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Reasoning Patterns in Galileo's Analysis of Machines and in Expert Protocols: Roles for Analogy, Imagery, and Mental Simulation

John J. Clement[1,2]

Abstract

Reasoning patterns found in Galileo's treatise on machines, *On Mechanics*, are compared with patterns identified in case studies of scientifically trained experts thinking aloud, and many similarities are found. At one level the primary patterns identified are ordered analogy sequences and special diagrammatic techniques to support them. At a deeper level I develop constructs to describe patterns that can support embodied, imagistic, mental simulations as a central underlying process. Additionally, a larger hypothesized pattern of 'progressive imagistic generalization'—Galileo's development of a model or mechanism that becomes more and more general with each machine while still being imagistically projectable into many machines—provides a way to think about his progress toward a modern explanatory model of torque. By unpacking his arguments, we gain an appreciation of his skillful ability to foster imagistic processes underlying scientific thinking.

Keywords Galileo · Imagery · Analogy · Mental simulation · Mechanisms · Visual argument

1 Introduction

In his treatise *On Mechanics* (Galilei ca. 1590/1960) Galileo consolidates, refines, and systematizes work on principles of machines by a number of earlier authors, as well as introducing some new proofs in the form of visual arguments. Galileo provides explanations for the force-magnifying power of machines in the following order: Balance, Steelyard, Prybar or Lever, Capstan, Windlass, Pulley (Fig. 1), Compound Pulley, and other machines. In each of the above cases except the first, he does this by working out how the machine is analogous to a particular lever or balance (e.g. lever BEC in the pulley in Fig. 1 with C moving up and B as fulcrum). In doing so he makes significantly further progress than his predecessors toward the modern concepts of torque (Cicarelli 2006), and conservation of energy (see Stillman Drake's introduction to *On Mechanics* in Galilei 1960). Others have pointed to his achievement in constructing proofs via mental modeling (Palmieri 2003), and reducing many complex systems by showing their equivalence to the balance, paving the way for an approach to science that embraces a 'Mechanical Universe' (Machamer 1998).[1]

1.1 Purposes: What Reasoning Patterns Underlie Visual Arguments by Analogy?

In this article I compare the reasoning patterns in *On Mechanics* with reasoning patterns identified in transcripts of modern day, scientifically trained experts thinking aloud about a similar problem (see Fig. 2). One can describe both the experts' patterns and those in *On Mechanics* as primarily involving 'visual arguments', but here I construct hypotheses for some of the mental processes underlying those 'visual arguments', including the use of embodied processes involving imagery and mental simulation, the role of which are still poorly understood.

In order to build up the hypotheses in a progressive manner, the plan for the article is to: (1) identify reasoning patterns for analogy use in an expert think aloud case study; (2) examine possible parallels in Galileo's treatise; (3) return to another expert case to examine finer grained processes of imagery and mental simulation use that can underlie

✉ John J. Clement
clement@educ.umass.edu

[1] Scientific Reasoning Research Institute, College of Natural Sciences and College of Education, Furcolo Hall, University of Massachusetts, Amherst, MA, USA

[2] Sunderland, MA, USA

[1] Other work related to this treatise empasizes the mathematical nature of Galileo's analogies (Daston 1984), its connection to other authors in the period (Meli 2006), and its pioneering use of kinematic diagrams (Cicarelli 2006).

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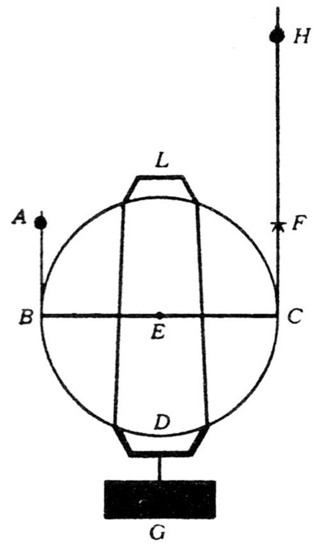

Fig. 1 Galileo's diagram of a pulley; 'A' is fixed, upward force at H (from Galilei 1960)

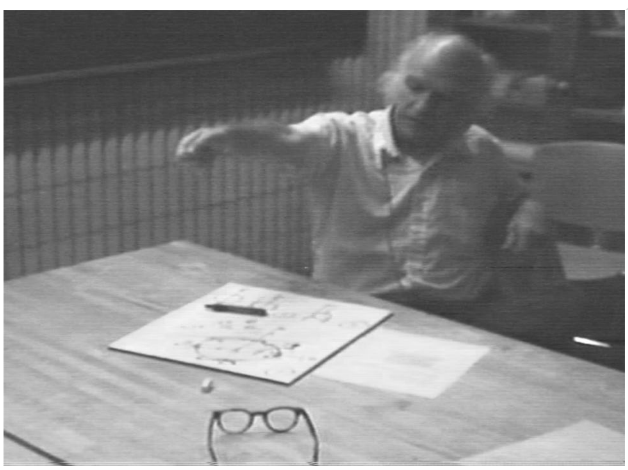

Fig. 2 Expert S7 gesture in Sisyphus Problem

analogies; (4) identify possible parallels in Galileo's treatise as well as new patterns.

2 Expert Think Aloud Case Study I

I will start from some reasoning patterns identified in a think-aloud study of expert scientists' use of analogies to solve explanation problems (Clement 2004, 2008). I follow Anzai and Simon (1979) in considering think aloud case

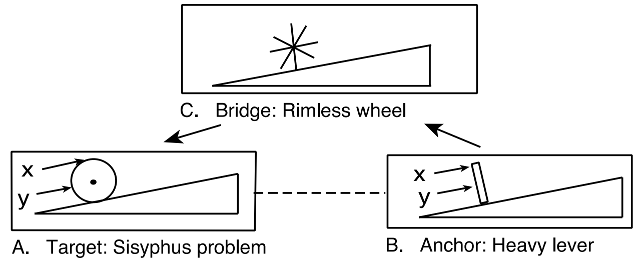

Fig. 3 Analogies for Sisyphus problem (adapted from Clement 2009a)

studies to be an important strategy for constraining initial modeling in an underdeveloped area. Subjects in the study were professors or advanced doctoral students in science or mathematics. I will focus here on the problem shown in Fig. 3a:

Sisyphus Problem Does it take the same amount of force to push a heavy wheel, say 4 ft. in diameter, uphill when you push [at X] parallel to the ground on the top of the wheel vs. pushing in the same direction [at Y] at the back of the wheel and even with the center (Fig. 3a)?

