Running Head: IMPROVING MATH INSTRUCTION AT SCALE

Towards an Empirically Grounded Theory of Action for Improving the Quality of Mathematics Teaching at Scale

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Abstract

Our purpose in this article is to propose a comprehensive, empirically grounded theory of action for improving the quality of mathematics teaching at scale. In doing so, we summarize current research findings that can inform efforts to improve the quality of mathematics instruction on a large scale, and identify questions that are yet to be addressed. We draw on an ongoing collaboration with mathematics teachers, school leaders, and district leaders in four urban school districts in the US. Each year, we make recommendations to the districts based on the data we collect about how they might revise their strategies for instructional improvement to make them more effective. The provisional theory of action that we report encompasses curriculum materials and district-developed instructional guidance instruments, formal and job-embedded teacher professional development, teacher networks, mathematics coaches' practices in providing job-embedded support for teachers' learning, school instructional leadership in mathematics, and district leaders' practices in supporting the development of school-level capacity for instructional improvement. In the final section of the article, we discuss areas in which additional studies are needed if research is to provide adequate guidance to school and district leaders who are attempting to support mathematics teachers' development of equitable, inquiry-oriented instructional practices.

Towards an Empirically Grounded Theory of Action for Improving the Quality of Mathematics Teaching at Scale

What does it take to improve the quality of mathematics instruction on a large scale? In this article, we address this question by proposing a comprehensive, empirically grounded theory of action for improving the quality of mathematics teaching at scale. In doing so, we draw on our ongoing collaboration with mathematics teachers, school leaders, and district leaders in four urban school districts in the US that serve a total of 360,000 students. Some components of the theory of action are provisional, given the lack of a solid research base on which to draw. In addition to discussing current research findings that are relevant to the challenge of improving the quality of mathematics teaching at scale, we suggest a range of pressing issues that are yet to be addressed.

Improving the quality of mathematics instruction across classrooms, schools, and broader educational jurisdictions is a pressing issue for both researchers and practitioners. School leaders in a number of countries are under increasing pressure to improve the quality of learning opportunities for students in mathematics (Even & Ball, 2010). However, the issue of how to support instructional improvement on a large scale continues to be under-researched (Coburn, 2003; Cohen, Moffitt, & Goldin, 2007; Stein, 2004). As a consequence, research can currently provide only limited guidance to district and school leaders who aim to improve the quality of mathematics teaching, and thereby the learning opportunities for large numbers of students.

In many respects, mathematics education researchers are well positioned to investigate the improvement of the quality of instruction at scale. At present, mathematics education researchers know a great deal about the development of students' learning and forms of instructional practices that support learning. Research in mathematics education and related fields has made considerable progress in recent years in documenting learning progressions in specific mathematical domains (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Lehrer & Lesh, 2003) and in identifying a common set of learning goals that focus on enduring, sophisticated understandings of central mathematical ideas (Kilpatrick, Swafford, & Findell, 2001). Recent research efforts have also delineated a set of instructional practices that support students' development of these mathematical ideas (Franke, Kazemi, &

Battey, 2007; National Council of Teachers of Mathematics, 2000). Additionally, in several countries, including the US, research-based instructional materials and associated resources have been developed that are aimed at supporting the enactment of these practices (Remillard, 2005; Senk & Thompson, 2003). The mathematics education research community has also learned a great deal about professional development that supports teachers' development of practices aimed at ambitious learning goals for students (Ball, Sleep, Boerst, & Bass, 2009; Kazemi & Hubbard, 2008; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). However, these advances have had limited impact on instruction in most US classrooms, which continues to focus primarily on performing procedures at the expense of understanding mathematical ideas and relationships (Stigler & Hiebert, 1999).

There have been a number of successful professional development programs in the US that have supported improvement in teachers' knowledge and instructional practices (Borko, 2004). These programs include Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996), Developing Mathematical Ideas (Bell, Wilson, Higgins, & McCoach, 2010), and Quantitative Understanding: Amplifying Student Achievement and Reasoning (Stein, Grover, & Henningsen, 1996). However, most programs have been researcher-led and there is little evidence that improvements in the quality of instruction are sustained after the research project ends. More generally, studies in educational policy indicate that large-scale improvement efforts in mathematics and other subject matter areas have rarely produced lasting changes in teachers' instructional practices (Elmore, 2004; Gamoran et al., 2003; McLaughlin, 2006). The central challenge for instructional improvement at scale is not merely whether researchers can support groups of teachers' learning, but whether researchers can support schools' and broader educational jurisdictions' development of the capacity to scaffold teachers' (and others') ongoing learning (Bryk, Sebring, Allensworth, Luppesco, & Easton, 2010; Elmore, 2004; Sebring, Allensworth, Bryk, Easton, & Luppesco, 2006). In other words, instructional improvement at scale is a problem of organizational learning as well as teacher learning.

In addressing the challenge of scale, it is important to acknowledge that the influence of teacher professional development on classroom practice is mediated by the school and broader settings in which teachers work and in which they reorganize their classroom practices (Cobb, McClain, Lamberg, & Dean, 2003; Coburn, 2005; Stein, 2004). As Coburn (2003) observed, "[B]ecause classrooms are situated in and inextricably linked to the broader school and system, teachers are better able to sustain change when there are mechanisms in place at multiple levels of the system to support their efforts" (p. 6). In other words, supporting instructional improvement across a large number of classrooms requires that the settings in which teachers work be organized to support their learning. This in turn implies that members of other role groups, including mathematics coaches and school leaders, will need to reorganize their practices. A comprehensive theory of action for instructional improvement at scale therefore aims to inform schools' and broader jurisdictions' development of the capacity to support and coordinate the learning of the members of multiple role groups.

Research on large-scale instructional improvement has traditionally been the province of educational policy and educational leadership. While much can be learned from these studies, most of this work does not take a position on what counts as high-quality teaching but instead operationalizes it in terms of increasing student test scores irrespective of the quality of the tests. One of the goals in this article is to illustrate that views on what counts as high-quality mathematics teaching matter when formulating strategies or policies for instructional improvement. As a first step, we note that the notion of instructional quality is grounded in a view of what is worth knowing and doing mathematically, and thus in an explicit set of goals for students' mathematical learning. The learning goals that we and the leaders in the four collaborating districts view as worthwhile include that students should develop conceptual understanding of key mathematical ideas and procedural fluency in a range of domains (e.g., number and operations, algebra, geometry, measurement, data analysis and probability), and that they should master increasingly sophisticated forms of mathematics argumentation (including methods of proof) and should learn to communicate their mathematical reasoning effectively by using and making connections between multiple representations (e.g., words, graphs, tables) (Kilpatrick, et al., 2001; National Council of Teachers of Mathematics, 2000).

A corresponding research-based vision of instruction that aims at these goals is articulated in the National Council of Teachers of Mathematics' (NCTM, 2000) *Principles and Standards for School Mathematics*. This vision is often referred to as a vision of *ambitious* teaching (Lampert, et al., 2010). In this vision, teachers support students to solve cognitively-demanding tasks (Stein, 2004); teachers press students to provide evidence for their reasoning and to make connections between their own and their peer's solution strategies (McClain, 2002); and teachers orchestrate whole class discussions in which they build on student contributions to support their understanding of key mathematical ideas (Franke, et al., 2007; Stein, Engle, Smith, & Hughes, 2008). Instructional practices of this type contrast sharply with typical teaching in most US classrooms and require teachers to anticipate and respond to students' thinking (Kazemi, Franke, & Lampert, 2009).

