# Development and Use of Learning Trajectories as Tools

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# A Longitudinal Examination of Children's Developing Knowledge of Measurement: Mathematical and Scientific Concept and Strategy Growth from Pre-K through Grade 5


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# Purpose of our project

- **Validating and revising hypothetical learning trajectories** for measurement of length, area and volume (Clements & Sarama, 2009)
- Engage students in critical mathematical and scientific concepts of measurement over multiple school years *to characterize shifts in strategy and reasoning* from level to level
- Inform curriculum design, professional development, and assessment projects
  - **ESPECIALLY plan to deliver LTs TO TEACHERS**!

# Our Theoretical Framework

Hierarchical Interactionism (Clements & Sarama, 2007) includes accounts of student cognition that culminate in learning trajectories (LTs) to describe cognitive development (innate and environmental interactions).

Hypothetical learning trajectory (Simon, 1995; Clements & Sarama, 2004). At each level there is:

- **A learning goal** (mathematical domain and topic)
- A **likely path for learning** (through levels of thinking)
- A **description of mental** *Actions on Objects*
- **Instruction that fits their present schemes**, given our understanding (model) of their actions on objects at that level.

# There is an essential connection between

# Teaching Experiment as an empirical context for work on the LTs

- For each teaching episode, the *teacher-researcher*:
  - **generates a testable hypothesis** about the level of sophistication exhibited by the student, and how the student reasons as they address tasks
  - must attempt to disregard or "**forget'' this hypothesis** during the teaching episode,
    - to focus on the interaction with the student(s)
    - to find what schemes are in place,
    - Describing and observing student's strategies, questions and statements
  - has the goal to **bring forth students' spontaneous schemes** and to foster students' successful assimilation (Steffe & Thompson, 2000).

# Methodological Context

- A longitudinal **teaching experiment** (design research)
  (Steffe & Thompson, 2000; Cobb & Gravemeijer, 2008)
- With 16 students from each school: one in IL, one in NY
- 3-5 teaching episodes per for 7 consecutive semesters
  - 15 to 25 minutes
  - 2-5 tasks
  - Video recordings by a witness accompanying the interviewer
- Year 1: An open-response **assessment** to all children in each school
- **Year 4:** Another **Assessment** (exit year)

# Tension between an actual LT (empirical) and a hypothetical LT

- Hypothetical Trajectory
  - Published or formalized LT
  - Predictive and Descriptive in most generalized way
- Empirical trajectory
  - exhibited by a student
  - Student's exhibit a range of aspects/levels throughout the years
  - Record of observed performance on tasks intended to address various levels
- The empirical LT changes often at first and gains stability.
- However, the theoretical LT is more generalized and needs extensive evidence before it can vary.

# How do you establish and validate a LT in a TE?

- **We predict** students' responses based on prior responses to instructional tasks at specific levels
- **We confirm** that tasks below the current level of sophistication are currently approachable (and that tasks more sophisticated than this level are too difficult presently).
- **We check** for separation among levels
  - that not all levels of thinking would be present at once; perhaps only one or two levels would be observable at once.
- Also, We look for evidence of **novel strategies**

# Different types of evidence are needed for different types of LT improvements

- The number of participants depends on the type of improvement
  - Rasch modeling **with large n** to make major changes in the sequence of levels, or
  - TE analysis **with small n** to adapt or clarify a level.
- Emerging questions led us to employ a range of methods: both clinical interview and broad assessments

# Types and examples of LT improvements (over 15 years)

| **Improvements for an entire level**<br><br>**Changes among levels** | | **Improvements for parts of a level**<br><br>**Changes within a level** | |
|---|---|---|---|
| Adding levels | Revising the flow or Collapsing | Clarifying levels | Expanding levels |
| ICPM, CAM for length trajectory | Relater and Repeater: LURR | EE, LURR | PRS |
| (Barrett et al. 2006): ICPM & CAM for length LT;<br>AERA Clements, Sarama & Van Dine (2011): Volume aspects | Szilagyi (2007) LURR –Unit relater and unit repeater;<br>(Sarama et al. 2011) Clarifying the path: SO, ILC, EE | MTL paper (Barrett et al. 2012) | AERA paper (Cullen et al., 2011): Area of blobs (non square units). |

