Building Mathematical Knowledge for Teaching Proof in Geometry

Geometry Proof Scaffold: A Pedagogical Framework for Teaching Proof

Sub-Goals	Descriptions	Competencies
Understanding Geometric Concepts	This sub-goal highlights the importance of understanding the building blocks of geometry.	1) Having accurate "mental pictures" of geometric concepts (i.e., having a concept image)
		2) Being able to verbally describe geometric concepts, ideally being fluent with one or more definitions of the concept (i.e., having or developing a concept definition)
		3) Determining examples and non-examples
		4) Understanding connections between classes of geometric objects, where they overlap, and how they are contained within other classes (i.e., understanding mathematical hierarchy)
Defining	This sub-goal highlights the nature of definitions, their logical structure, how they are written, and how they are used.	1) Writing a "good" definition (includes necessary and sufficient properties)
		2) Knowing definitions are not unique (i.e., geometric objects can have different definitions)
		3) Understanding how to write and use definitions as biconditionals
Coordinating Geometric Modalities	This sub-goal highlights the ways in which the mathematics register draws on a range of modalities.	1) Translating between language and diagram
		2) Translating between diagram and symbolic notation
		3) Translating between language and symbolic notation
Conjecturing	This sub-goal recognizes that conjecturing is an important part of mathematics and proving.	1) Understanding that empirical reasoning can be used to develop a conjecture but that it is not sufficient proof of the conjecture
		2) Being able to turn a conjecture into a testable conditional statement.
		3) Seeking out counterexamples to test conjectures and knowing that only one counterexample is needed to disprove a conjecture
		4) Understanding that when testing a conjecture, you are testing it for every case so you might begin by writing: "All," "Every, or "For any"
Drawing Conclusions	This sub-goal presents the idea of an open-ended task that leads to conclusions that can be drawn from given statements and/or a diagram.	1) Understanding what can and cannot be assumed from a diagram
		2) Knowing when and how definitions and/or "given" information can be used to draw a conclusion from a statement about a mathematical object
		3) Using postulates, definitions, and theorems (or combinations of these) to draw valid conclusions from some given information
Understanding Common Sub-arguments	This sub-goal recognizes that there are common short sequences of statements and reasons that are used frequently in proofs and that these pieces may appear relatively unchanged from one proof to the next.	1) Recognizing a sub-argument as a branch of proof and how it fits into the larger proof
		2) Understanding what valid conclusions can be drawn from a given statement and how those make a sub-argument (i.e., knowing some commonly occurring sub-arguments)
		3) Understanding how to write a sub-argument using acceptable notation and language (where "acceptable" is typically determined by the teacher)
Understanding Theorems	This sub-goal highlights the nature of theorems, their logical structure, how they are written, and how they are used.	1) Interpreting a theorem statement to determine the hypothesis and conclusion, and, if needed, providing an appropriate diagram
		2) If applicable, marking a diagram that satisfies the hypothesis of a proof
		3) Rewriting a theorem written in words in symbols and vice versa
		4) Understanding that a theorem is not a theorem until it has been proven
		5) Understanding that one cannot use the conclusions of the theorem itself to prove the conclusions of that theorem (i.e., avoiding circular reasoning)
		6) Understanding that theorems are mathematical statements that are only sometimes biconditionals
		7) Understanding the connection between logic and a theorem, for example, how to write the contrapositive of a conditional statement
Understanding the Nature of Proof	This sub-goal highlights the nature of proof, proof structure, and how the laws of logic are applied.	1) Understanding that the only way to sanction the truth of a conjecture is through deductive proof (rather empirical reasoning)
		2) Exploring a pathway for constructing a proof (i.e., the problem solving aspect of proving)
		3) Understanding that proofs are constructed using axioms, postulates, definitions, and theorems and that they follow the laws of logic
		4) Knowing what language is acceptable to use and how to write up a proof
		5) Recognizing that if you prove that something is true for one particular geometric object, then it is true for all of them