A central goal of ambitious teaching is that learning opportunities are distributed equitably (Lampert & Graziani, 2009; NCTM, 2000). In this context, equity implies that all students should be able to participate substantially in all phases of classroom activities. However, in the course of collaboration with the four districts, it has become clear that research in mathematics education currently provides only limited guidance about concrete instructional practices that result in equitable learning opportunities. Although there is a sizable body of literature on equity and mathematics achievement, only a small proportion of these studies focus on classroom teaching and learning; of those studies, only a handful describe and provide empirical evidence for concrete instructional practices that support all students' substantial participation in each phase of classroom lessons (Boaler & Staples, 2008; Franke, et al., 2007; Gutiérrez, 2002; Moschkovich, 2007).

In the context of our current work, we conducted an exhaustive review of over 800 articles that focused on equity and mathematics learning and analyzed video-recordings of high-quality classroom instruction in order to identify a set of instructional practices that aim specifically at supporting struggling students' substantial participate in inquiry-oriented instruction (Jackson & Cobb, 2010). The criteria we used to identify these practices were that 1) there is empirical evidence that they can support all students' substantial participation in various phases of ambitious mathematics lessons, and 2) they are potentially learnable by teachers in the context of high-quality PD. Given the communities served by the collaborating districts, we focused specifically on groups of low-performing African American students and English Language Learners (ELLs). The practices concern introducing tasks so that all students can engage in them mathematically and planning and leading whole class discussions to support all students' participation (Jackson & Cobb, 2010; Stein, et al., 2008). We refer to this elaborated vision of high-quality mathematics teaching as "ambitious and equitable" to highlight the equity dimension that is often taken for granted in conceptualizations of inquiry-oriented instruction (Jackson & Cobb, 2010).

In what follows, the learning goals we have outlined and a vision of ambitious and equitable instruction orient our discussion of research that seeks to inform the improvement of mathematics instruction at scale.

We organize our discussion of current research and future directions by considering key components of a comprehensive *theory of action* for instructional improvement in mathematics at the level of a large school district. The five components are: a coherent instructional system for ambitious and equitable instruction that encompasses both formal and job-embedded teacher professional development; teacher networks; mathematics coaches' practices in providing job-embedded support for teachers' learning; school leaders' practices as instructional leaders in mathematics; and district leaders' practices in supporting the development of school-level capacity for instructional improvement. As our focus in the remainder of this article will be on instructional improvement at the level of large US school districts, we give an overview of the US educational system before discussing the components of a comprehensive theory of action. We anticipate that most of these components will be relevant to non-US readers but acknowledge that the appropriate organizational unit or educational jurisdiction beyond the school will differ depending on the structure of the educational system in a particular country.

The US Educational System

The US educational system is decentralized, and there is a long history of the local control of schooling. Each US state is divided into a number of independent school districts. In rural areas, districts might serve less than 1,000 students whereas a number of urban districts serve more than 100,000 students. In the context of the US educational system, when we speak of scale we have in mind the improvement of mathematics teaching and learning in urban districts as they are the largest jurisdictions in which it is feasible to design for improvement in the quality of instruction (Supovitz, 2006).

Large school districts such as those with which we are collaborating have a central office whose staff are responsible for selecting curricula and for providing teacher professional development in various subject matter areas including mathematics. In this article, we use the designation *district leaders* to refer to members of the central office staff whose responsibilities focus on either classroom instruction or school leadership. We use the term *district mathematics specialists* to refer to central office staff whose responsibilities focus specifically on the teaching and learning of mathematics.

The role of the US national government in education has been quite limited historically when compared with most other industrialized countries. However, in 2001, the US Congress with the overwhelming support of both Republicans and Democrats passed a national policy called the No Child Left Behind (NCLB) act. The intent of NCLB is to enable all students to meet high performance standards in language arts and mathematics. States are given financial incentives to design and enact the three central components of NCLB policy: content standards, tests aligned with the standards, and mechanisms for holding schools accountable for increasing test scores. In addition to improving the overall mathematics achievement of all students, states are provided financial incentives to close gaps in achievement between sub-populations of students. Historically, students of color, students from economically disadvantaged backgrounds, and students for whom English is not their first language have performed at significantly lower levels than white students and students from economically advantaged backgrounds on mathematics assessments (Darling-Hammond, 2007).

Most impartial commentators consider that NCLB policy is flawed in two important respects. First, most states lacked the capacity to respond effectively to the assessment and accountability mandates of the policy (Elmore, 2004). As a consequence, the tests that most states selected to assess student achievement focus on low-level skills rather than central mathematical ideas (Shepard, 2002). Second, it is becoming increasingly clear that most district and school leaders have little if any knowledge of how to respond effectively to state accountability policies. As Elmore (2006) observed, most urban districts in the US are not equipped to respond to these pressures in ways that increase access to ambitious mathematics instruction for groups of students who are typically identified as "low-performing." Many districts are instead implementing strategies that involve "gaming the system" or "teaching to the test" (Heilig & Darling-Hammond, 2008). In the schools and districts that Elmore (2006) studied, for example, standards-based reform had become assessment-driven reform (cf. Resnick & Zurawsky, 2005). Elmore found that only a minority of schools and districts have developed at least a moderately "worked out" strategy for improvement that has the potential to both motivate and support teachers' improvement of their instructional practices.

The four districts with which we are collaborating are typical of urban districts in most respects and have to cope with a number of challenges including significant numbers of low-performing students, limited funding and high teacher turnover. However, they are atypical in one respect: they are amongst the minority identified by Elmore and are responding to high-stakes accountability pressures not by pushing teachers to “teach for the test,” but by supporting teachers’ development of inquiry-oriented instructional practices that aim at rigorous learning goals for students. In addition, district leaders have designed relatively coherent supports for teachers’ learning.

Particularly over the last few decades, it has become increasingly common for US school districts to adopt curricular programs to guide instruction in mathematics (Remillard, 2005). Available mathematics curricula vary significantly in terms of the goals for students’ learning, nature of tasks, sequencing of tasks, and support provided for teachers. The US National Science Foundation funded the development of several inquiry-oriented mathematics curricula that aim at the rigorous learning goals specified in the NCTM *Standards*. The four districts with which we are collaborating are all attempting to satisfy accountability demands by supporting teachers’ effective use of inquiry-oriented curricula of this type.

Investigating and Supporting Large-scale Instructional Improvement

The key components of a theory of action for instructional improvement at scale were identified in the course of our current work with the four collaborating districts. The overall goal of this work is to test, revise, and elaborate a comprehensive set of hypotheses and conjectures about district and school supports for improving classroom instructional practice aimed at rigorous goals for students’ mathematical learning. The participants in each of the four districts include 30 middle-school mathematics teachers who teach 12-14 year-old students from between six and ten schools, and 20 school and district leaders.