# Examples of LT improvements

| **Improvements for an entire level Changes of the trajectory levels** | | **Improvements for the pieces in a level Changes within a level** | |
|---|---|---|---|
| Adding a level | Revising the flow or Collapsing | Clarifying | Expanding |
| ICPM, CAM for length trajectory OR Volume LT | Relater and Repeater: LURR | EE, LURR | PRS |

# Example: Adding a level

- (JRME (2006), Barrett & Battista (in press))
  - Moving from 4-part account: levels 1, 2a, 2b and 3
  - 5-part account: levels 1, 2a, 2b, 3a and 3b
- What is the evidence for splitting levels?
  - Variability that was systematically dichotomous, yet supersedes prior levels (2b).
- Methodology that resulted in additions:
  - Structured, task-based analysis with cross-sectional sampling from grades 2 through 11

# Barrett (2006); Barrett and Battista (in press)

## Previous Length LT

- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Length Measurer
- Conceptual Ruler Measurer

- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Coordinated, Integrated Abstract Measurer with Derived Units

# Examples of LT improvements

| Improvements for an entire level<br>Changes of the trajectory levels | | Improvements for the pieces in a level<br>Changes within a level | |
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| ICPM, CAM for length trajectory | Relater and Repeater: LURR | EE, LURR | PRS |

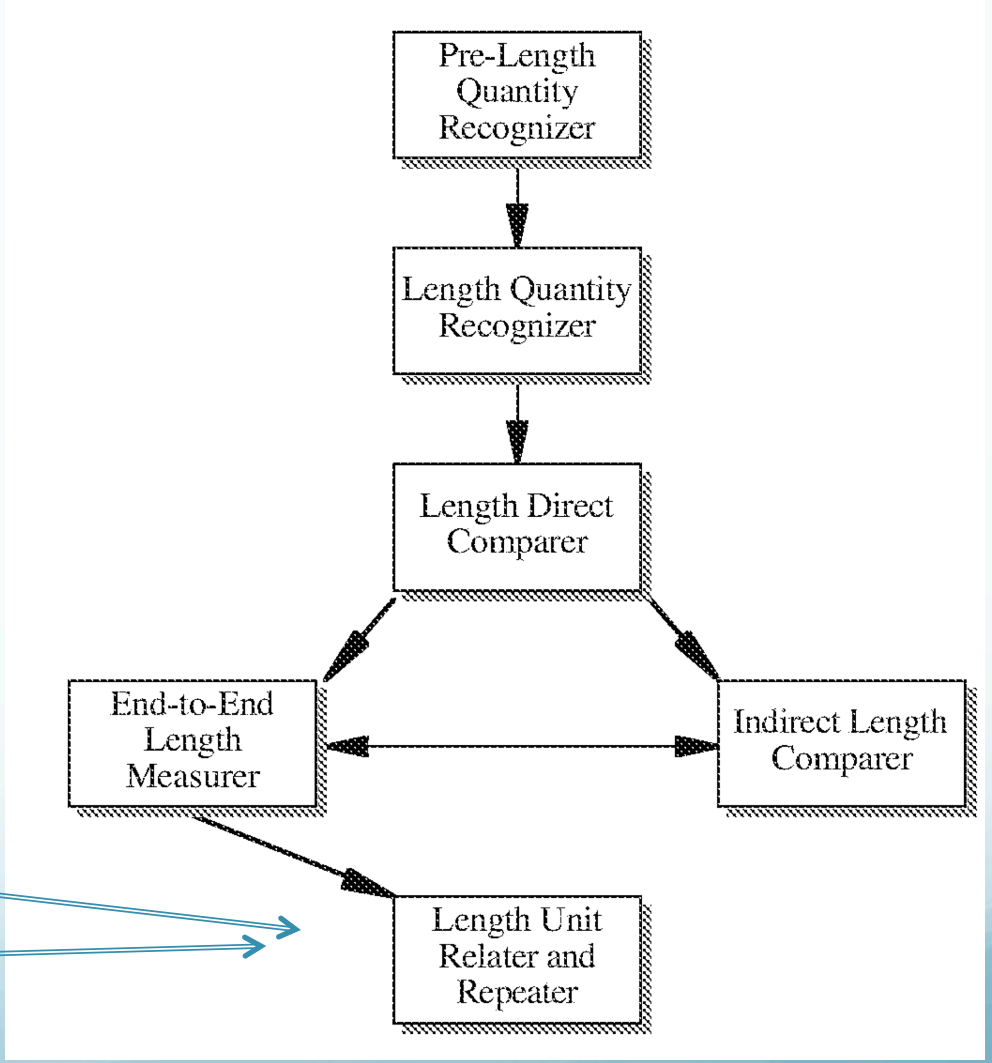
# Revise Length LT: collapse URR and widen the route of the LT

