CAREER: Mathematically Captivating Learning Experiences (MCLE) Project

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Project Goals

Our goal is to explore how the design of mathematical content of high school lessons can inspire student curiosity, wonder, and excitement, and overall shift student attitudes toward mathematics.

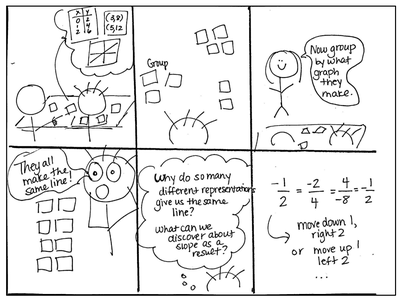
Working with six experienced teachers, at 3 high schools in the Boston area, we have designed and tested 17 *mathematically captivating lesson experiences* (MCLEs).

Design Approach

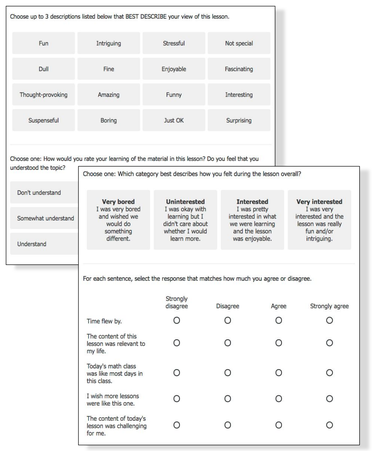
The MCLEs were designed using the mathematical story framework: an approach that interprets mathematical sequences (i.e., lessons) as a form of narrative for how it captures attention, inspires questions, and offers surprises (Dietiker, 2013, 2015).

So far, the MCLEs went through two design-and-test cycles in which we observe and film the lesson enactments. To test the MCLEs, we give students a post-lesson survey. We also interview between 2-5 students about their experiences in the lesson.

We use the same protocols for non-MCLEs (non-special lessons selected throughout the school year with the same students and teacher), and then compare and contrast student reported experiences in MCLEs with non-MCLEs.



Data Collection Tool - the Lesson Experience Survey


Choose up to 3 descriptions listed below that BEST DESCRIBE your view of this lesson.

Fun | Intriguing | Stressful | Not special
Dull | Fine | Enjoyable | Fascinating
Thought-provoking | Amazing | Funny | Interesting
Suspenseful | Boring | Just OK | Surprising

Choose one: How would you rate your learning of the material in this lesson? Do you feel that you understood the topic?

Don't understand | Somewhat understand | Understand

Choose one: Which category best describes how you felt during the lesson overall?

Very bored I was very bored and wished we would do something different.

Uninterested I was okay with learning but I didn't care about whether I would learn more.

Interested I was pretty interested in what we were learning and the lesson was enjoyable.

Very interested I was very interested and the lesson was really fun and/or intriguing.

For each sentence, select the response that matches how much you agree or disagree.

	Strongly disagree	Disagree	Agree	Strongly agree
Time flew by.	○	○	○	○
The content of this lesson was relevant to my life.	○	○	○	○
Today's math class was like most days in this class.	○	○	○	○
I wish more lessons were like this one.	○	○	○	○
The content of today's lesson was challenging for me.	○	○	○	○

Comparison Group of Lessons: Captivating vs. Non-Captivating

Lesson Topics and Interest Measures for Captivating and Non-captivating Lessons per Teacher

			Captivating Lessons			Non-Captivating Lessons		
Teacher	Course	Grades	Topic	Avg. Interest	Avg. Positive Descriptor	Topic	Avg. Interest	Avg. Positive Descriptor
Ms. Cherry	Alg 2H	10	Extraneous solutions	2.89	1.61	Systems of ineqs	2.71	1.12
Mr. Palm	AP Calc	12	Deriv. of exp. functions	2.82	2.00	Product rule	2.60	1.33
Ms. Elm	Math 3H	10, 11	Rational root theorem	3.19	2.05	Percent change	2.65	0.96
Ms. Spruce	Math 1	9	Equivalent linear functions	3.69	2.62	Graphing lin ineqs	2.86	1.79
Mr. Ash	Alg 2	11	Intro to imaginary numbers	2.80	1.50	Properties of logs	2.69	1.38
Ms. Willow	Alg 2	10	Logarithm identities	2.90	1.27	Inverse functions	2.36	1.09

	Weighted Average Positive Descriptors			Weighted Average Student Lesson Interest Measure		
Class	Non-MCLE	MCLE	Δ	Non-MCLE	MCLE	Δ
A1	1.87	2.10	+0.23	3.04	3.33	+0.29
B1	1.42	1.40	- 0.02	2.67	2.80	+0.13
B2	0.73	1.29	+0.56	2.33	2.62	+0.29
C1	1.13	1.45	+0.32	2.64	2.89	+0.25
C2	2.00	2.17	+0.17	2.81	2.94	+0.13
D1	1.55	1.50	- 0.05	2.74	2.77	+0.03
E1	1.34	1.74	+0.40	2.85	2.98	+0.21
F1	1.33	1.90	+0.57	2.60	2.92	+0.32

Proportion of Descriptors Selected by Students after Each Lesson, with Dominant Aesthetic Qualities for Each Lesson Highlighted

For MCLE Lessons, the dominant aesthetic qualities were **more positive and less negative overall** when compared with lessons that were not designed with the Mathematical Story Framework.