Thus far, we have completed three annual rounds of data collection and analysis. Each October, we interview leaders in each district to document their current strategies for supporting the improvement of middle-school mathematics instruction. In January-March of each year, we document how these strategies are actually playing out in schools and classrooms. The data we collect include: audio-recorded interviews conducted with the 200 participants that focus on the school and district settings in which the teachers and instructional leaders work (e.g., formal and informal sources of support, to who they are accountable and what they are accountable for); on-line surveys for teachers, coaches, and school leaders; video-recordings of two consecutive lessons in the 120 participating teachers' classrooms, coded using the *Instructional Quality Assessment* (IQA) (Crosson, Junker, Matsumura, & Resnick, 2003); teachers' and coaches' scores on the *Mathematics Knowledge for Teaching* (MKT) instrument (Hill, Schilling, & Ball, 2004); video-recordings of select district professional development; audio-recordings of teacher collaborative planning meetings; and an on-line assessment of teacher networks completed by all 300 middle-school mathematics teachers in the participating schools. In addition, the districts have provided us access to mathematics and reading achievement data for students in the participating teachers' classrooms. The interviews and online surveys were designed to measure each construct in our hypotheses and conjectures about school and district supports for instructional improvement (downloadable at http://bit.ly/MISTtools). Thus far, we have achieved almost 100% participation in our data collection efforts in each district each year.

Each February-May, we analyze transcripts of the 200 interviews to identify and explain gaps between each district's intended and implemented improvement strategies. On this basis, we develop a detailed report for leaders in each district in which we share our findings and make actionable recommendations on how they might adjust their improvement strategies to make them more effective. Each May, we visit the districts to discuss our findings and recommendations with district leaders. Unsolicited, the leaders in all four districts have told us that this is the most valuable research in which they have participated. The interviews conducted the following October reveal that the district leaders are indeed acting on our recommendations to a remarkable degree (Cobb & Jackson, 2010). As a consequence, we are, in effect, conducting four parallel design experiments at the level of large districts in which we are testing and revising our hypotheses about supports for instructional improvement at scale (Cobb & Smith, 2008).

Key Components of a Comprehensive Theory of Action for Improving the Quality of Mathematics Instruction at Scale

Our initial hypotheses and conjectures about district and school supports for instructional improvement (reported in Cobb and Smith, 2008) were relatively abstract. We refined and elaborated these hypotheses while conducting the three rounds of data collection, analysis, and feedback by pushing to the level of concrete, potentially learnable practices for members of specific role groups (e.g., teachers, coaches, school leaders). The resulting theory of action for district-level instructional improvement comprises the following five components: coherent instructional system, teacher networks, coaching, school instructional leadership, and district instructional leadership. Although we present each component separately, we contend that instructional improvement at scale requires the simultaneous coordination of all five components.

Coherent Instructional System

The first component of the theory of action concerns the construction of a coherent instructional system for supporting mathematics teachers' development of ambitious and equitable teaching practices. Based on Newmann et al.'s (2001) and Bryk et al.'s (2010) findings and our work (Cobb & Jackson, 2010), a coherent instructional system includes the following:

1) explicit goals for students' mathematical learning;
2) a detailed vision of high-quality instruction that specifies concrete instructional practices that will lead to the attainment of the learning goals;
3) instructional materials and associated tools designed to support teachers' development of these practices;
4) district teacher professional development that focuses on the specific practices, is organized around the above materials, and is sustained over time;
5) school-based professional learning communities (PLCs) that provide ongoing opportunities for mathematics teachers to discuss, rehearse, and adapt the practices that are introduced in district professional development;
6) assessments aligned with the goals for students' mathematical learning that can inform the ongoing improvement of instruction and identify students who are currently struggling; and
7) additional supports for struggling students to enable them to succeed in mainstream classes.

In the mathematics education research literature, various aspects of this system are often investigated separately. For example, research on the design of sequences of instructional tasks typically draws on research on student learning and fails to make contact with research on either mathematics teaching or teacher professional development. However, our ongoing analyses indicate that instructional improvement at scale is only possible in practice when district leaders deliberately coordinate the above elements so that they constitute a system in the true sense of the term.

Explicit goals for students' mathematical learning. When attempting to improve instruction at any level of scale, it is imperative to identify the learning goals toward which the instruction aims (Hiebert & Morris, 2009; Jansen, Bartell, & Berk, 2009; Wiggins & McTighe, 1998). In Jansen et al.'s (2009) terms, the learning goals should be 1) *targeted*, or "sufficiently well specified to suggest the interventions for supporting learners in achieving them and to indicate the types of evidence needed to determine if the goals have been achieved," and 2) *shared*, or "mutually understood and committed to by all participants in the knowledge-building process" (p. 525). Our current work suggests the importance of district leaders supporting teachers and members of other role groups in coming to understand the goals for students' mathematical learning. As we describe when we discuss district leadership, if different units within the district central office have different goals for students' learning, it is likely that they will, in turn, hold members of different role groups (e.g., teachers, school leaders) accountable for developing practices that are at odds with each other. These goals (in conjunction with a vision of high-quality instruction) should drive the design of the remaining elements of the instructional system.

Detailed vision of high-quality mathematics instruction. A second aspect of a coherent instructional system is a detailed vision of high-quality instruction that specifies concrete instructional practices that will lead to the attainment of the learning goals. This vision articulates the goals for teachers' learning. In the course of our current work, we have found it important that the guiding vision of instruction specifies a relatively small set of high-leverage instructional practices (Ball, et al., 2009) that are learnable in the context of high-quality professional development. We have come also to appreciate the value of specifying practices that are specific to particular phases of lessons (e.g., introducing a task to ensure all students are able to engage in solving the task, orchestrating a whole class discussion that furthers all students understandings) and for which there is evidence that the practices support all students' substantial participation (i.e., can lead to equitable learning opportunities). This set of concrete, high-leverage practices orients the design of teachers' professional development, and should also orient the delineation of high-quality coaching and school instructional leadership practices that aim to support teachers' learning (as described below).

Instructional materials and associated tools designed to support teachers' development of these practices. A third aspect of a coherent instructional system is the provision of instructional materials and tools designed to support teachers' development of the focal practices. As we have indicated, it is common practice for US districts to adopt a single mathematics text. The districts with which we work have all adopted middle-grades mathematics texts that aim at rigorous goals for students, and have developed inquiry-oriented visions of ambitious instruction. The findings of a number of studies indicate that inquiry-oriented curricula, when implemented effectively, improve students' conceptual understanding and problem solving more than instruction using traditional curricula, while having no negative effect on the development of procedural fluency (Cai, Nie, & Moyer, in press; Schoenfeld, 2002). However, there is also strong evidence that teachers' use of a text of this type does not ensure that they will enact the intended practices in their classrooms (Remillard, 2005; Riordan & Noyce, 2001; Tarr et al., 2008). For example, it is common for US teachers to proceduralize the cognitively demanding tasks in these texts when they introduce them to students, thus reducing the cognitive demand (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein, Smith, Henningsen, & Silver, 2000) and making it unlikely that the subsequent whole class discussion will support students' development of conceptual understanding. In this regard, Tarr et al. (2008) found that the classroom learning environment impacts the effectiveness of inquiry-oriented curricula, with an achievement advantage occurring only in classrooms where students explain their solutions, multiple strategies are encouraged, and lessons foster conceptual understanding. Investigations of the implementation of inquiry-oriented curricula also indicate that although subpopulation achievement gaps tend to diminish, they are not eliminated (Riordan & Noyce, 2001; Schoenfeld, 2002). This suggests that these curricula can be a valuable resource for addressing issues of equity but are not by themselves sufficient (Darling-Hammond, 2007).