Szilagyi, Sarama & Clements (in press): using an IRT methodology

## Previous Length LT

- Pre-Length Quantity Recognizer
- Length Quantity Recognizer
- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater
- Length Repeater

## Updated Length LT



# Examples of LT improvements

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# Paper on *Clarifying* the Length LT
(Barrett et al., 2012)

## Previous Length LT

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- Length Direct Comparer
- Indirect Length Comparer
- Serial Orderer to 6+
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Length Measurer
- Conceptual Ruler Measurer

# e.g., End-to-End Level (prior to 2012)

| Developmental progression | Conceptual: Actions on Objects | Instructional Tasks |
|---|---|---|
| Age 6: **End-to-End** Length Measurer (**EE):** Lays units end-to-end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later in this level. Needs a complete set of units to span a length. | An implicit concept that lengths can be composed as repetitions of shorter lengths underlies a scheme of laying lengths end to end. This initially only applied to small numbers of units. The scheme improves by attending more explicitly to covering distance or composing a length with parts. | Have the child create a ruler and mark it with ticks and numerals to match units (in or cm). Ask students to guess objects by telling them a length, with only one unit to model it. Use measuring software that snaps to whole number values of units to report length. |

# End-to-End Level (Updated, 2012)

| *Developmental Progression* | *Conceptual Structures and Strategies* | *Instructional Tasks* |
|---|---|---|
| Age 6: **End-to-End** Length Measurer **(EE):** Lays units end-to-end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later in this level. Needs a complete set of units to span a length.<br><br>*[This level is concurrent with Serial Orderer to 6+.]* | An implicit concept that lengths can be composed as repetitions of shorter lengths underlies a scheme of laying lengths end to end. This initially only applied to small numbers of units. The scheme *is enhanced by the growing conception of length measuring as sweeping through large units coordinated with composing a length with parts (unit sticks). The scheme may be curtailed as sets of objects are internally presented as images that are symbolized by re-tracing the set using only one unit, or by mere pointing and sweeping in a coordinated set of actions (leading toward URR at the next level).*<br><br>*An Ordering Scheme is organized in a hierarchy (initially implicit) for an ordered series of objects, eventually supporting a graduating sequence scheme.* | *1) Provide incomplete sets of linear objects to span the length of an object to measure.*<br>*2) Use relatively large objects as units (and build a ruler with pen length units).*<br>*3) Compare two objects that must be compared indirectly using only shorter objects.*<br>***4)** Provide the student with a contiguous set of yellow strips taped in a row to find length for comparisons.*<br>*5)* Draw a ruler and mark it with ticks and numerals to match units (in or cm). |

# Examples of LT improvements

| **Improvements for an entire level Changes of the trajectory levels** | | **Improvements for the pieces in a level Changes within a level** | |
|---|---|---|---|
| Adding a level | Revising the flow or Collapsing | Clarifying | Expanding |
| ICPM, CAM for length trajectory | Relater and Repeater: LURR | EE, LURR | PRS: Area LT<br><br>VURR: Volume LT |

