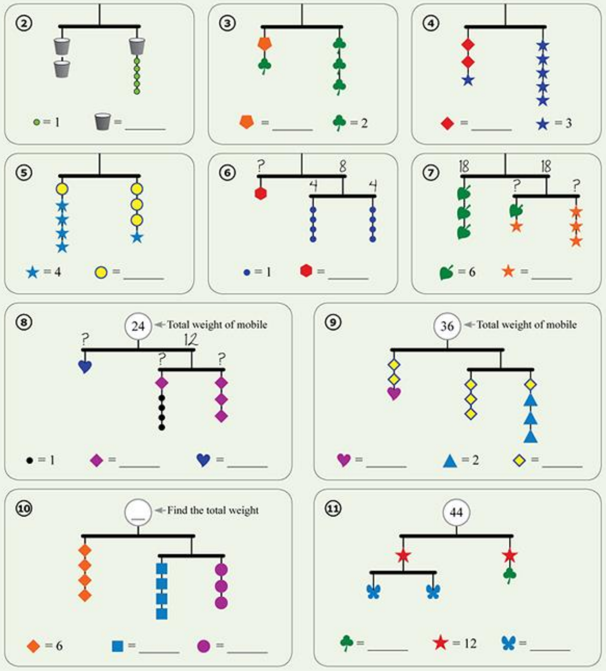
Michael: I get it! To make it balance, that bucket on the right has to weigh the same as the 3 ones on the left. That's how the chunks of stuff match up!

Algebraic Habits of Mind: *Seeking and Using Structure*

Jay looks at the mobile and sees that a whole chunk of the left side (the first three marbles) matches with one piece on the right side. Switching between looking at individual objects and groups of objects can make problems simpler to solve.



Every beam in these mobiles is balanced. The strings and the beams weigh nothing. Find the weight of each shape.



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3) *Algebraic Habits of Mind:* **Communicating with precision**

Challenges to proficiency in algebra

- Develop and practice using mathematical language and academic discourse
- Expressing what they know with confidence

How *Transition to Algebra* bridges the gap

- Models discussions; encourages students to explain their reasoning
- Develops and refines academic language

CCSS, SMP#6: *Attend to precision;* **SMP#3** *Construct Viable Arguments and Critique the Reasoning of Others*

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Dialogues

- *Implementing the Mathematical Practice Standards* (IMPS)

Dividing fractions	Multiplying fractions	Adding fractions Anita's way

mathpractices.edc.org

Who?

Paul Goldenberg, Al Cuoco, Mark Driscoll, June Mark, Deborah Spencer, Katherine Schwinden, Victor Mateas, Johannah Nikula, Matt McLeod, Jane Kang, Mary Fries

Advisors: *Diane Briars, Dan Chazan, Brad Findell, Bill McCallum, Barbara Reys, Mike Shaugnessy*