It is important to clarify that teachers are expected to use the adopted text to address state mathematics standards. Districts frequently develop so-called curriculum frameworks to assist teachers in making this coordination when they plan for instruction. In their most elementary forms, curriculum frameworks do little more than specify the sequencing and pacing of instruction both within and across grade levels. However, some districts, including two with which we are collaborating, have developed elaborate curriculum frameworks that include information regarding student solutions to anticipate, what is likely to be linguistically demanding for students whose first language is not English in particular lessons, and strategies teachers might use to meet the needs of these and other groups of students.

Newmann et al. (2001) argue that the use of a common curriculum framework is a key element of a coherent instructional system. However, there is little if any research on how teachers actually use such frameworks. Our work indicates that districts typically provide limited professional development on using the framework, and most teachers use the framework developed by their district as a pacing guide. They rarely draw on other information in the framework to plan for instruction, including anticipating students' solutions or how to differentiate instruction. Changing the pacing of instruction involves adjusting current instructional practices whereas the effective use of other information in curriculum frameworks requires that teachers reorganize their current instructional practices. These observations suggest that teachers need sustained support to learn how to use frameworks productively.

Taken together, the findings we have discussed indicate that effective use of inquiry-oriented curricula and associated curriculum frameworks requires significant teacher learning, and that the equity dimension of ambitious teaching requires explicit attention. Ongoing professional development is required if teachers are to learn how to use these resources effectively as part of the process of developing the intended instructional practices (e.g., introducing tasks to support all students' substantial engagement while maintaining the cognitive challenge).

District teacher professional development. We purposefully separate teacher professional development that a district offers for all district mathematics teachers at particular grade levels from school-based professional development. Typically, teachers are released from teaching to attend district-wide professional development for a few days each year, whereas school-based professional development is often ongoing and involves only the mathematics teachers in a particular school.

Research literature suggests that professional development only impacts teacher knowledge and/or practices when it is sustained over time, involves the same group of teachers working together, is focused on issues central to instruction, and is organized around the tasks that teachers use in their classrooms (Darling-Hammond, Wei, & Orphanos, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Kazemi & Franke, 2004; Little, 2003). Current research in mathematics teacher education also indicates the value of organizing professional development around a limited set of concrete, high-leverage practices (Ball, et al., 2009; Grossman et al., 2009; Lampert, et al., 2010; Lampert & Graziani, 2009). In addition, recent work by Grossman and colleagues (Grossman, et al., 2009; Grossman & McDonald, 2008) indicates the value of distinguishing between two types of activities, both of which contribute to effective professional development: *pedagogies of investigation* and *pedagogies of enactment*. Pedagogies of investigation involve analyzing and critiquing representations of practice such as student work and video-cases of teaching (Borko, Jacobs, Eiteljorg, & Pittman, 2009; Sherin & Han, 2004). Pedagogies of enactment involve planning for, rehearsing, and enacting high-leverage practices in a graduated sequence of increasingly complex settings (e.g., teaching other teachers who play the role of students, working with a small groups of students, teaching an entire class). Pedagogies of investigation are far more common in mathematics teacher professional development than pedagogies of enactment. However Grossman et al. argue convincingly that both are necessary if teachers are to develop ambitious forms of practice.

Because large numbers of teachers typically participate in district professional development, we conjecture that it is better suited for pedagogies of investigation whereas school-based professional development is suited for both types of pedagogies. The importance of coordinating the two forms of professional development has become apparent in our current work such that they focus on the same set of high-leverage practices and are organized around the same instructional materials. For example, teachers might first analyze video-recordings of teachers introducing cognitively demanding tasks in district professional development, and then enact introducing similar tasks with their colleagues in school-based professional development. This coordinated approach focuses on supporting teachers' development of the same concrete practices over time, using materials that are central to their instruction.

Research in mathematics education has begun to made headway in delineating effective professional development practices (Borko, et al., 2009; Elliott et al., 2009). This work suggests the value of supporting facilitators' development of what Coburn and Russell (2008) call "routines of interaction." Routines of interaction are questions that professional development leaders pose routinely to press participants on a specific set of key issues (e.g., identifying the key mathematical ideas in tasks, identifying aspects of the task scenario that might be unfamiliar to some students, anticipating student solutions). Coburn and Russell present evidence that coaches subsequently pressed teachers on the same issues on which they had been pressed in coach professional development, and that teachers then began pressing each other on these issues. We therefore conjecture that the routines of interaction in district professional development will influence the nature and depth of interactions in school-based professional development and thus the extent to which the latter supports teachers' development of ambitious and equitable instructional practices.

School-based professional learning communities. Professional learning communities (PLCs) in which the mathematics teachers at a school meet together on a regular basis to work on problems of practice are a central aspect of school-based teacher professional development. It is becoming common for US districts to mandate that schools schedule time during the school day for teacher collaboration. These are costly initiatives, given that PLCs vary in the extent to which they support instructional improvement (Little, 1993). A growing number of studies indicate that when PLCs function well, they provide opportunities for teachers to address problems that arise in the course of instruction, integrate ideas and tools introduced in district professional development into practice, and rehearse specific practices (Bell, et al., 2010; Cobb, Zhao, & Dean, 2009). As we have indicated, we conjecture that the work of PLCs should follow up on district professional development, should focus on a set of high-leverage instructional practices, and should involve pedagogies of enactment as well as pedagogies of investigation if it is to support the participating teachers' development of ambitious and equitable instructional practices.

To date, research has focused primarily on researcher-led PLCs (Horn, 2005; Little, 2002). This work has identified features of productive PLCs, which include a focus on students' mathematical sensemaking, sustained attention to problems of practice, and clear principles about the work of learning and teaching (Horn, 2007; Horn & Little, 2010). Current research also provides some guidance on the types of activities in which teachers might engage (Kazemi, 2008; Lewis, Perry, & Murata, 2006). In addition, this work indicates the importance of leadership for PLCs in setting an agenda, initiating and guiding activities, and enacting routines of interaction. As we clarify below when we discuss mathematics coaching, we view coaches as the most likely candidates for providing this leadership.

For the most part, current research on PLCs has treated them as existing in an institutional vacuum and has not taken account of the school and district settings in which the participating teachers work. However, several studies indicate that these settings can profoundly affect the extent to which PLCs are productive for teacher learning (Cobb & McClain, 2006; Stein & Coburn, 2008). For example, one of the basic requirements for a PLC to be productive is that the participating teachers have deprivatized their instructional practices and are willing to discuss openly the problems that they encounter in practice. Cobb et al. (2003) reported a case in which school leaders made frequent classroom visits in order to monitor that instruction addressed state mathematics objectives and that students were on task. In this setting, teaching was highly privatized and it was 18 months before the teacher group became a genuine PLC with a common agenda that focused on problems of practice (Dean, 2005). These studies indicate the importance of taking into account the school and district settings and, in particular, clarifying the role of school leaders in organizing the conditions for PLCs (e.g., scheduling time for meetings) and shaping PLC interactions (e.g., the focus of their classroom observations and the intent of the feedback that they give teachers).