The subjects included in this paper were not physicists and did not use standard 'homework problem' methods on this target problem. They had to use methods outside of their scientific specialty. Here then, we are looking at *adaptive expertise* for model construction on the frontier of one's personal knowledge, rather than disciplinary expertise.

Subject S2 compared the wheel to the analogous case of raising a very heavy (say 4 ft. long) lever hinged to the hill, shown in Fig. 3b. He felt that pushing at the point at the top of the lever would require less force than pushing at the middle, and then inferred by analogy that that the wheel as a disk would also be easier to push at the top (the correct answer for the wheel).

2.1 Anchoring Case

Here, the lever plays the role of what I will call an *Anchor* (a confident base case for the analogy). When analogies are generated spontaneously, it is often not easy for the subject to find an analogous case whose behavior they can predict with confidence. In fact though, S2 was confident of his prediction for the lever, but gave no evidence of using any formal physics knowledge to derive it. I interpreted his process as accessing a confident perceptual-motor schema that he had acquired from practical experience in using levers. However there is a second condition for the success of an analogy: confidence in the *validity of the analogy relationship* (relevant similarity relationship) between the base anchor and target. If this confidence is high, then the subject can

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transfer a finding from the analogous anchor case to the target case with confidence.

2.2 Validity of the Analogy Relation

In fact, even though S2 was sure about the answer for his anchor, he was *not* sure of the validity of the analogy relationship between the lever and the wheel in this case, asking questions such as: where is the 'fulcrum' for the wheel? Is it at the center or at the bottom or nowhere? This uncertainty is symbolized by the dotted line in Fig. 3—he was not sure whether analogy relation 1 is valid.

2.3 Bridging Cases

S2 eventually reduced this uncertainty by generating an intermediate analogy or *Bridging Case* in the form of the spokes of a wagon wheel without a rim (shown in Fig. 3c)—of the same size as the original wheel. This suggests that one think about the wheel as a collection of many levers, and this intermediate bridging case significantly increased his confidence in his ability to apply his answer from the lever anchor to the wheel. In the bridging case the spoke that is touching the ground, can be seen as a lever with its fulcrum at the ground. This means that the entire wheel of spokes can be seen at any one time as equivalent to a single lever, supporting the analogy of C to B on the right hand side BC of the bridge in Fig. 3. By breaking the problem of confirming a "farther" analogy into the problem of confirming two "closer" analogies, such a bridge can apparently make it easier to develop confidence that the wheel does work like the lever in Fig. 3b.

Given a target problem A and an anchoring case B, a Bridging Case C is defined here as occurring when the subject finds or generates an intermediate case C that is 'closer' to A than B and also closer to B than A. 'Closer' may be thought of in terms of the number of discrete features shared or alternatively in terms of being perceptually closer in some analog way, such as closer in shape. The bridging case appeared to help this subject transfer the result from the lever case to the original target problem by confirming the validity of that analogy relationship. The value of bridging analogies has been documented previously in a number of expert problem contexts and in instructional applications (Clement 1993, 2008).

2.4 Overlay Diagrams

The Spokes bridge and the Wheel were shown separately in Fig. 3 and a for clarity of introduction, but in fact in the interview the subject drew and inscribed the single lever to fit on top of and aligned within the circular wheel in the subject's original drawing in Fig. 3a of the target problem.

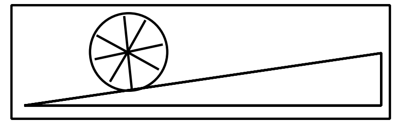

Fig. 4 S2's final overlay diagram for wheel

This created what I call an *Overlay Diagram* with the lever overlaid on the wheel. Later he also drew the spoked wheel without a rim on top of the original circular wheel drawing, as shown in Fig. 4, to create a second Overlay Diagram. Figure 4 looks like a normal wagon wheel, but he spoke about comparing the way the spokes would roll on their own to the way the original wheel would roll. The spokes are drawn at the same size and location as the original wheel, and this may make it easy to sense that the way the rimless spoked wheel rolls on its own can be seen as similar to the way the original wheel rolls in the same exact location, and make it easier to see the bottom of the wheel as a fulcrum. In sum, in this first case study I have identified: (1) two basic but essential processes in using an analogy, finding a *confident anchoring source case* and becoming *confident of the validity of the analogy relation*; and (2) two subprocesses that can help with the latter validating process, *bridging analogies* and *overlay diagrams*.

3 Reasoning Patterns in *On Mechanics*

One can ask whether there are reasoning patterns in Galileo's treatise *On Mechanics*. Whereas the expert examined was thinking aloud, Galileo had time to assemble (partly from other sources) a refined and consolidated argument in the treatise, so we cannot claim to follow his spontaneous thought processes. Rather, this article will examine a possible analogy between processes used on-the-fly by modern experts and the processes Galileo appears to be trying to elicit, instinctively and implicitly, in his reader.

As mentioned, the first part of Galileo's' treatise treats machines in the following order: Balance and Steelyard (a type of balance); Prybar or Lever; Windlass; Capstan; and Pulley. His reasoning rests primarily on analogies between these cases, and I will argue that that their order has been carefully designed. Commenting on the history of the idea of theoretical mechanisms in science, Machamer et al. (2000) write: "The modern idea of explaining with mechanisms became current in the seventeenth century when Galileo articulated a geometrico-mechanical form of explanation based on Archimedes' simple machines (p. 15)".

