Development and Use of Learning Trajectories as Tools

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A Longitudinal Examination of Children’s Developing Knowledge of Measurement: Mathematical and Scientific Concept and Strategy Growth from Pre-K through Grade 5

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    - Julie Sarama, Ph.D.

- **Current research team members**
  - Craig Cullen, Ph.D.
  - Doug Van Dine, Melike Kara, Jennifer McDonel, Lisa Napora, Amanda Miller, and Cheryl Eames
Purpose of our project

- Validating and revising hypothetical learning trajectories for measurement of length, area and volume (Clements & Sarama, 2009)

- Engage students in critical mathematical and scientific concepts of measurement over multiple school years to characterize shifts in strategy and reasoning from level to level

- Inform curriculum design, professional development, and assessment projects

- ESPECIALLY plan to deliver LTs TO TEACHERS!
Our Theoretical Framework

Hierarchical Interactionism (Clements & Sarama, 2007) includes accounts of student cognition that culminate in learning trajectories (LTs) to describe cognitive development (innate and environmental interactions).

Hypothetical learning trajectory (Simon, 1995; Clements & Sarama, 2004). At each level there is:

- A learning goal (mathematical domain and topic)
- A likely path for learning (through levels of thinking)
- A description of mental Actions on Objects
- Instruction that fits their present schemes, given our understanding (model) of their actions on objects at that level.
There is an essential connection between

an instructional task

Each level in the Learning Trajectory
Teaching Experiment as an empirical context for work on the LTs

- For each teaching episode, the teacher-researcher:
  - generates a testable hypothesis about the level of sophistication exhibited by the student, and how the student reasons as they address tasks
  - must attempt to disregard or “forget” this hypothesis during the teaching episode,
    - to focus on the interaction with the student(s)
    - to find what schemes are in place,
    - Describing and observing student’s strategies, questions and statements
  - has the goal to bring forth students’ spontaneous schemes and to foster students’ successful assimilation (Steffe & Thompson, 2000).
Methodological Context

- A longitudinal teaching experiment (design research) (Steffe & Thompson, 2000; Cobb & Gravemeijer, 2008)
- With 16 students from each school: one in IL, one in NY
- 3-5 teaching episodes per for 7 consecutive semesters
  - 15 to 25 minutes
  - 2-5 tasks
  - Video recordings by a witness accompanying the interviewer
- Year 1: An open-response assessment to all children in each school
- Year 4: Another Assessment (exit year)
Tension between an actual LT (empirical) and a hypothetical LT

- Hypothetical Trajectory
  - Published or formalized LT
  - Predictive and Descriptive in most generalized way

- Empirical trajectory
  - exhibited by a student
  - Student’s exhibit a range of aspects/levels throughout the years
  - Record of observed performance on tasks intended to address various levels

- The empirical LT changes often at first and gains stability.

- However, the theoretical LT is more generalized and needs extensive evidence before it can vary.
How do you establish and validate a LT in a TE?

- **We predict** students’ responses based on prior responses to instructional tasks at specific levels.
- **We confirm** that tasks below the current level of sophistication are currently approachable (and that tasks more sophisticated than this level are too difficult presently).
- **We check** for separation among levels
  - that not all levels of thinking would be present at once; perhaps only one or two levels would be observable at once.
- Also, **We look for evidence of novel strategies**
Different types of evidence are needed for different types of LT improvements

- The number of participants depends on the type of improvement
  - Rasch modeling with large n to make major changes in the sequence of levels, or
  - TE analysis with small n to adapt or clarify a level.

- Emerging questions led us to employ a range of methods: both clinical interview and broad assessments
<table>
<thead>
<tr>
<th>Types and examples of LT improvements (over 15 years)</th>
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<tr>
<td><strong>Improvements for an entire level</strong></td>
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<td><strong>Changes among levels</strong></td>
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<tr>
<td>Adding levels</td>
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<tr>
<td>ICPM, CAM for length trajectory</td>
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<td>(Barrett et al. 2006): ICPM &amp; CAM for length LT;</td>
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<td>AERA Clements, Sarama &amp; Van Dine (2011): Volume</td>
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### Examples of LT improvements

| Improvements for an entire level
| Changes of the trajectory levels | Improvements for the pieces in a level
| Changes within a level |
|---------------------------------|----------------------------------|
| **Adding a level** | **Revising the flow or Collapsing** | **Clarifying** | **Expanding** |
| ICPM, CAM for length trajectory OR Volume LT | Relater and Repeater: LURR | EE, LURR | PRS |
Example: Adding a level

- (JRME (2006), Barrett & Battista (in press))
  - Moving from 4-part account: levels 1, 2a, 2b and 3
  - 5-part account: levels 1, 2a, 2b, 3a and 3b

- What is the evidence for splitting levels?
  - Variability that was systematically dichotomous, yet supersedes prior levels (2b).