Assessments aligned with the goals for students' mathematical learning. A sixth element of a coherent instructional system concerns assessments that can inform the ongoing improvement of instruction and identify students who are currently struggling and may need additional support (Newmann, et al., 2001). In the context of high-stakes accountability in the US, assessment often connotes the administration of state achievement tests designed to determine the extent to which schools and districts are making adequate yearly progress in increasing student achievement. As we have noted, state mathematics achievement tests often assess procedural competency but not conceptual understanding (Shepard, 2002), and therefore cannot adequately inform instructional improvement that aims at ambitious learning goals (Schoenfeld, 2002). It is common for districts to develop their own assessments (frequently called benchmark assessments) that are administered several times during the school year. However, these assessments typically focus on procedural competence and are designed to predict whether students will be successful on state assessments.

One of the districts with which we are collaborating is implementing a formative assessment system that provides a useful model of assessments designed to inform instructional improvement. The district has established professional learning communities in each school and expects mathematics teachers to analyze student work to identify patterns and variation in student thinking, to identify aspects of instruction that might have contributed to the patterns and variation, and to plan future instruction to further all students' understandings of a particular mathematical idea (Kazemi & Franke, 2004; Little, Gearhart, Curry, & Kafka, 2003). In addition, this district has developed a series of assessments that are aligned with the ambitious learning goals of each unit of the adopted curriculum. Teachers are expected to administer these assessments midway through each unit rather than at the end, and to adjust their instruction based on an analysis of the results conducted with colleagues in professional learning community meetings (Shepard, 2006). In addition, these assessments can be used to identify students who are struggling not merely in terms of being assessed as proficient on the state test but with respect to the districts' ambitious learning goals.

In the context of high-stakes accountability, it is rare for a district to make this level of commitment to assessments that are explicitly designed to inform instructional improvement. However, assessments of this type appear to be a critical aspect of a coherent instructional system. The experience of the collaborating district indicates that their development requires the collaboration of the Curriculum and Instruction (C&I) unit (responsible for curriculum and teaching) and the Research, Evaluation, and Accountability (REA) unit within the district central office. We have found in our work that members of C&I typically have expertise in designing useful formative assessment tasks, whereas members of REA typically have expertise in validating student assessments and in developing data management systems.[1] Both forms of expertise are necessary when developing and administering district-wide formative assessments.

Additional supports for struggling students. A seventh aspect of a coherent instructional program concerns additional supports for struggling students that aim at supporting them to succeed in mainstream mathematics classes (Newmann, et al.,

[1] If members of REA do not have expertise in validating student assessments and in developing data management systems, it will be necessary for district leaders to work with outside consultants who have this expertise.

2001). It is common for districts to provide supplemental supports for struggling students (e.g., tutoring, additional or "double dose" mathematics classes). A recent study found that "nearly half of large [US] urban districts report double-period math instruction as the most common form of support for students with lower skills" (Durwood, Krone, & Mazzeo, 2010, p. 7). However, the research base is extremely thin regarding the nature of instruction in additional mathematics classes for struggling students (Nomi & Allensworth, 2009). Based on our work with the four collaborating districts, we strongly suspect that most supplemental instruction focuses on the procedural competencies assessed by state tests.

In our view, the goal of supplemental supports should be more ambitious and should aim to enable struggling students to participate substantially in mainstream classroom instruction. Consistent with Durwood et al.'s (2010) recent findings, our observations indicate that teachers who provide supplemental support should receive professional development and associated curricular resources that are specific to meeting the needs of students who are currently struggling in mathematics. This proposal implicates district leaders in Special Education (and the Office of English Language Learners, if ELLs tend to struggle in mathematics in a particular district) in collaborating with C&I to plan professional development and develop appropriate instructional resources to guide supplemental services. The feasibility of this proposal rests on district leaders in Special Education and in the Office of ELL viewing rigorous goals for mathematical learning and visions of ambitious instruction as appropriate for the sub-populations of students their offices serve.

Teacher Networks

In addition to supporting individual teachers' learning, a coherent instructional system can support the development of strong professional relationships among mathematics teachers in a school (Bryk et al., 2010). The resulting trust, mutual accountability for student learning, and access to others' expertise are at least as important as teachers' perceptions of the value of the instructional reform in driving instructional improvement (Bryk & Schneider, 2002; Frank, Zhao, & Borman, 2004; Penuel, Riel, Krause, & Frank, 2009; Spillane & Thompson, 1997). In this respect, teachers' social networks are a key support for school-wide instructional improvement (Coburn, 2001; Penuel, Frank, & Krause, 2006) and therefore constitute the second component of our proposed theory of action.

Although teacher networks are emergent phenomena and cannot simply be mandated into existence (Smylie & Evans, 2006; Spillane, Reiser, & Gomez, 2006), district and school improvement policies can influence the conditions under which teachers decide whether to turn to a colleague for instructional advice and the types of advice they seek (Coburn & Russell, 2008). In this regard, the network analyses that we have conducted in our current work indicate that school-based PLCs can facilitate the emergence of teacher networks (Garrison, Smith, Cobb, & Green, 2011).

The extent to which a teacher network does in fact support the participating teachers' learning depends crucially on the nature of their interactions with one another. Building on the work of Coburn and Russell (2008), we have found it essential to distinguish between low-depth interactions that focus on "surface structures and procedures" (e.g., sharing materials, pacing) and high-depth interactions that focus on "underlying pedagogical principles of the approach, the nature of the mathematics and how students learn" (e.g., discussing different solution strategies to mathematical problems, analyzing student work) (p. 212). We are currently investigating the conjecture that networks in which a significant proportion of interactions are of high depth provide greater learning opportunities for teachers than networks in which most interactions are of low depth.

Our network analyses indicate that at least one member of the network, which might include a coach or an instructional leader, needs to have developed relatively sophisticated knowledge for teaching and/or accomplished instructional practices if networks are to support teacher learning (Sun & Frank, 2011). These analyses reveal that teachers' interactions with more accomplished colleagues are related to significant improvements in their mathematical knowledge for teaching and in the quality of their instructional practices. We conjecture that the extent to which network interactions support instructional improvement is also related to the nature of activities in which teachers engage during district and school-based professional development and, in particular, the role of coaches in pressing teachers on specific high-leverage issues.

Mathematics Coaching

The third component of the proposed theory of action concerns mathematics coaching.[2] Although US districts are increasingly using coaches as a primary means of supporting teachers' learning, the designs of their coaching programs vary considerably. For example, one of the districts we work with is implementing a school-based coaching design in which a mathematics teacher in each middle-grades school serves as a part-time coach (i.e., the coach works with colleagues half of the day and teaches the other half of the day). Another district has created a cadre of full-time coaches, each of whom serves three or four schools. In addition, there is variation in the extent to which coaches are expected to work with individual teachers in their classrooms or to work with groups of teachers. Given the importance that we attribute to ensuring that PLCs are productive, we currently recommend to the four collaborating districts that they give priority to coaches leading PLC meetings.

Coaches can potentially be more knowledgeable colleagues in teacher networks and can guide the development of PLCs (Frank, et al., 2004; Louis & Kruse, 1995; Penuel, et al., 2009). However, our initial analyses indicate that districts have to overcome a number of challenges in order to support the development of coaches' expertise as more knowledgeable others, and to ensure that this expertise is deployed effectively (Gibbons, 2011a, 2011b; Gibbons & Cobb, 2010). Current research on researcher-led PLCs provides some insight into the types of activities in which coaches should engage with groups of teachers to support their learning. However, the research base on how coaches might work with individual teachers in their classrooms and on what constitutes high-quality coach professional development is extremely thin.