After giving justifications for the proportional relationships in the simple Balance and its two moments of force produced by two weights hung at different distances, Galileo

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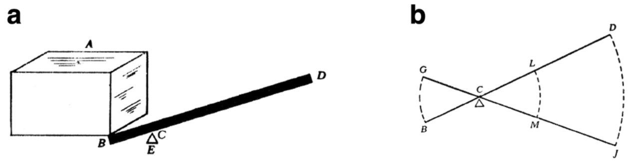

Fig. 5 **a** Galileo's Prybar, **b** Galileo's skeletal diagram for a Lever (from Galilei 1960)

Fig. 6 Windlass (circa 1380, Chesterfield Borough Council – Chesterfield Museum Service)

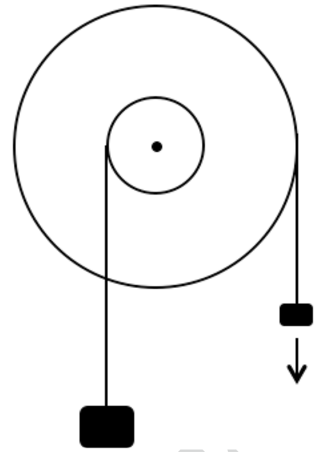

Fig. 7 Windlass diagram

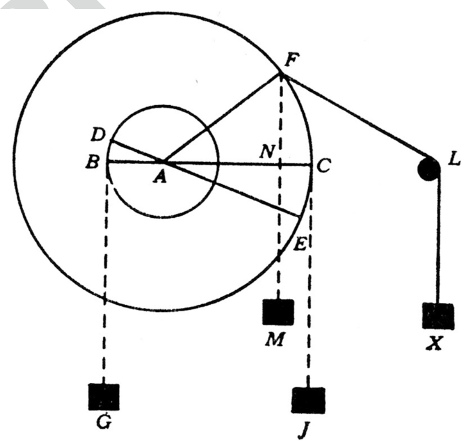

Fig. 8 Galileo's diagram for the Windlass, with J described as a small weight that can raise a larger weight G (from Galilei 1960)

states that the Prybar and abstract Lever in Fig. 5a, b are equivalent to such a balance. The principle machines discussed here, the Windlass and Capstan (as used on a ship), as well as the Pulley, are then analyzed primarily by analogy to the Lever and its counter-balancing moments of force. Therefore I will refer to the Lever and its two moments as his *anchor* or confidently understood source case, for understanding the Windlass, Capstan and Pulley, even though it has very different surface features from those machines. That both Galileo and expert S2 happen to use the analogy of a lever is interesting. But that is not the main focus here as much as are the similar subsequent reasoning patterns they use to support their analogy.

The medieval Windlass in Fig. 6 could be operated by a man walking inside, or when atop a cathedral under construction, by attaching a weight to a long rope wrapped around the outside of the wheel. Galileo's analysis is of the latter arrangement, as shown in my Fig. 7, with a small weight able to raise a larger weight.

Galileo analyzes the Windlass as being equivalent to the Lever in Fig. 5b. He argues that the Windlass is "nothing but a perpetual lever"; "uniting together as it were infinite levers". He does so by reducing the windlass to an equivalent lever BAC in Fig. 8, enabling him to show why it multiplies force, and to calculate by how much.

3.1 Use of an Overlay Diagram for the Windlass

His only actual diagram of the Windlass in the text is not the Fig. 7 I have drawn but rather Fig. 8. He refers to a large weight G on a rope and a smaller weight J able to raise G. He refers to BAC as a lever and this therefore appears to be an *overlay diagram* with the lever BAC drawn on top of the Windlass. The diagram suggests that the effect of the lever on the rope at point B would be the same as the effect of the Windlass on the rope at point B, at least for small increments. This is similar to expert S2's projection of a single lever onto the wheel in his overlay diagram. (He also treats line FLX in Fig. 8 to show that the ropes need not always be vertical to have the same effect but that will not be discussed

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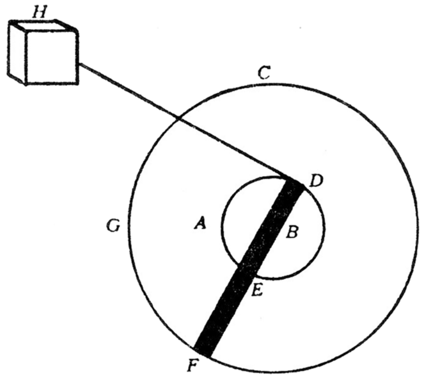

Fig. 9 Galileo's diagram for the Capstan, showing lever DBF (from Galilei 1960)

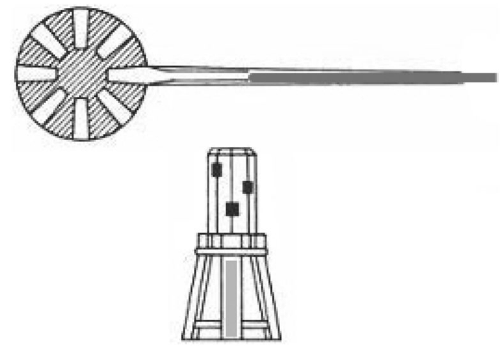

Fig. 10 Ship's Capstan (from the side) and removeable pushbar (from above) contemporary to Galileo

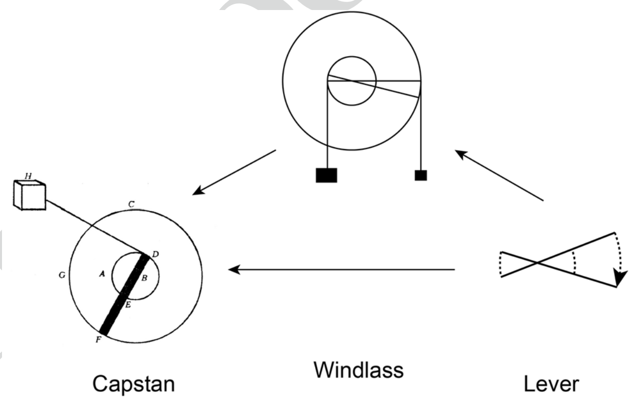

Fig. 11 Visual arguments for the analysis of the Capstan

here.) I infer that another purpose in using an *overlay diagram* is quantitative—to aid the reader in seeing how the windlass is equivalent to a lever with the same dimensions—it serves a metric geometry purpose in aligning dimensions of the two systems that should be the same length.