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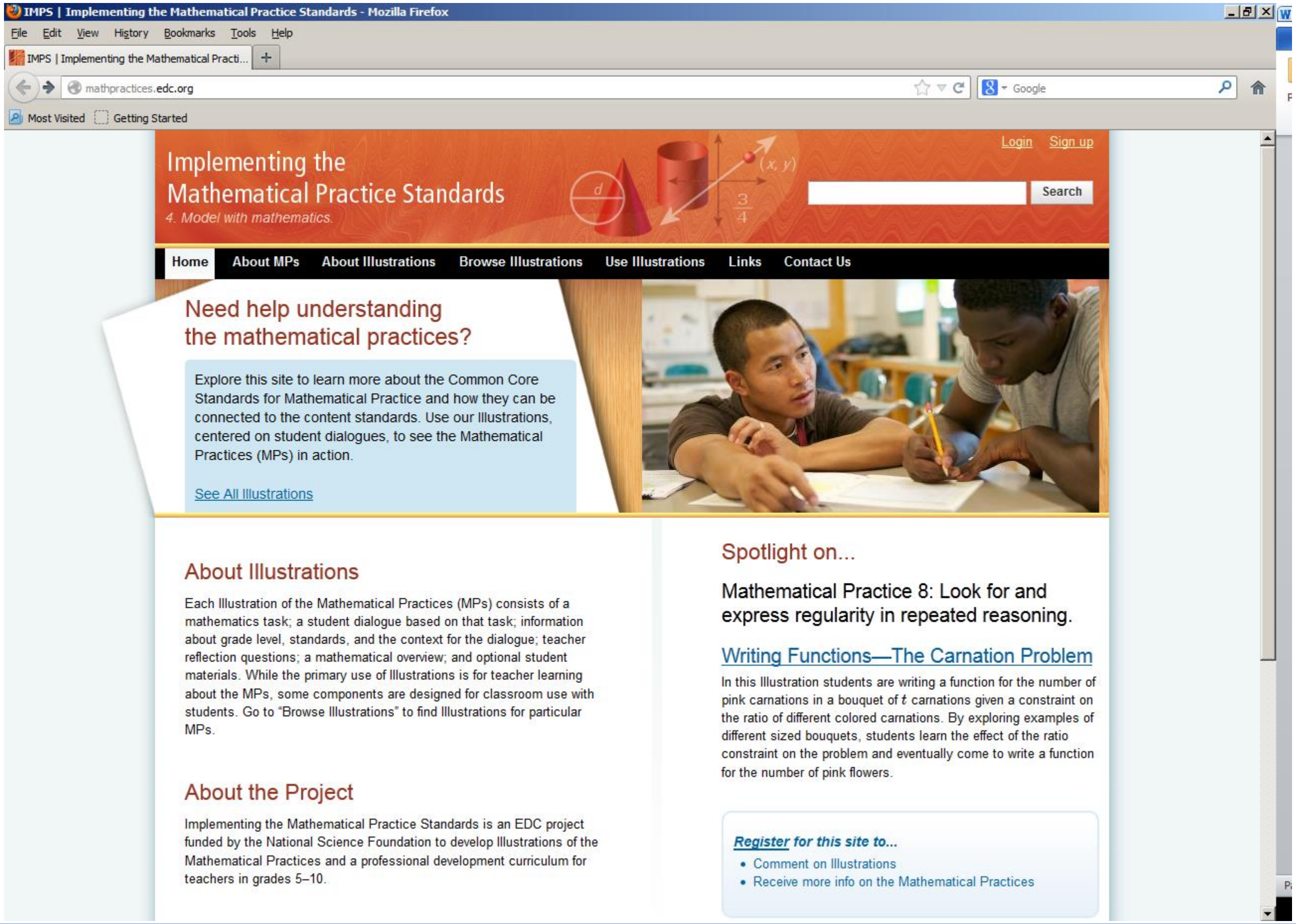
Project Goals

- Increase awareness of the Standards for Mathematical Practice (SMP)
- Support understanding of the SMP connected to content standards
- Cultivate teachers’ capacity to identify these SMP in student thinking
- Develop teachers’ ability to plan instruction to support these SMP

mathpractices.edc.org



Illustrations

- Student dialogues clarify the *meaning* of SMP by showing what a conversation among students engaging in SMP might look like.
  - Embedded in the context of specific mathematical content.
  - Model productive mathematical discourse.
  - Strategically chosen student characters.

- Each dialogue is accompanied by supporting materials:
  - A mathematical problem
  - Teacher discussion/reflection questions
  - A mathematical overview
  - Follow-up activities and discussion questions for students