Current research on the development of complex practices suggest that it is critical that novices co-participate in activities that approximate as closely as possible the targeted practices with more accomplished others (Bruner, 1996; Forman, 2003; Lave & Wenger, 1991). In the case of one-on-one coaching, this implies that high-leverage practices are ones in which the teacher (i.e., the novice) co-participates with the coach (i.e., the more accomplished other) in activities central to teaching. Based on our work with the districts and on research on teacher learning, we therefore

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[2] We recognize that districts might not employ coaches as a strategy for supporting instructional improvement at scale. Our key point is that people who have already developed accomplished instructional practices work with teachers directly on their classroom practices.

conjecture that the following are high-leverage coaching activities: a) co-teaching and b) enacting the coaching cycle of jointly planning a lesson, observing the enactment of the lesson, and then jointly analyzing the lesson (Bradley, 2007; Neufeld & Roper, 2003; Olson & Barrett, 2004). Co-teaching and the enactment of the coaching cycle would ideally foreground a limited set of concrete practices that are the focus of professional development. It is noteworthy that, to this point, coaches in the four collaborating districts spend the bulk of their time observing instruction and giving feedback when they work with individual teachers. This coaching practice might be useful at specific points in teachers' development (e.g., when they have become relatively accomplished in enacting particular instructional practices and need assistance in fine-tuning those practices). However, there is little reason to believe that the practice of observing and providing feedback will, by itself, be sufficient to support teachers' development of the focal instructional practices.

As we have indicated, coaches who have already developed accomplished instructional practices themselves are well positioned to lead groups of mathematics teachers in PLCs. Research in teacher professional development suggests that potentially productive PLC activities include doing mathematics problems and comparing solution strategies, analyzing student work and classroom video-recordings, rehearsing high-leverage instructional practices, and debriefing challenges in implementation (Ball, et al., 2009; Borko, et al., 2009; Kazemi & Hubbard, 2008; Sherin & Han, 2004). Coburn and Russell's (2008) findings indicate that it is also important for coaches to enact specific routines of interaction (e.g., pressing teachers to identify the mathematical relationships that students need to understand in order to engage in a given task productively) when working both with groups of teachers and with individual teachers.

The findings of a number of studies and our observations indicate the development of relatively accomplished instructional practices is a necessary but not sufficient condition for developing effective coaching practices (Borko, et al., 2009; Elliott, et al., 2009). As is the case for teachers, coaches' learning needs to be scaffolded by co-participating in activities central to coaching with more accomplished others, namely district mathematics specialists. We also conjecture that coach professional development should be ongoing, should include both pedagogies of investigation and of enactment, and should focus on specific coaching practices. For example, coaches might work with district mathematics specialists on how to support teachers in learning to introduce tasks, or to orchestrate a whole class discussion effectively. As we have indicated, it is also important that district mathematics specialists enact specific routines of interactions with coaches, and then support them in enacting these same routines with teachers.

Coach professional development should also support coaches in working one-on-one with teachers in their classrooms. For example, a district mathematics specialist might enact a coaching cycle with the coach while the coach acts as the teacher. The district mathematics specialist would co-plan a lesson with the coach, the coach would teach the lesson, and then they would jointly analyze the lesson, ideally focusing on a limited set of instructional practices. The district mathematics specialist might then scaffold the coach's enactment of the coaching cycle with one or more teachers. In doing so, the mathematics specialist and coach would co-plan the work with individual teachers and would debrief after each enactment of the cycle with a teacher. The intent of these activities is to enable the coach to co-participate in one-on-one coaching practices with a more accomplished other.

School Instructional Leadership

The fourth component of the theory of action for improving the quality of mathematics instruction at scale concerns school instructional leadership. Historically, the principalship in US schools has focused on administration and management (e.g., scheduling classes, school finances, student discipline, relations with the community served by the school) (Elmore, 2000; Glennan & Resnick, 2004; Honig, 2006; Leithwood, Louis, Anderson, & Wahlstrom, 2004). However, increasing accountability demands have resulted in the wide spread expectation that principals should act as instructional leaders in mathematics and other disciplines (Fink & Resnick, 2001; Murphy, Elliott, Goldring, & Porter, 2007; Robinson, Lloyd, & Rowe, 2008). There is also evidence that the principal's role as an instructional leader can be critical in driving instructional improvement efforts (Bryk, et al., 2010; Spillane, Hallett, & Diamond, 2003).

Current research provides contradictory guidance on school instructional leadership. More specifically, a consensus is yet to emerge on what principals need to know and do in order to be effective instructional leaders in mathematics. Some researchers argue that it is sufficient for school leaders to understand general, content-independent principles of learning and instruction (Resnick & Glennan, 2002;

Resnick & Zurawsky, 2005) whereas other researchers contend that school leaders need a deep understanding of mathematics, students' mathematical learning, and teacher learning (Nelson & Sassi, 2005; Stein & Nelson, 2003). Our initial findings suggest that professional development based on the first view is too abstract and that most school leaders are not able to connect general principles to concrete instructional practices (Cobb & Jackson, 2010). It also appears that the provision of professional development based on the second view is beyond the capacity of most districts.

All four districts in our study are attempting to support school leaders' (principals' and/or assistant principals') development as instructional leaders in mathematics, and all have provided extensive professional development for school leaders. In the context of our current work, we have elaborated a set of leadership practices that have emerged as feasible goals for school leaders' learning. We have also elaborated a set of conjectures for how to support school leaders' learning, based in large part on a design experiment in which we designed, led, and analyzed a series of professional development sessions specific to instructional leadership in mathematics. The professional development included three half-day sessions for 80 school leaders and mathematics coaches in one of the four districts in August-October 2009. The goals of the sessions were to support principals in distinguishing between cognitively low- and high-demand mathematics tasks and in recognizing the value of key aspects of ambitious instruction (e.g., whole class discussions that support the development of conceptual understanding). The results from the pilot are encouraging. School leaders' ability to distinguish between high- and low-level mathematics tasks increased significantly as a consequence of the sessions (Colby, Gibbons, Henrick, Wong, & Boston, 2010). Their ability to recognize key aspects of ambitious mathematics instruction also moderately increased.

Based on our current work, we envision a distribution of instructional leadership where coaches and district mathematics specialists are primarily responsible for supporting teachers' learning, and school leaders press and hold teachers accountable for developing the intended instructional practices (Elmore, 2006; Printy & Marks, 2006; Spillane, Halverson, & Diamond, 2004). Our findings indicate that it is not feasible for school leaders to serve as primary supports for mathematics teachers' learning. However, initial findings also indicate that it is feasible and indeed necessary for school leaders to communicate instructional expectations to teachers that are aligned with the instructional vision guiding the improvement effort, and to ensure that teachers are provided with supports to enable them to meet those expectations. More specifically, we have identified two concrete instructional leadership practices aimed at pressing teachers to develop the intended forms of practice and providing teachers with adequate support: 1) observing mathematics instruction and providing feedback, and 2) participating in mathematics PLCs. In additional, current research indicates that school leaders play a critical role in enabling coaches to support teachers' learning effectively.

By observing instruction and providing teachers with informed feedback, school leaders can both communicate expectations and hold teachers accountable for improving classroom instruction. It is important that the feedback be specific to particular phases of lessons and to instructional practices on which teacher professional development focuses. However, the extent to which school leaders' feedback accomplishes these goals depends crucially on the nature of professional development principals have received (see below).