3.2 Use of an Overlay Diagram for the Capstan

Galileo also analyzes a ship's Capstan in terms of a lever FBD as shown in his Fig. 9. Whereas Fig. 8 is a side view, Fig. 9 is a view from above of a horizontal instrument. He says:

> From the instrument [Windlass] just explained, that which is called the capstan does not much differ as to form; indeed, it does not differ at all except in mode of application, the windlass being arranged and moved vertically, and the capstan being worked horizontally.... [there] the lever FBD comes to be formed.. (Galilei ca. 1590/1960, p. 161)

This can also be seen as an overlay diagram where the action of the Lever can be seen as similar to the action of the Capstan, as F moves around the circle.[2] In sum, he appears to draw analogies from the Lever to the Windlass to the Capstan and to use two overlay diagrams for the Windlass and the Capstan.

3.3 The Windlass as a Bridge to the Capstan

I will also interpret the manner in which Galileo has ordered his discussion as carefully setting up a *bridging analogy* between the Lever and the Capstan, by drawing diagrams of these in carefully chosen orientations and by placing the Windlass analysis in between the Lever and Capstan, as symbolized in my Fig. 11. The Lever is closer to the Windlass than it is to the Capstan in similarity since both are shown operating in a vertical plane. And the Windlass is closer to the Capstan than is the Lever, since both involve circular and concentric structures, and ropes. In this sense the Windlass is an intermediate case, or bridge, to the Capstan from the Lever. This should make it easier to see that the Capstan operates in the same way as the Windlass, and the Windlass operates in the same way as the Lever.

These considerations suggest that rather than simply pronouncing that each case reduces to the Lever, or to a ratio of diameters, Galileo is making multiple visual arguments in his analysis of how to think about the Capstan, as shown in Fig. 11. One line of argument via the bridge says that the Lever is strongly analogous to the Windlass, and the Windlass is strongly analogous to the Capstan. The other line of

[2] A possible objection to saying it is an overlay diagram is that lever FBD could be interpreted as a drawing of one of the arms that is inserted in the capstan. But as shown in Fig. 10, of a design from Galileo's era, a real arm would not be inserted as far as B in Fig. 9 and certainly not to D. So Lever FBD in Fig. 9 serves as a more theoretical overlay diagram for purposes of analysis.

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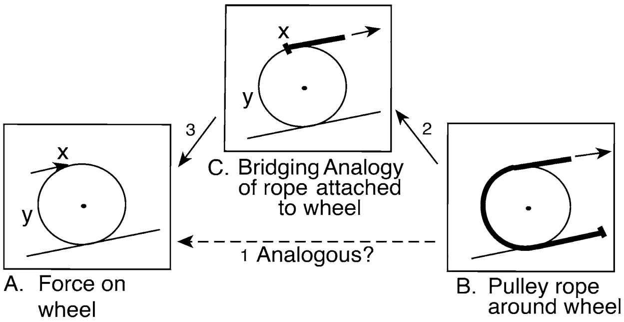

Fig. 12 Pulley analogy and bridging case for Wheel Problem (adapted from Clement 2009a)

argument uses an overlay diagram to show directly how the Capstan can be seen in terms of an equivalent Lever (the lower horizontal arrow of Fig. 11).

In sum, the treatise appears to use a confident *anchor*, and attends to validating *analogy relations* by using a *bridging case* and *overlay diagrams* in ways very similar to expert S2's reasoning patterns described earlier.

4 Expert Think Aloud Case Study II

4.1 Why Do Bridging Analogies and Overlay Diagrams Help?

A deeper, more fine-grained hypothesis concerning the function of Galileo's overlay diagrams, and why they may help, is this: by using the diagram to support the reader's embodied processes of *dynamic imagery* in a *mental simulation* (see Tricket and Trafton 2007; Clement 1994, 2008) of the lever along with a mental simulation of another system like the windlass, and *comparing those simulations*, one can sense if they are operating to multiply force in the same way. To develop the constructs and vocabulary for examining this hypothesis, I will describe gestures and statements from another subject that suggest the use of visual (and kinesthetic) imagery and mental simulations in reasoning about the Sisyphus Problem described at the beginning of section two.

4.2 Imagery and Imagistic Simulation

Subject S7 used a different anchoring case, bridging case, and overlay diagram from that of S2 described earlier, as follows. He appears to think imagistically about pulling on the wheel to roll it up the hill shown in Fig. 12a.

> S7: I'm imagining something that's extraordinarily heavy...... (holds both hands out as if pulling something and shakes them slightly) and I've got my full power available- and where would I apply that? My instinct tells me [it is easier to apply force at the top of the wheel at] X.., but again it's in terms of a pull and not a push. I'd have to get a grip. Assuming that's not a problem, then pulling should be the same as pushing.

One can point to the three underlined segments of this passage to introduce indicators that I take as evidence for his use of imagistic simulations. The first segment underlined is an *imagery report*. These occur when a subject spontaneously uses terms like "imagining," "picturing," or "feeling what it's like to manipulate" a situation. (The term imagery is used in a broad sense here that includes all perceptual modes plus kinesthetic imagery of actions).

The second underlined segment is a *depictive gesture* (depicting objects, forces, locations, or movements of entities) from the video tape. The third segment is a *dynamic imagery report* (imagery report involving movement or forces). Each indicator provides evidence for imagery use (denoted by underlining in transcripts in this study.) Taken together with the subject's new predictions, the observations above can be hypothesized to be the product of an *imagistic simulation process* wherein a somewhat general perceptual motor schema (here 'pulling') assimilates the image of a particular object and produces expectations about its behavior in a subsequent dynamic image (Clement 1994, 2008, 2009b).

4.3 Pulley Analogy

The subject then continues thinking about the wheel in Fig. 12a by generating an analogy to a pulley:

> And you're over here pulling like this [at X].
> That feels like you're on the outside of a pulley pulling up. (Illustrated in Fig. 12b).

I take this as a kinesthetic imagery report that applies to both the original wheel case and the pulley analogy case, giving some evidence that he is doing an imagistic simulation of both cases. S7 imagines his runnable anchor of a pulley in a rather odd position, laying it on its side diagonally on the ramp in Fig. 12b, with one end of the rope fixed to the ramp and pulling on the other end.