About the Illustrations as a Set

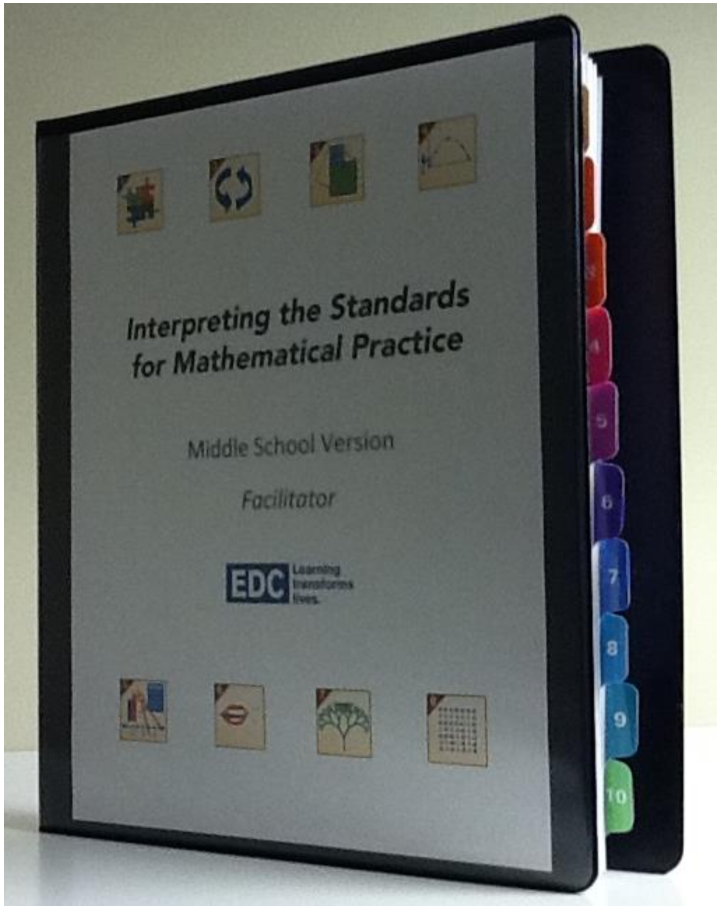
- 31 Illustrations developed and reviewed to date
- Multiple SMP identified in each dialogue
- Several Illustrations that address each SMP
- Grade levels from 5-10
- Number, algebra, geometry, data and statistics
- Range of mathematical tasks, some more open-ended

Professional Development

Materials for 20 Hour
Professional Development Course
(MS & HS Versions)

Three Main Activity Types:

- Doing and Discussing Mathematics
- Analyzing Artifacts of Student Thinking
- Connecting to Classroom Practice



Professional Development Main Components

- **Doing and Discussing Mathematics**
  - Exploring IMPS mathematics tasks as mathematical learners
  - Discussing own use of standards for mathematical practice (MPs)
- **Analyzing Artifacts of Student Thinking**
  - Dialogues that are part of the Illustrations
  - Video or written work from participants' students based on IMPS tasks
  - Video or written work from sample students (provided with PD materials)
- **Connecting to Classroom Practice**
  - Anticipating and planning for student engagement in MPs
  - Planning around IMPS tasks
  - Adapting tasks from teachers' own curricula

Professional Development Structure

- Ten two-hour sessions
- Sessions can be grouped into larger chunks
- Flow of sessions organized by content strand
- All SMP highlighted over time through Illustrations
- Options for middle school and high school groups

Professional Development Materials

Facilitator Guide

- Session agendas and goals
- Materials and prep
- Activity instructions
- Discussion questions/prompts
- Facilitator Notes

Participant Materials

- Math tasks and dialogues
- Planning protocols
- Student work analysis protocols

Sample student work/video

Assessing Secondary Teachers' Algebraic Habits of Mind

Partners:

- Boston University (Glenn Stevens)
- EDC (Sarah Sword, Al Cuoco, Miriam Gates, Jane Kang)
- St. Olaf College (Ryota Matsuura)

Generalizing from Repeated Reasoning

A high school Algebra class is considering the following:

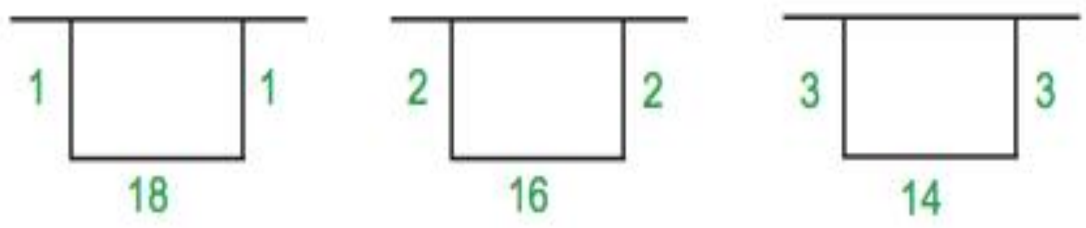
Barry has 20 m of wire mesh. He would like to enclose as large as possible a rectangular chicken coop using a barn wall as one of the sides. What values for the length and width give him the largest area?

Generalizing from Repeated Reasoning

T: Just kind of look at it. If the length is just 1, then what does the width have to be?

S: Is it 9? Oh, it has to be 18. I thought there were four sides.

T: So when the length is 1, the width has to be 18, since they have to add up to 20. What if the length is 2? What if it's 3?



Generalizing from Repeated Reasoning

Students begin to recognize a linear pattern in widths.
The teacher asks for the width if the length is x.
Students first say $x - 2$, then $2x$.
The teacher suggests they check $2x$ by substituting 4 for x.

T: So if $x = 4$,the width will be 8. Is that true?

S: No, you have to plus something.

T: How do you figure it out? What is it supposed to be if it's 4?

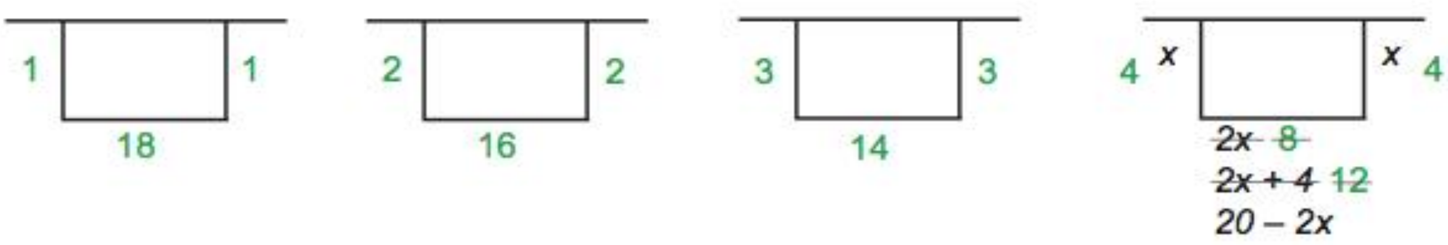
S: 12.

Generalizing from Repeated Reasoning

Students then suggest $2x + 4$ (to get 12), but they realize their error after substituting 5 for x.

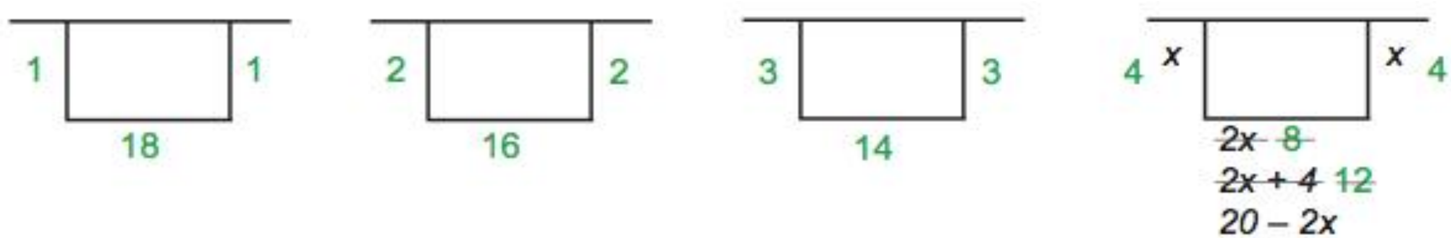
T: So we know it's actually going down by 2. [Student name], why did you pick $2x$?