		Positive												Neutral		Negative		
Topic	Interest	Suspenseful	Amazing	Fascinating	Fun	Funny	Enjoyable	Satisfying	Intriguing	Surprising	Thought-provoking	Fine	Just OK	Frustrating	Not special	Dull	Boring
MCLEs																	
Equivalent linear functions	3.69	23%	8%	8%	62%	38%	62%			31%	31%	15%		23%			
Geometric transformations	3.43	14%	21%	14%	29%		64%	21%	36%	7%	36%	43%	14%				
Rational Root Theorem	3.19	5%	5%	24%	24%	5%	38%	19%	24%	14%	48%	57%	29%		10%		
Volume of solids	3.14	7%			64%	21%	57%	21%	21%	21%	21%	29%	21%	14%			
Polynomial division	2.94		22%	33%	50%	6%	33%	11%	17%	6%	39%	44%	22%	6%	6%		6%
Exploring tile patterns	2.93		7%	13%	13%		27%		27%	13%	33%	47%	60%	40%	13%	7%	
Logarithmic identities	2.90	20%		10%	20%	10%		10%	40%		40%	50%	40%	10%	20%	10%	20%
Extraneous solutions	2.89		6%	6%	11%		28%	6%	39%	6%	61%	61%	33%	17%	11%	6%	11%
Function horizontal translation	2.88			6%	24%		24%	29%	35%		12%	76%	35%	18%	18%	12%	12%
Derivatives of exponential fns.	2.82			9%	9%		27%	27%	64%		64%	45%	27%			9%	18%
Imaginary number introduction	2.80			20%	7%	7%	20%	7%	20%	33%	60%	47%	27%	20%	13%	13%	7%
Area of trapezoids	2.80		10%		10%		30%	20%	20%	10%	40%	60%	10%	20%	30%	10%	30%
Riemann sums and integrals intro	2.79				7%		21%	14%	29%	14%	50%	50%	36%	29%	21%	7%	21%
Introduction to the unit circle	2.77		5%		18%		27%	18%	27%	9%	41%	45%	50%	9%	14%	18%	18%
Logarithmic identities	2.73						27%	7%	33%	13%	47%	87%	40%		27%	7%	13%
Introduction to inverses	2.36		9%	18%	9%	18%	9%	9%	9%	9%	18%	55%	55%		45%	18%	18%
Non-MCLEs																	
Scatter plots and trend lines	3.18		12%	6%	53%	6%	59%		12%	29%	18%	59%	35%			6%	6%
Logarithm basics	3.04	4%	4%	8%	20%		44%	28%	16%	12%	32%	60%	44%		20%		8%
Graphing linear inequalities	2.86		14%	21%	29%	7%	64%	7%	7%		29%	64%	29%	7%	7%	7%	7%
Rational expressions (test prep)	2.81		25%	13%	19%	13%	50%	6%	25%	6%	44%	13%	31%	31%	6%	6%	13%
Exponential decay (with dice)	2.80	7%			27%		60%	20%	27%	7%	27%	73%	20%		20%	7%	7%
Systems of inequalities	2.71			6%			24%	24%	18%		41%	82%	59%	18%	12%	12%	6%
Props. of logs, solving exp. eqns.	2.69	13%		6%		6%	25%	6%	38%	6%	38%	63%	31%	13%	25%	19%	13%
Surface area of cones	2.67				25%		42%	25%	25%	8%	17%	42%	33%		25%	25%	33%
Percent change (review)	2.65				17%	4%	22%	13%	4%	9%	30%	65%	30%	13%	48%	43%	
Differentiating with Product Rule	2.60			7%	13%	7%	20%	7%	27%		53%	67%	53%	13%	20%	7%	7%
The unit circle	2.54			8%	8%	8%	15%	15%	15%	15%	31%	69%	62%		23%	15%	15%
Inverse functions	2.36			9%	18%			18%		9%	9%	45%	45%	27%	36%	27%	55%
Logarithm change of base rule	2.18				8%			8%		17%	25%	33%	50%	33%	42%	25%	33%

Characteristics of Captivating vs. Non-Captivating

	Captivating mean (sd)	Non-Captivating mean (sd)
Overall Structure of the Mathematical Plot		
Number of acts	13.67 (4.37)	13.00 (2.90)
Number of formulated questions	52.17 (6.65)	46.83 (10.76)
Characteristics of the Formulated Questions		
Mean arc length (in acts) *	**3.35 (0.56)**	**2.05 (0.83)**
Mean arc length as proportion of story*	**0.26 (0.05)**	**0.16 (0.03)**
Max arc length as proportion of story*	**0.89 (0.13)**	**0.57 (0.24)**
Proportion of extended story arcs*	**0.52 (0.09)**	**0.35 (0.09)**
Percent of story arcs with uncoded acts*	**0.34 (0.10)**	**0.16 (0.10)**
Percent of formulated question disclosed	0.70 (0.16)	0.69 (0.13)
Characteristics of the Acts		
Mean number of questions open per act*	**13.33 (3.46)**	**7.12 (1.51)**
Mean degree of inquiry per act*	**9.73 (2.92)**	**3.50 (1.62)**
Percent of acts in story arcs with codes	0.85 (0.05)	0.92 (0.06)
Mean number of coded questions per act*	**9.24 (2.63)**	**5.53 (1.28)**