4.4 Bridge Between Pulley and Wheel

Elsewhere he indicates his confidence that the pulley will cut the force in half, however, the subject is still unsure of whether the pulley is a valid analogy for the wheel. A main difference between cases A and B in Fig. 12 is that the rope extends in a curve around the wheel in B and one is unsure of how it applies forces to the wheel, calling the analogy into question. He then generates a creative

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Table 1 Bridging sequence for S7's pulley analogy

S7: "Seems clear that- (silently holds both hands out as if pulling a rope for 4 sec.)... So we attach a rope to one of the teeth [as in Fig. 12c]	1-Attached rope
now it becomes more like the pulley problem (holds r. hand out as if pulling a rope for 3 sec. as in Fig. 3)	2-Pulley
The teeth at the bottom are playing the role of-	3-Attached Rope
the pulley doesn't look so bad after all	4-Pulley
And you hang on for all you're worth up there, to keep it from rolling"	5-Attached rope

bridging analogy case of a rope attached to the top of a gear-toothed wheel at X in Fig. 12c. (Adding gear teeth to a wheel is a standard technique in physics problem solving for insuring the condition of no slipping, but attaching a rope is nonstandard.)

> 162 So we attach a rope to one of the teeth [as in Fig. 12c], (gestures as shown in Fig. 2) now it becomes more like the pulley problem [as in Fig. 12b].

Note the similarity between the bridging diagram in Fig. 12 and that for *On Mechanics* in Fig. 11. We can examine details of the bridging strategy as well as the presence of imagery indicators (underlined) in the more detailed transcript in Table 1. The right hand column indicates which of the three cases in Fig. 12 he is referring to.

Here the subject appears to be asking whether he would see and feel the same behavior in cases B and C in Fig. 12 by imagining pulling on them and examining the force and how the wheel will move, and his answer is affirmative. The bridging strategy plus running the imagistic simulations appear to increase his confidence in the pulley analogy.

This passage motivates asking whether a subject would be capable of comparing a mental simulation of pulling the wheel with an attached rope to a second mental simulation of pulling on a pulley to decide whether they work in same way—comparing via a 'Dual Simulation'. We can think of pulling on a rope or using a pulley as perceptual motor actions. These can be controlled by perceptual motor control schemas that are in parallel control of many muscles, as opposed to discrete symbol structures. Can analogies occur at this presymbolic, embodied level? (See also, Tweney 1996.)

I interpret the subject as evaluating the analogy between cases B and C in Fig. 12 via a Dual Simulation—via vicarious, imagistic perceptual motor actions: running imagistic simulations of the anchoring case (Pulley) and bridging case (rope attached to wheel) and comparing or projecting one onto the other to evaluate whether they are analogous with respect to the forces required. This method would be heuristic—not guaranteed to work—but it would be very direct and may yield a valuable kind of grounded confidence at a perceptual motor level. Features of the transcript that support this Dual simulation hypothesis are listed in Table 2:

Using Table 2, one finds evidence for dual simulations from the transcript in Table 1 includes: Depictive gestures in both Cases C & B in Lines 1 & 2, global comparison statement in line 2, alternating reference to the cases.

After confirming the analogy between cases in Fig. 12b and c on the right side of the bridge in Fig. 12, he continues below by evaluating the analogy between A and C on the left hand side of the bridge, as follows (with underlined imagery indicators):

> 163 S7: Seems a lot easier than getting down here behind it [at "Y" in Fig. 12a] and pushing. Why? because of that coupling pulley effect. It seems like it would be a lot easier to hold it here [rope near X in Fig. 12c] for a few minutes (Holds hands outstretched as if pulling a rope, shown in Fig. 2) than it would be to get behind it [at Y in Fig. 12a… yeah, my confidence here is much higher now, that it's right… [easier to apply force at X in Fig. 12a].
> [164 S7: And so the pull—it just felt right with the pulley feeling. Now pushing (lays extended finger on paper pointing up slope to the left of X in Fig. 12a and moves it toward X) uh,.. it's got to be the same problem…
> 178 I: Do you have a sense of where your increased confidence is coming from?
> 179 S7: It's the pulley analogy starting to feel right.

S7 now appears to have gradually transferred perceptual motor intuitions about pulleys to the original problem. In line 163 the subject appears to focus on whether a force applied to the wheel at Y in Fig. 12a and a pulling a rope attached at X in Fig. 12c "feel" the same as he performs an imagistic simulation of each case in alternating fashion.

Table 2 Evidence types for dual simulation

Visual and kinesthetic Imagery indicators (underlined): depictive gestures, imagery reports, dynamic imagery reports
Verbal description of movements, actions, or other dynamics for each case
Alternating references to the cases
Global statements comparing two cases, e.g. "A is like B"
Verbal comparison of movements, actions, or other dynamics
Use of an overlay diagram

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From the indicators in Table 2, this is a verbal comparison of force actions accompanied by imagery indicators. This provides some evidence for another dual simulation, as does the statement "it just felt right with the pulley feeling," a global comparison statement referring to kinesthetic imagery. So the transfer of confidence from the anchoring Pulley case appears to have been completed by using dual simulation comparisons on each side of the bridging case C in Fig. 12.

4.5 S7's Internal Overlay Diagrams

Although I have drawn three cases in Fig. 12 for clarity and to illustrate the bridging strategy, in fact S7 stared only at Fig. 12a while talking about the three cases; he did not actually draw 12c or b. Rather he stares at and points to Fig. 12a. Because of this I infer that he thinks of a purely imaginary overlay for the pulley operating on top of the drawing of the wheel. He also imagines the bridging case of the rope attached to the wheel overlayed on top of the drawing of the wheel. Because he stares at the drawing I assume that he is imaging these cases all being of the same size as the original wheel. This suggests that the dual simulations just discussed were actually done with an *'internal overlay diagram'* (image), in a mental 'overlay simulation' supported by a single external diagram of the original wheel. These constructs may add to our understanding of the role of imagery in analogical reasoning (Clement 2004, 2008, 2009a).

4.6 What Are the Functions of Bridging and Overlay Diagrams?

While it appears to be very helpful to subjects, inventing a bridging case in itself is an incomplete strategy for analogy evaluation, since each half of the bridge is a new analogy pair to be evaluated (e.g. analogy relations 2 and 3 in Fig. 12). This raises the paradox of why experts bother to consider bridging cases at all, since they seem to create more work by the necessity to evaluate two analogy relations rather than one.