- Methodology that resulted in additions:
  - Structured, task-based analysis with cross-sectional sampling from grades 2 through 11
Barrett (2006); Barrett and Battista (in press)

Previous Length LT
- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Length Measurer
- Conceptual Ruler Measurer

Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Coordinated, Integrated Abstract Measurer with Derived Units
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<td>PRS</td>
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Revise Length LT: collapse URR and widen the route of the LT

Szilagyi, Sarama & Clements (in press): using an IRT methodology

### Previous Length LT

- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater
- Length Repeater
## Examples of LT improvements

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### Adding a level
- Revising the flow or Collapsing
- ICPM, CAM for length trajectory
- Relater and Repeater: LURR
- EE, LURR

### Improvements for the pieces in a level
- Clarifying
- Expanding
- PRS
Paper on *Clarifying the Length LT*  
(Barrett et al., 2012)

**Previous Length LT**
- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- **Serial Orderer to 6+**
- **End-to-End Length Measurer**
- **Length Unit Relater and Repeater**
- Length Measurer
- Conceptual Ruler Measurer
e.g., End-to-End Level  
(prior to 2012)

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<th>Developmental progression</th>
<th>Conceptual: Actions on Objects</th>
<th>Instructional Tasks</th>
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<td>Age 5: Indirect Length Comparer (ILC): Compares the length of two objects by representing them with a third object. May assign a length by guessing or moving along a length while counting without equal length units. May use a ruler, but often lacks understanding or skill. A mental image of a particular length can be built, maintained, and (to a simple degree) manipulated. With the immediate perceptual support of some of the objects, such images can be compared. A counting scheme operates on an intuitive unit of space or of movement. To shift toward End-to-End: children should talk about numbers for lengths that they can compare indirectly. Use physical or drawn units along objects to compare. Focus on long thin units and help them count to make comparisons. Accentuate the linear aspect of any object, and use thin, long objects as units that can be accumulated.</td>
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<td>Age 6: End-to-End Length Measurer (EE): Lays units end-to-end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later in this level. Needs a complete set of units to span a length. An implicit concept that lengths can be composed as repetitions of shorter lengths underlies a scheme of laying lengths end to end. This initially only applied to small numbers of units. The scheme improves by attending more explicitly to covering distance or composing a length with parts.</td>
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<td>Age 7: Length Unit Relater and Repeater (URR): Measures by repeated use of a unit (initially may be imprecise). Relates size and number of units explicitly, but may use units of varying lengths. Can add lengths to obtain the length of a whole. Iterates a single unit to measure. Uses rulers with minimal guidance. Action schemes include the ability to iterate a mental unit along a perceptually available object. The image of each placement can be maintained while the physical unit is moved to the next iterative position. With the support of a perceptual context, scheme can predict that fewer larger units will be required to measure an object's length. These action schemes allow counting-addition schemes to help measure. Pretend to gap or overlap units as they are repeated to challenge consistent measures. Have the child create a ruler and mark it with ticks and numerals to match units (in or cm). Ask students to guess objects by telling them a length, with only one unit to model it. Use measuring software that snaps to whole number values of units to report length.</td>
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# End-to-End Level (Updated, 2012)

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<td>An implicit concept that lengths can be composed as repetitions of shorter lengths underlies a scheme of laying lengths end to end. This initially only applied to small numbers of units. The scheme is enhanced by the growing conception of length measuring as sweeping through large units coordinated with composing a length with parts (unit sticks). The scheme may be curtailed as sets of objects are internally presented as images that are symbolized by re-tracing the set using only one unit, or by mere pointing and sweeping in a coordinated set of actions (leading toward URR at the next level).</td>
<td>1) Provide incomplete sets of linear objects to span the length of an object to measure. 2) Use relatively large objects as units (and build a ruler with pen length units). 3) Compare two objects that must be compared indirectly using only shorter objects. 4) Provide the student with a contiguous set of yellow strips taped in a row to find length for comparisons. 5) Draw a ruler and mark it with ticks and numerals to match units (in or cm).</td>
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<td>VURR: Volume LT</td>
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Expanding the Area LT (humble theory?)

(e.g., Change the Area Row and Column Structurer row of the Area LT)

- Include student strategies of decomposing and recomposing partial squares to make whole units. Amend the highest level of the initial HLT for area, the Array Structurer level.
- An instructional task, *The Comparison of Blobs* (task 2) should be added to the tasks and
- Include work on rectilinear arrays that are not “square” to the mental actions on objects in the Array Structurer level.
Task 2

- The students were asked, “How does the area of this small blob compare to the large blob?” If the student gave a qualitative comparison, we followed up with the prompt, “How much bigger?”

- This task was designed to promote shifts between units, sub units and further sub units.
Are there falsifiable claims here?

- ZDM article (Sarama et al., 2011) is an example of falsifying a claim about an LT sequence for length:
  - now there is a parallel sub-sequence for SO, EE and ILC.

- Although one may establish a trajectory, it is not necessarily a completed product.

- It may be useful in spite of its tentative nature.

- Are the students described by the LT set of levels and is the sequence predictive? Is it productive for analysis?
Modifying a trajectory

- Can we extend a sequence (progression levels)?
- Challenge the sequence?
  - Add levels or collapse levels?
  - Should there be sub routes in every LT?


- Extend the instructional task set for each level in a given LT. Can these become public collections of task examples?
Some Challenges in using progressions or trajectories:

- How do we characterize learning trajectories as tools to clarify terms and focus our work?
- See each LT as:
  - a design tool
  - an analytical tool
  - an object to be modified, extended
- We need to generate alternative LTs to challenge the comprehensive nature of any LT for its domain
- What are the terms in use within LT and LP research that should be clarified and formalized?