S: Because there are two sides for the x. Oh, you minus 20 to give you the width.



Generalizing from Repeated Reasoning

Students begin to articulate that to find the width, they had been multiplying the length by 2 and then subtracting that from 20, which is the amount of wire that was initially given.



Driving Research Question

What mathematical habits of mind do secondary teachers use, how do they use them, and how can we measure them?

Instrument Development

To investigate our research question, we've been developing:

- Detailed definition of MHoM, based on literature, our own experiences as mathematicians, and classroom observations.
- An observation framework for understanding the nature of teachers' use of MHoM in their classroom work.
- A paper and pencil (P&P) assessment that measures how teachers use MHoM when doing mathematics for themselves.

Note: We have developed all three components together.

Remarks

- Our original intent was to assess our *own* PD programs and learn how to strengthen their impact.
- We are creating instruments for research and development, *not* for teacher evaluation. The instruments are intended to help researchers, district leaders, and PD developers better understand and meet the mathematical needs of secondary teachers.
- Our research is centered on understanding the nature of MHoM and the roles these habits play in teaching. We recognize that MHoM constitute just one aspect of a broad spectrum of knowledge and skills that teachers bring to their profession.

A Focus on Three Habits

Our current focus is on three categories of MHoM:

- **EXPR.** Engaging with one's experiences
- **STRC.** Making use of structure to solve problems
- **LANG.** Using mathematical language precisely

Remark: Focusing on three habits has allowed us to create a P&P assessment that is not too burdensome to use and has focused our observation work. Eventually, we will investigate other habits, too.

Connection to CSSM

Our three mathematical habits are closely related to the following Common Core Standards for Mathematical Practice:

- **MP2.** Reason abstractly and quantitatively
- **MP6.** Attend to precision
- **MP7.** Look for and make use of structure
- **MP8.** Look for and express regularity in repeated reasoning

Features of the P&P Assessment

- Assessment measures how secondary teachers use mathematical habits of mind when doing mathematics.
- Items are accessible: most secondary teachers can solve them, or at least begin to solve them.
- Coding focuses on the *approach*, not on “the correct solution.”
- Assessment items are drawn from multiple sources, including our classroom observation work.
- Field tested to date with over 500 teachers. (Field tests are on-going.)

Early Pilot Data (*n*=39)

Initial Reliability/Validity	Result
Internal Consistency	0.87 (Cronbach's Alpha) 0.90 (Guttman Lambda 6)
Construct Validity	Convergent and divergent validity based on initial PCA results
Item Discrimination Analysis	"High" group of participants outscored "low" participants with statistical significance ($p < .05$)

Sample Item: Maximum Value

Find the maximum value of the function

$$f(x) = 11 - (3x - 4)^2.$$

Notes

- Though most teachers obtained the same (correct) answer, there were vast variations in their approaches.
- These various approaches came in “clumps,” as our advisors (assessment experts) and research literature had told us to expect.
- Using these responses, we developed a rubric that allows us to code *how* each teacher solved the problem.

Sample Code: SQR

$f(x) = 11 - (3x-4)^2$. Anything squared is ≥ 0.

Therefore, $11 - (\text{stuff squared})$ must be ≤ 11. So 11 is the max.

Sample Code: SYMM

$$f(x) = 11 - (3x-4)^2$$
$$= -9x^2 + 24x - 5$$

x-coord. of vertex:

$$\frac{-b}{2a} = \frac{-24}{2(-9)} = \frac{-24}{-18} = \frac{4}{3}$$

$$f\left(\tfrac{4}{3}\right) = 11 - \left(3\left(\tfrac{4}{3}\right) - 4\right)^2$$
$$= 11 - (4-4)^2$$
$$= \boxed{11}$$

max value is 11.