The human imagery system is limited in its capacity to imagine complex objects or collections (Kosslyn 1980). I hypothesize that a major function of bridging (and of overlay diagrams) is to support the embodied process of dual simulation. An intermediate bridging case supports dual simulation by creating pairs of cases that are closer together visually than the original analogy, making their behavior easier to compare in dual simulations than in the original analogy. Internal or external overlay diagrams support dual simulation by reducing the load on the perceptual motor imagery system and placing two cases in close juxtaposition so that their movements or actions may be compared easily.

5 Could These Same Imagistic Processes Underlie the Reasoning Patterns in *On Mechanics*?

5.1 How the Treatise Supports Imagistic Processes

The question in the heading above is harder to answer than with expert tapes since we are not dealing with a real time protocol and have no access to gestures. However, the imagistic processes we have just identified in experts are consistent with not only the overlay diagrams and bridging sequence in *On Mechanics* described earlier but with some of the language in Galileo's treatise. For the diagram of the Windlass in Fig. 8, he says:

> (1) If we think of the lever BAC supported at the point A, and the weight G hanging from the Point B, the force being placed at C, it is evident that by transferring the lever to the position DAE, the weight G will rise through the distance BD but that it cannot continue to be elevated much more...
> (2) It will be necessary to fix it [the rope] in this position with some other support, and return the lever to its previous place BAC; then, taking hold of the weight again, to raise it once more through a similar height BD... Doing the same thing many times, the raising of the weight may be accomplished ..[but it is] not very convenient.
> (3) Hence this difficulty has been overcome by finding a way of uniting together as it were infinite levers, perpetuating the operation without any interruption whatever...
> (4) Now since the axle [inner circle] always turns with the wheel [outer circle], the cords which sustain the weights always hang tangent to the circumferences of the wheel and axle, maintaining a similar position and relation to the distances BA and AC. Thus the motion will come to be perpetuated, the weight J descending and constraining G to rise (Ibid., pp. 159–160).

5.1.1 Anchor Simulation

In segment (1) he is in effect asking his reader to run an imagistic simulation of the lever in action.

5.1.2 Dual Simulation

In segment (2) he first gives a dynamic description of an equivalent series of lever actions, by reattaching a slightly lower location on the rope GB to the lever repeatedly, inviting the reader to mentally simulate repeated movements of the lever while staring at the overlay diagram in Fig. 8.

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He then invites the reader (in segment 4 above) to mentally simulate the motion of the windlass in the same drawing. These show that the windlass operates dynamically in the same way as the repeated lever actions. From Table 2, this is a verbal similarity comparison of movements and actions for the two cases. I take segments 2, 3 and 4 as a direct blow by blow description of a dual simulation, that describes the equivalence of the multiple actions of a lever and then the actions of the windlass, with the help of an overlay diagram.[3] We certainly cannot know that fostering 'dual simulation', in so many words, was a conscious strategy of Galileo's, but we can hypothesize that it is an important source of the effectiveness of the treatise in convincing others, and refer to it as at least an implicit strategy in the treatise.

5.1.3 Overlay Supports Dual Simulation As Well As Quantitative Alignment

Because of the geometry, it is hard to see a circular machine as analogous to a straight lever, but Galileo does it here, and expert S2 did it with the rimless spoked wheel discussed earlier. I see the overlay diagrams for both S2 and Galileo as a support for the internal dual simulation comparison between a small action of the simple lever model and a small action of the circular target system. Seeing the two actions as equivalent boosts confidence in the transfer of the qualitative idea of force multiplication from the lever to a circular system.

However, Galileo is also aiming for a mathematical level of precision in this treatise that will (stated in modern terms) produce correct ratios for various effort and load forces as equal to the ratio of load and effort arms in the lever. The Overlay diagrams in the treatise also yield a second product: a strong hypotheses about the appropriate lengths of the equivalent lever arms through the spatial alignment of the lever and machines like the windlass.

5.1.4 Imagistic Function of Bridging

The Windlass was described earlier as a bridge between the Lever and the Capstan. One can ask, what if Galileo had jumped from his discussion of the vertically oriented Lever to the horizontally oriented Capstan instead of to the Windlass?

It is very convenient for the Windlass argument above that the Windlass operates in the same vertical orientation as the vertically oriented Lever that Galileo analyzed early

[3] This could then lead to what Gentner (1983) and others would call a mapping of features from the base of an analogy to the target. The interplay between embodied imagistic simulation and discrete propositional mapping is an interesting unresolved issue (see Clement 2009a, 2008).

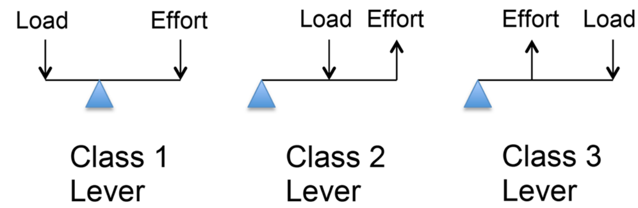

Fig. 13 Modern terminology for lever types

on in the treatise. This matching orientation, along with the overlay diagram, allows the reader to mentally simulate small movements of the wheel juxtaposed with small movements of the lever raising a weight. This would have been more difficult with the Capstan. Hence, one can also see the bridging intermediate case of the Windlass as having the function of supporting dual simulations. The bridging case promotes dual simulation by making the actions of two analogous cases (the Lever and the Windlass) easy to compare visually. It also makes it easy to compare simulations of the Windlass and the Capstan.

5.1.5 Diagrammatic Strategies for Enhancing Dual Simulations

There are additional imagistic properties of Galileo's diagrams that have the appearance of being designed to enhance analogy evaluation via dual simulation. First, the drawings are mostly *skeletal line diagrams* showing only the most important features for comparison. Secondly, there are seemingly intentional *imagery and simulation enhancement techniques* in the form of visual similarities between diagrams to enhance dual simulations. Galileo's diagram for the Capstan in Fig. 9 is a strange mixture. Circle CGF is not part of the machine and yet it appears in the drawing. Also the lever is drawn perpendicular to the straight rope, whereas that would usually not be the case for the actual push-bar. These features have the effect of maximizing the visual similarity of the Capstan diagram in Fig. 9 to the diagram of the Windlass in Fig. 8. In the diagrams in Fig. 11 all of the levers operate in a clockwise manner, and all are class 1 levers as pictured (with the fulcrum between the effort point and the load point; see Fig. 13). In contrast, the lever drawn overlaid on the capstan would be a class 2 lever if he had drawn the rope meeting the inner wheel at E. The Lever and Windlass are also drawn in the same orientation (shorter moment on the left). These visual similarities seem irrelevant for abstract arguments but could enhance (make it easier to perform) dual simulations of the cases operating in the same way, by reducing the cognitive load on the imagery system, as would the use of skeletal diagrams (cf. Clement 2008 on imagery enhancement).