School leaders' participation in mathematics PLCs signals the importance of teacher collaboration, enables school leaders to hold teachers accountable for using collaborative time productively, and constitutes a context for school leaders' learning, thus better positioning them to procure appropriate resources for teachers. In this regard, a meta-analysis conducted by Robinson, Lloyd, and Rowe (2008) found that school leaders' participation in teacher professional development is strongly associated with improvements in the quality of instruction and student achievement.

The findings of several studies, including our own, indicate that coach effectiveness depends on school leaders assuming shared responsibility for instructional improvement with coaches (Gibbons & Cobb, 2010; Mangin, 2007; Matsumura, Sartoris, Bickel, & Garnier, 2009). This requires that school leaders understand the district-wide goals for students' mathematical learning and the guiding vision of high-quality instruction, and appreciate the critical role of coaches in supporting teachers' learning. In the context of our current work, we have documented several cases in which principals assigned additional duties to coaches that took them away from their work with teachers (e.g., analyzing data to identify struggling students, tutoring struggling students). However, observations indicate that principals protect coaches' time when they understand the coaches' role in the improvement effort.

The development of shared responsibility for instructional improvement is facilitated if school leaders and coaches meet regularly to share their observations about the quality of teachers' instructional practices, discuss how work with teachers is progressing, jointly select teachers with whom the coach should work, and plan for future work with groups of teachers. These meetings provide opportunities for the school leader to both communicate expectations to the coach, and hold the coach accountable for working with individual and groups of teachers individually as planned. These meetings can also give rise to opportunities for school leaders to deepen their understanding of high-quality mathematics instruction and the means of supporting teachers' learning (Cobb & Jackson, 2010). To further support school leaders' learning, we recommend to the four collaborating districts that coaches and school leaders observe instruction together and then discuss their observations and the nature of feedback that they might give the teacher. Clearly, it is important that teachers understand that the purpose of the observations is not evaluative in nature, lest the observations jeopardize the coach's relationship with teachers.

It is a non-trivial undertaking for school leaders, most of whom are not mathematics specialists, to develop the instructional leadership practices we have described. The districts in which we work have provided extensive professional development, and although school leaders in the districts now observe instruction for an average of two hours each day and provide feedback to teachers, most are not able to distinguish between strong and weak inquiry-oriented lessons (Cobb & Jackson, 2010).

The principles of high-quality professional development that we have discussed should guide the design of professional development for school leaders as well as for teachers and coaches. These principles include that professional development should involve ongoing work with more accomplished others that focuses on concrete, high-leverage practices, involves pedagogies of investigation and enactment, and a routine press on a small number of key issues. The feedback that we give to the collaborating districts is based on the conjecture that professional development with the following foci will support school leaders' development of the intended instructional leadership practices.

First, if school leaders are to effectively and realistically press teachers to improve the quality of instruction, professional development for school leaders should enable them to recognize the instructional practices that are the focus of teacher professional development, and to distinguish between low- and high-quality enactments of those practices. We conjecture that a consistent emphasis on instructional practices across teacher, coach, and school leader professional development will contribute to the development of compatible visions of high-quality instruction and to the alignment of supports for teachers' learning.

Second, we conjecture that professional development should attend explicitly to how to provide feedback to teachers that communicates expectations for ambitious and equitable instruction. This might involve school leaders and district mathematics specialists observing instruction or watching video-recordings of specific phases of lessons and discussing the feedback they would provide with the goal of improving its quality.

Third, professional development should build on school leaders' developing understanding of high-quality mathematics instruction by clarifying the role of coaches and PLCs in supporting teachers' development of ambitious and equitable instructional practices. We have documented several cases in which a school leader has taken over the agenda for PLC meetings to the detriment of the participating teachers' learning. We therefore conjecture that it is important to give particular attention to how the distribution of instructional leadership between coaches and school leaders should follow the contours of their complementary areas of expertise (Elmore, 2006). The pilot professional development sessions that we conducted in one of the collaborating districts indicated that mathematics coaches' participation in professional development with principals can support their development of productive professional relationships.

District Instructional Leadership

The fifth and final component of the theory of action for improving the quality of mathematics instruction concerns district instructional leadership. The work of several district leaders has emerged as particularly significant in the course of our work with the four collaborating districts. These include the Superintendent, who is in charge of the entire district, the Chief Academic Officer (CAO) who is typically responsible for matters relating to curriculum and instruction in all content areas, and the leaders of several central office units including Curriculum and Instruction (C&I, responsible for teacher and coach professional development), Leadership (responsible for supporting and assessing school leaders), English Language Learners, Special

Education, and Research, Evaluation, and Accountability (REA). The literature on the role of central office units in supporting instructional improvement is almost non-existent (Honig & Copland, 2009; Louis, 2008; Rorrer, Skrla, & Scheurich, 2008). However, at the outset of our current work, we conjectured that the relationship between central office units would influence the success of the collaborating districts' instructional improvement efforts. This has proved to be the case. The alignment of the agendas of C&I and Leadership appear to be particularly critical (Cobb & Jackson, 2010). In the following paragraphs, we share our findings about the role of districts leaders in supporting instructional improvement.

First, it appears critical that district leaders in the central office units that we have listed share both goals for students' mathematical learning and a vision of ambitious and equitable instruction (i.e., goals for teachers' learning). This is particularly important for leaders of C&I and Leadership. If leaders in different units have different goals for students' learning, it is likely that they will, in turn, hold members of different role groups (e.g., teachers, school leaders) accountable for developing practices that are at odds with each other. For example, in one of the collaborating districts, we have found that while the efforts of leaders in C&I focus on supporting teachers' and coaches' development of ambitious practices, leaders in Leadership hold principals accountable primarily for the improvement of students' mathematics achievement scores. In turn, principals communicate these expectations to teachers, and do not press teachers to improve the quality of instruction. Additionally, principals direct resources toward providing supplemental supports for struggling students that are not aligned with mainstream classroom instruction (e.g., directing coaches to coordinate tutoring programs that focus on basic computational skills rather than working with teachers to improve the quality of instruction).

Second, how district leaders frame the problem of improving student mathematics achievement influences what they hold school leaders, coaches, and teachers accountable for, and thus the prospects for district-wide instructional improvement. In the context of high-stakes accountability in the US, district leaders are under considerable pressure to improve students' achievement on state mathematics assessments, and are frequently hired and fired on this basis. If a CAO, for example, frames the problem of increasing mathematics achievement as one of fundamentally improving the quality of instruction, he or she might view mathematics instruction that is compatible with the NCTM *Standards* and that addresses state objectives as a viable way to both meet NCLB mandates and ensure that instruction attends to conceptual as well as procedural goals. We refer to this response as reflecting an *instructional improvement* orientation because it focuses on the quality of teachers' instructional practices and entails the provision of professional development and job-embedded supports for teachers' learning. Alternatively, a CAO might attempt to satisfy NCLB mandates by ensuring that instruction focuses on state objectives, and by providing students who have not met particular objectives with additional instruction or tutoring that focuses on those objectives. We refer to this response as reflecting an *instructional management* orientation because it focuses on redeploying the district's current instructional resources and does not attempt to improve the quality of those resources.