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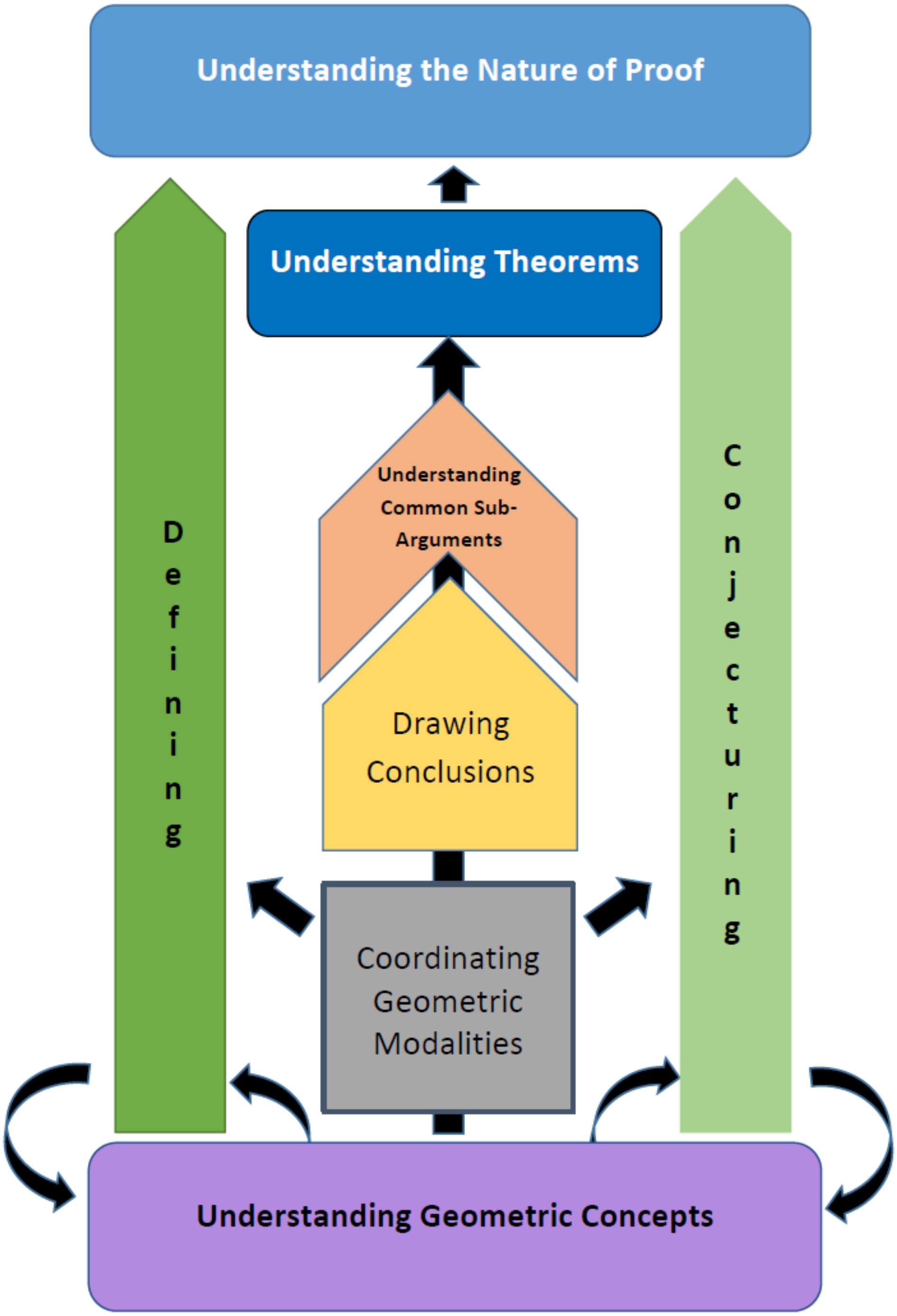
A Visual Representation of the Geometry Proof Scaffold



Proof in Secondary Classrooms Project (PISC) **– PI: Michelle Cirillo, University of Delaware**
NCTM Regional Meeting October 3, 2019 1:00 – 2:00

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