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In sum, the concepts of imagistic simulation and dual simulation processes, help us understand the role of bridging cases, overlay diagrams, and diagrammatic imagery enhancement techniques in Galileo's treatise in addition to their roles in the expert protocols.

6 Combining the Patterns from *On Mechanics* and Expert Protocols

I set out to compare the reasoning patterns in expert think aloud protocols and in Galileo's *On Mechanics* as a way of generating constructs to describe some of the processes underlying visual arguments by analogy. Although the expert record is of spontaneous, unedited reasoning patterns, and Galileo is presenting more carefully chosen patterns, 'edited down' for presentation, nevertheless some strong similarities have been found between the two. A theory of how the different processes identified serve each other is shown in Fig. 14. There rectangles indicate cases and their representations; ovals indicate processes; and dotted rectangles indicate outcomes.

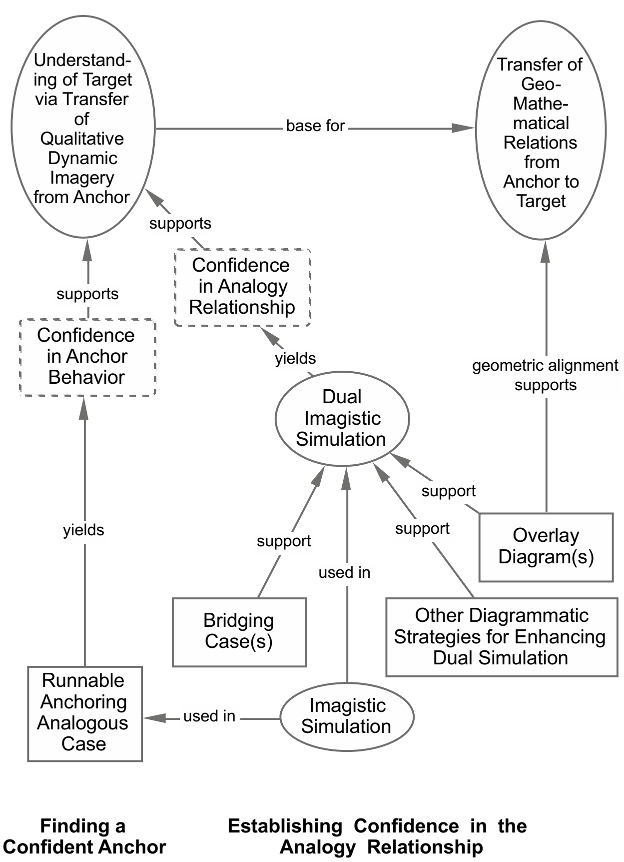

Fig. 14 Reasoning patterns in using analogies. (Key: Rectangles indicate cases and their representations; ovals indicate processes; dotted rectangles indicate outcomes.)

In the first two sections after the introduction, I introduced the concepts of a confident anchoring case, and confidence in the analogy relationship as two key outcomes sought in a successful analogy, shown as two main branches pointing to the upper left in Fig. 14. I identified bridging cases and overlay diagrams as two interesting techniques used for validating analogy relationships. In section three, I used S7's gestures and other imagery indicators as initial grounding for hypothesizing the other basic processes in Fig. 14: imagery, imagistic simulation, and dual simulation use by an expert. This provided some constructs for hypothesizing in section four how Galileo's bridging cases, overlay diagrams, and skeletal diagrams with similar orientations could all help his readers validate an analogy relationship by supporting an underlying dual simulation process—a process of comparing and perceiving imagistic similarities in the operations of two dynamic systems.

6.1 Dual Simulation Can Contribute to Conceptual Understanding

The lever analogy is not just providing the 'right answer' for predictions, but also providing a form of conceptual understanding via a satisfying explanation for how forces can be multiplied by various machines. Coming from the expert protocols, I hypothesize that dual simulation is key to this, allowing the reader to project and see lever actions within other machines—e.g. to see the windlass moving and acting as a lever moving and acting. Imagery is transferred to the windlass, not just results. That is, if one has perceptual motor knowledge schemas for what it feels like and looks like to use a lever, and one can *project* an image of the lever into the windlass in the right orientation, then some aspects of those imagistic lever intuition schemas can be transferred and adapted to understanding the windlass. This is consistent with Galileo's (1960) language: he does not say the windlass and capstan *'are like'* a lever, rather he says the windlass and capstan "*are* nothing but a perpetual lever" (p. 159). Machamer and Woody (1994) discuss a related, broader construct of 'intelligibility' wherein target cases become intelligible because they can be 'seen' as cases of what I am calling an anchor.

6.2 'Progressive Generalization Hypothesis': Progression from a Specific Analogy Toward a General Explanatory Model of Torque

Galileo goes on to use the lever to analyze other machines, including the Pulley in Fig. 1 (lever BEC in the pulley in Fig. 1 with C moving up and B as fulcrum), multiple levers in Compound Pulleys, and others, but I do not have space to analyze them here. By basing his explanations of the Windlass, Capstan, and Pulley on the Lever and its two moments, and by projecting the skeletal image of a lever model into

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Table 3 Progression from a specific analogy toward a general explanatory model

Device	Predominant model applied	Added properties of expanded domain of model	Old limitations of model removed
1. Balance (and Steelyard) (in *On Mechanics*)	Two counter-balancing moments of force	Linear device, 2 moments of force from weights, static in use, vertical orientation	
2. Prybar Lever	Vertical lever (class 1) and its two moments	Manual force input, output; purposeful motion	Static device, weights as only forces
3. Windlass	Vertical lever (class 1) and moments	Circular, fixed axle	Linear form
4. Capstan	Lever (class 1) and moments	Horizontal	Verticality
5. Pulley and compound pulley	Multi-class general lever and moments	Any orientation, portable axle, Type 2 as well as Type 1 lever possible	Spatial orientation, Type 1 only
Beams (in *Two New Sciences*)	Bent lever and moments	Forces inside materials	Force exerted by a tool or machine

those more complex cases in skeletal overlay diagrams, Galileo may have taken the first steps toward evolving the Lever and its two moments into the modern concept of torque, and equilibrium from counterbalancing torques, as a general explanatory model. To examine this hypothesis and track the development of the model, the progression of the first five machines in *On Mechanics* dealt with here is shown in column 1 of Table 3. Column 2 shows the predominant model applied to each machine. Column 3 shows how the domain of the lever model expands to encompass machines with more and more properties as he moves gradually further away from the anchors of the Lever and Balance.