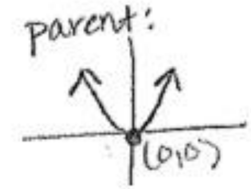
Sample Code: TRANS

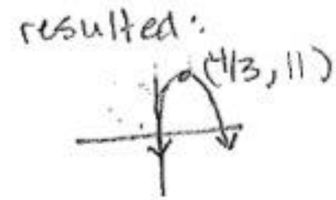
Parent function is $f(x) = x^2$.

$f(x) = 11-(3x-4)^2$ is a matter of translations from the parent function.

$f(x) = -(3x-4)^2 + 11$ so it has shifted up 11 units$^{(+11)}$, is flipped about the x-axis$^{(-)}$, and shifted right 4/3 units$^{(3x-4)}$.

$\boxed{\text{Maximum value point is at } (\tfrac{4}{3}, 11).}$





Sample Code: CALC

$$f(x) = 11 - (9x^2 - 12x - 12x + 16) = -9x^2 + 24x - 5$$

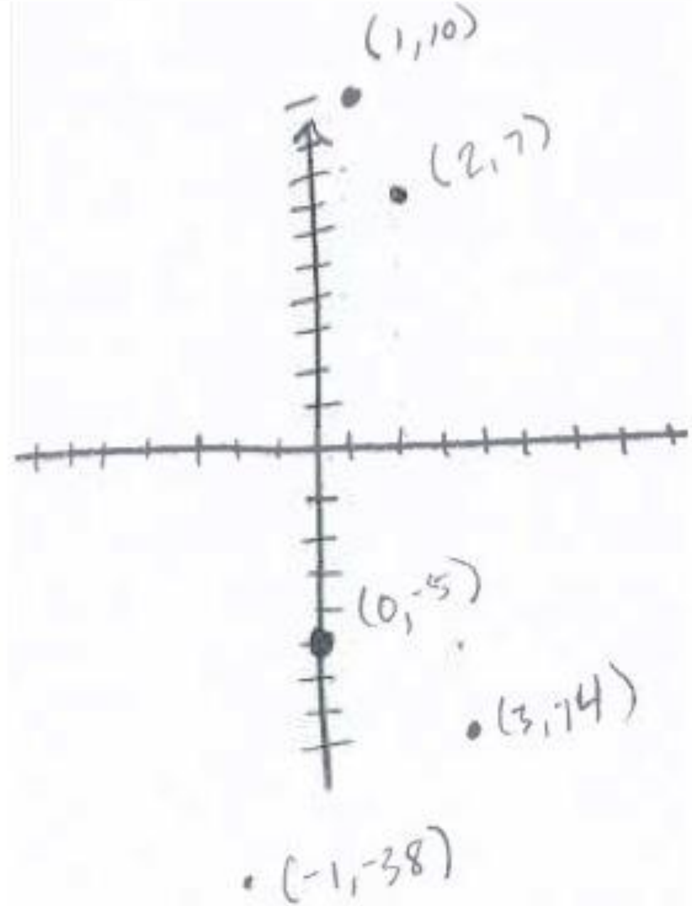
$$f'(x) = -18x + 24 \qquad \text{when } x = \frac{4}{3}$$

$$-18x + 24 = 0 \quad x = \frac{4}{3} \qquad f\left(\frac{4}{3}\right) = 11 - \left(3\left(\frac{4}{3}\right) - 4\right)^2 = 11$$

Sample Code: PNTS

x	$11-(3x-4)^2$	y
-2	$11-(-6-4)^2$	-89
-1	$11-(-3-4)^2$	-38
0	$11-(-4)^2$	-5
1	$11-(3-4)^2$	10
2	$11-(6-4)^2$	7
3	$11-(9-4)^2$	-14
4	$11-(12-4)^2$	-53

max will be somewhere in this range.



Final Note

Assessment development is ongoing – we are currently field testing with a group of 200+ teachers. If you're interested in participating, let us know:

ssword@edc.org

QUESTIONS?