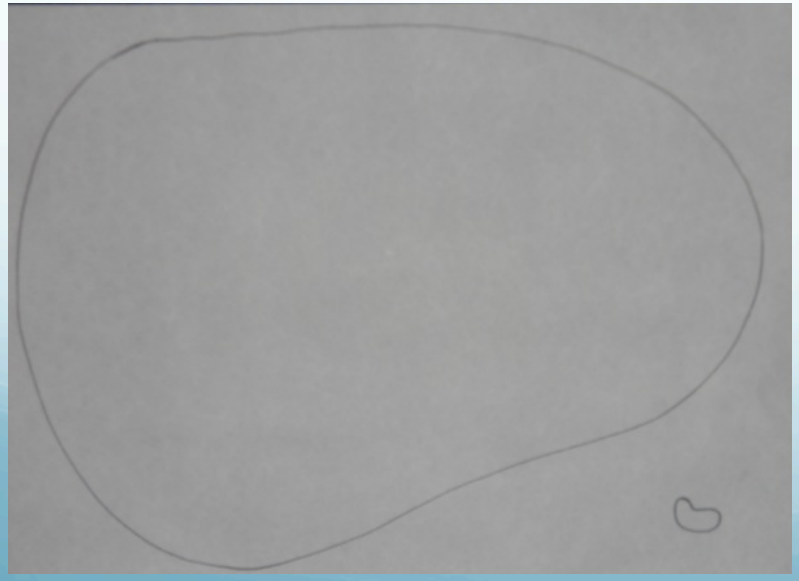
# Expanding the Area LT (humble theory?)

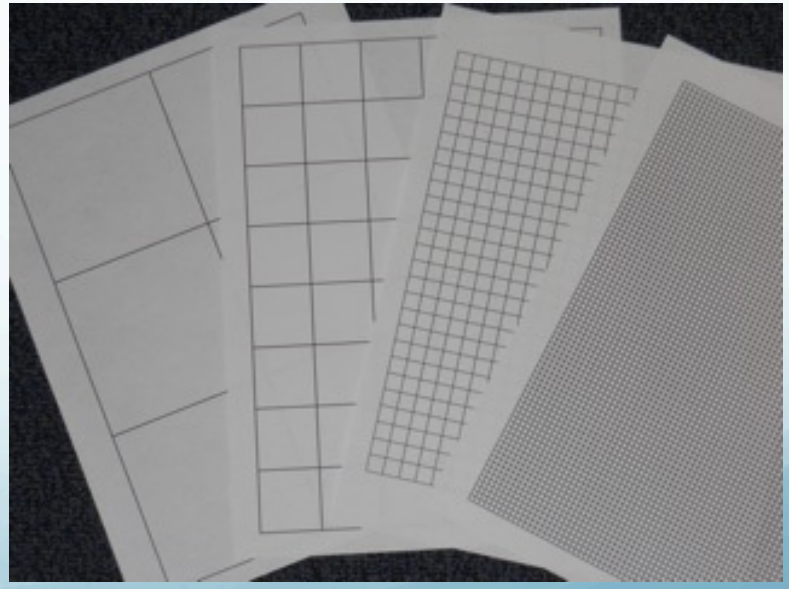
(e.g., Change the Area Row and Column Structurer row of the Area LT)

- Include student strategies of decomposing and recomposing partial squares to make whole units. Amend the highest level of the initial HLT for area, the Array Structurer level.
- An instructional task, *The Comparison of Blobs* (task 2) should be added to the tasks and
- Include work on rectilinear arrays that are not "square" to the mental actions on objects in the Array Structurer level.

# Task 2

- The students were asked, “How does the area of this small blob compare to the large blob?” If the student gave a qualitative comparison, we followed up with the prompt, “How much bigger?”
- This task was designed to promote shifts between units, sub units and further sub units.





# Are there falsifiable claims here?

- ZDM article (Sarama et al., 2011) is an example of falsifying a claim about an LT sequence for length:
  - now there is a parallel sub-sequence for SO, EE and ILC.
- Although one may establish a trajectory, it is not necessarily a completed product.
- It *may* be useful in spite of its tentative nature.
- Are the students described by the LT set of levels and is the sequence predictive? Is it productive for analysis?

# Modifying a trajectory

- Can we extend a sequence (progression levels)?
- Challenge the sequence?
  - Add levels or collapse levels?
  - Should there be sub routes in every LT?

  (what metaphor helps? A river? Layered soil? A series of routes through a canyon? (J. Confrey, June 14, 2012))
- Extend the instructional task set for each level in a given LT. Can these become public collections of task examples?

# Some Challenges in using progressions or trajectories:

- How do we characterize learning trajectories as tools to clarify terms and focus our work?
- See each LT as:
  - a design tool
  - an analytical tool
  - an object to be modified, extended
- We need to generate alternative LTs to challenge the comprehensive nature of any LT for its domain

- What are the terms in use within LT and LP research that should be clarified and formalized?