As illustrated in columns 3 and 4, an anchoring case like the Prybar lever and its two moments can grow toward a *general explanatory model*, or mechanism, when detailed surface features like linear shape, and vertical orientation have been removed. This could happen when the lever analogy is applied to many systems in many orientations, requiring its flexible use. For example the top three rows only deal with vertical orientations, (even his definition of 'moment' early in the treatise is only in terms of vertical forces), but when he comes to the Capstan, he applies the lever model to horizontal forces, which removes the vertical feature we see as irrelevant today, making the concept more general. (However, it involves more than 'removing' since he is also building up ways to handle forces acting in a continuously moving object, and eventually forces non-perpendicular to the lever arm and non-type 1 levers.)

I call this a 'progressive generalization hypothesis' for how certain select analogous cases may become more general explanatory models, capable of being projected as a hidden mechanism into many different systems, as they are 'elevated' to a more general plane, stripped of inessential features, and refined, with a significantly expanded domain of application. In this view, what we see in 'On Mechanics' is the beginning of this process. In analyzing the Pulley and Compound Pulleys later in the treatise he uses both class 1 levers and class 2, (Fig. 13), suggesting the formation of a generalized lever model noted in column 2 of Table 3. These are significant steps toward the modern concept of torque. Although we cannot assume that Table 3 represents the time ordered sequence of Galileo's development, we can at the very least speculate that the treatise may have had something like these effects on his students and colleagues.

In a related vein, Nersessian (2008) speaks of Maxwell's need to abstract generic features of analogies like gear trains with idler wheels before he applied them to his theories of the electromagnetic field, and to eventually remove the concrete anchoring analogy completely. That removal was mandated by the move to an essentially non-Newtonian domain. In the present case I refer to progressive imagistic generalization happening in a less grandiose domain, where some skeletal aspects of the imagery of the anchoring case are retained in the model, allowing us to examine the imagistic processes promoted by the treatise in validating each extension.

6.2.1 Runnable Explanatory Model

This leads to the view of a conceptual understanding of a concept like torque as a general, *runnable explanatory model*—runnable in the sense of being capable of generating imagistic simulations—that could be overlayed and projected onto or into a multitude of different target cases. An interesting feature of such a model is that it is abstract in the sense of being general, but still concrete in the sense of being skeletally imageable. In the present case the lever model is also *intuitively grounded* (qualitatively) for many in the sense that it is developed from a confidently self evaluated and self evident set of practical schemas for using levers. (Other sources of grounding for the quantitative relationships in the balance and the lever are discussed in Palmieri 2003).

Later in his career, in writing "Two New Sciences", the lever model is extended further when he uses a bent

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lever and its two moments as a hidden explanatory model to analyze the resistance of beams to breaking, an even bolder extension of the domain of the lever model to the interior of a solid (Galilei ca. 1638/1954). Thus we can see Table 3 as depicting a gradual process of generalization that begins with a simple, concrete, and specific analogous case, and works toward a general and less concrete but still imageable explanatory mechanism. If dual simulation helps one understand how a specific concrete analogy explains a specific target case, it may also help us understand how a generalized, runnable, explanatory model explains a multitude of target cases. Such a model could be projected into a target system, much in the same way that an anchoring analogy is projected onto a target system in an overlay diagram to perform a dual simulation. This brings us to the view that Galileo is also making progress here toward a new form of explaining via general models or mechanisms, similar to the view of Machamer et al. (2000). Imagistic simulation constructs should help us develop a more fine grained description of these processes.

7 Conclusion

A number of constructs for viewing the role of imagery in analogical reasoning, shown at the bottom of Fig. 14, have been developed that can be seen as reasoning patterns in expert protocols and in Galileo's *On Mechanics*. These allow one to unpack the 'visual arguments' that Galileo uses to make the analogies at the center of his reasoning convincing. At one level the primary patterns identified were ordered analogy sequences and special diagrammatic techniques to support them. At a deeper level constructs were developed to describe embodied, imagistic, mental simulations, dual simulations, and transfer of dynamic imagery from one system to another as central underlying processes. In addition, 'progressive imagistic generalization'—his development of a model that is more and more general while still being imagistic—provides a way to think about his growth toward a modern explanatory model of torque. One practical implication of these patterns is the strong possibility that science educators and curriculum developers would benefit from studying the techniques used in *On Mechanics, a*s Machamer and Woody (1994) have also recommended.

Whereas many of the constructs in Fig. 14 originated from the expert studies, the progressive generalization hypothesis (of certain select analogies developing into a general model) and the construct of simulation enhancement techniques in comparing diagrams have reached fruition for this author in the present study of Galileo, so constructs have come from both sources. Galileo's precedent of fostering projected imagistic simulations of the lever or balance and their two moments into other machines can be seen as an important piece of his contribution to establishing a mechanistic view of science.

Galileo's 'visual arguments' may be so basic and so intuitive as to make each step seem unworthy of attention; as physics lecturers are wont to say, "it is clearly obvious that...". Consequently we have had a dearth of constructs to use to describe such visual arguments. But by comparing his arguments to those of experts and unpacking them, we gain an additional appreciation of his skillful ability to foster imagistic processes underlying scientific thinking.

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Compliance with Ethical Standards

Conflict of interest The author declares that he has no conflict of interest.

Ethical Approval All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. Informed consent was obtained from all individual participants included in the study.

Informed Consent Informed consent was obtained from all individual participants included in the study.

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