In our work, we have found that C&I Departments tend to adopt an instructional improvement orientation whereas Leadership Departments frequently adopt an instructional management orientation (Cobb & Jackson, 2010). Our analyses indicate that instructional improvement and instructional management are both important but need to be tightly coordinated so that instructional management aims specifically at enabling struggling students to succeed in their regular mathematics classes. We have found that in many schools, additional mathematics classes for struggling students aim solely at procedural goals for students' learning, and are thus unlikely to support students to succeed in their mainstream mathematics classes.

Third, we have found that it is crucial that district leaders, particularly the Superintendant, CAO, and leaders in the offices of C&I, Leadership, and ELL view instructional improvement at scale from a *learning perspective* (Hubbard, Mehan, & Stein, 2008). In other words, they recognize that achieving an ambitious and equitable vision of mathematics instruction across district classrooms is not merely a matter of ensuring compliance with district policies, but instead requires significant learning on the part of teachers, coaches, and principals. Additionally, they view it as their responsibility to lead the design and implementation of supports for teachers', coaches', and principals' learning.

Our observations indicate that district leaders who take a learning perspective typically recognize those in the district who have expertise in supporting others' development as teachers or instructional leaders in mathematics, and capitalize on that expertise. For example, the CAO in one of our districts regularly draws on the expertise of district mathematics specialists when formulating policies that are specific to instructional improvement in mathematics. The CAO recognizes that he does not have the specific knowledge to make decisions regarding, for example, teacher professional development, and draws on the expertise of those who do. The extent to which district leaders take a learning perspective also has implications for the extent to which instructional improvement and instructional management are coordinated. In our experience, leaders who take a learning perspective are more likely to view the problem of increasing mathematics achievement as one that requires aligning additional supports with the learning goals of the overall instructional improvement effort.

Fourth, we have found that it is critical that leaders in various central office units have regular opportunities to collaborate together on the design and implementation of instructional improvement policies. Throughout this article, we have emphasized the importance of ensuring that all of the supports for instructional improvement are aligned and are aimed at an explicit set of goals for students' mathematical learning and a concrete vision of high-quality mathematics instruction (i.e., an explicit set of goals for teachers' learning). This is only possible if leaders of key central office units regularly communicate and collaborate around the work of improvement. As we have indicated, it is essential that leaders in C&I and Leadership work together to coordinate both supports for teachers', coaches', and principals' learning, and what they are held accountable for. We also conjecture that the relationship between the offices of C&I and the ELL are central to instructional improvement in districts that serve large numbers of ELLs. Leaders in the Office of ELL are knowledgeable about what it likely to be linguistically demanding for ELLs as well as instructional adjustments that might support ELLs to participate substantially in instruction; however, they are unlikely to be knowledgeable about mathematics instruction. On the other hand, the district mathematics specialists are unlikely knowledgeable about supporting ELLs. It would therefore seem important that the leaders in both offices work together to develop a shared vision of ambitious and equitable mathematics instruction, and to plan professional development and the revision of instructional materials (e.g., curriculum frameworks) accordingly.

Discussion and Conclusion

In this article, we have proposed an empirically grounded theory of action for improving the quality of mathematics instruction at scale. The theory of action comprises five components: a coherent instructional system for ambitious and equitable instruction that encompasses both formal and job-embedded teacher professional development; teacher networks; mathematics coaches' practices in supporting teachers' learning; school leaders' practices as instructional leaders in mathematics; and district leaders' practices in supporting the development of school-level capacity for instructional improvement. This theory of action is specific to the US educational context and reflects both the decentralized structure of the US educational system and the demands of high-stakes accountability. We anticipate that the specific components of the theory of action and the more general approach of framing instructional improvement at scale as a problem of organizational learning will both prove relevant when considering the improvement of mathematics instruction at scale in the context of other educational systems.

We contend that all five components of the proposed theory of action are necessary for large-scale instructional improvement; the prospects for achieving and sustaining instructional improvement diminish significantly if any one of the components is neglected. As we have attempted to illustrate, improving instruction at scale involves aligning supports for the learning of members of multiple role groups. For example, we would question an improvement strategy that focuses on high-quality curriculum materials, teacher professional development, and mathematics coaching but does not attend to school leaders' development as instructional leaders. Such a strategy is unlikely to be effective because it is unlikely that school leaders will either press teachers to develop the intended practices or support coaches' work with teachers.

Due to space limitations, we have not given adequate attention to the development of tools designed to support the members of various role groups in reorganizing their practices. Although we included curriculum frameworks for teachers as a key element of a coherent instructional system, we did not discuss tools for coaches and school leaders. Initial findings from our work suggest that it is important that any tool used by teachers or instructional leaders be aligned with the guiding vision of high-quality instruction. For example, we have found that the aspects of instruction that school leaders attend to when making classroom observations and giving feedback to teachers is significantly influenced by the classroom observation protocol they use. We have documented cases in which a lack of alignment with the vision of high-quality instruction impedes the extent to which school leaders' observations and feedback communicate appropriate expectations for instructional improvement to teachers. However, as we have made clear, the provision of tools will not, by itself, support the reorganization of practice. Carefully designed professional development that focuses on learning to use the tools in the intended ways (and thus on developing the intended practices) is essential. We refer the reader to Cobb and Jackson (2010) for a detailed discussion of the design of tools aimed at supporting instructional improvement at scale, and the design of professional development specific to supporting various role groups in learning to use the tools.

We view the theory of action we have proposed as a work in progress and have indicated the aspects of the theory that are provisional and subject to revision. We intend to further refine these aspects of the theory as we conduct future empirical analyses. In the course of this work, it has become apparent that research in mathematics education, educational leadership, and educational policy can provide only limited guidance in several areas that are central to improving the quality of mathematics instruction at scale. In particular, additional research is needed to specify: a) concrete forms of instructional practice aimed at rigorous learning goals that result in equitable learning opportunities and that are learnable in the context of high-quality professional development; b) how teachers typically use curriculum frameworks and related tools, and how professional development can be designed and implemented to support them in using such tools effectively; c) how district and school-based teacher professional development can be coordinated to support teachers' development of ambitious and equitable instructional practices (i.e., the coordination of pedagogies of investigation and enactment across the two settings); d) effective leadership of school-based PLCs that aim at ambitious and equitable teaching; e) the design and implementation of additional supports for struggling students that support their participation in ambitious and equitable mainstream instruction; f) the influence of teachers' participation in teacher networks on their instructional practices; g) high-leverage coaching practices for working with individual and groups of teachers; h) what school leaders need to know and be able to do to support the improvement of mathematics instruction in schools; i) the design and implementation of high-quality professional development for coaches and school leaders; and j) the orientations and practices of district office leaders that support the development of school-level capacity for instructional improvement.

This daunting list of areas where additional studies are needed reflects the limited attention that researchers in mathematics education and related fields have given to instructional improvement at scale. As we noted at the beginning of this article, the history of large-scale improvement efforts that involved significant changes in teachers' instructional practices is primarily one of failure. We contend that this unfortunate record is due in large part to the inability of research to inform the design and implementation of comprehensive systems of supports aimed at building and sustaining district and school capacity for instructional improvement. In the course of our work, it has become only too apparent that district leaders necessarily have to venture into uncharted territory when they formulate and attempt to implement instructional improvement policies. In our view, significant progress on the issues that we have listed will require the collaboration of researchers in mathematics education, educational leadership, and educational policy.

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