Note: *Reflects a statistically significant difference (alpha < .05)

Relationship of Story Characteristics with Student Interest

Independent Variable	Intercept	Slope	Standardized slope	R^2
Characteristics of the Formulated Question				
Mean story arc length (in acts)	2.35	0.19	0.18	0.28
Mean arc length as proportion of story	**2.02**	**4.04**	**0.27**	**0.66**
Max arc length as proportion of story	2.35	0.68	0.17	0.26
Proportion of extended story arcs	2.23	1.41	0.17	0.27
Percent of story arcs with uncoded acts	2.48	1.45	0.19	0.34
Characteristics of the Acts				
Mean number of questions open per act	**2.18**	**0.07**	**0.29**	**0.66**
Mean inquiry per act	**2.38**	**0.07**	**0.28**	**0.72**
Mean number of coded questions per act	2.22	0.09	0.25	0.50

MCLEs Shifted the Quality of Questions

	MCLE mean (SD)	non-MCLE mean (SD)
Average Number of Questions per Lesson		
Number of teacher questions per lesson	39.2 (5)	30.3 (8.5)
Number of student questions per lesson	12.8 (7.3)	16.5 (6.8)
Proportion of Teacher Questions by Type		
EMMR *	23.8 (10.4)	1.3 (2.1)
GIPF*	27.2 (12.7)	50.7 (16)
SIPF*	0.4 (0.97)	5.9 (5.8)
PET	18.7 (16.2)	6.4 (5.2)
PSWP	29.6 (15.7)	35.6 (19.2)
IT	0.38 (0.9)	0.0 (0.0)
Proportion of Student Questions by Type		
EMMR *	25.8 (24.1)	0.06 (1.6)
GIPF	19.4 (13.5)	38.9 (17.8)
SIPF	6.2 (13.4)	11.6 (18)
PET	18.5 (14.1)	23.5 (15.8)
PSWP	23.4 (14.4)	22.4 (14.8)
IT	6.7 (10.3)	3.02 (5.7)

Note: *Reflects a statistically significant difference (alpha < .05)

EMMR (Exploring mathematical meanings and/or relationships); GIPF (Gathering information, Procedural and Factual); SIPF (Struggling with information, Procedural and Factual); PET (Probing for an explanation of thinking); PSWP (Problem Solving Without known Procedure); IT (Inserting Terminology)

Read more

Dietiker, L., Singh, R., Riling, M., & Nieves, H. I. (2020). What makes a mathematics lesson interesting to students? Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 391–399. https://doi.org/https://doi.org/10.51272/pmena.42.2020

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Sample Comparison of Mathematical Plots

Ms. Spruce's Captivating Mathematical Story - Linear Functions

KSy2o3p4M	1	2	3	4	5	6	7	8	9	10	11
1 Do Now question.	1543										
2 What are some mathematical ideas that could be represented by an equation?	24										
3 What can the parts of a linear equation tell us about a real-life situation?	154						54				
4 What is a linear function?		15									
5 What does the equation of a linear function look like?		15D									
6 What does m stand for in Y=mx+b?		15D									
7 What does b stand for in Y=mx+b?		15D									
8 Which groups of cards are similar?			1654	5	5	75	5	54	54	5	D
9 Why do certain cards go together?			154	5	5	5	5	4		54	5
10 Why are N and 1 a match?			154	5				5		5	D
11 How do you know that two cards aren't a match?			154	7							8
12 How does considering (0,6) help us find matches?			15	5		5				5	
13 What is the 'starting point' of a linear function?			15	5			5				
14 How does considering travel behavior help us find matches?			15	5	5		5				5
15 What might G be a match with?			25					5	5	5	D
16 What could be negative about a graph?			25	5							
17 How does a rate help find the y-intercept when the graph doesn't include it?			15								
18 If two cards have one point in common, does that mean they go together?			15			5				5	
19 How many cards can be in a group?				1	5		5	5			D
20 What pattern do the y-values follow in Table N?				25							
21 How do the patterns in the y-values help find a match?				25							
22 How do you calculate the rate between two points?				1		5					4
23 Will finding the rate of the graph give us matches?				1	5	5	5				5
24 How does A match with K?				25				5		5	D
25 If the values on two cards are different, could they match?				27		5		5			D
26 Why are E, M, and C in the same group?					15	7		5			D
27 Why are E, M, and F a better match than E, M, and C?						157					
28 Why are F and D a group?						157	5	5			D
29 Do all points need to match up for two cards to be similar?						15	5				5
30 Can one card match with two other cards?						15		4			D
31 Why does F go with D and E?						175	5	5			D
32 Why are D and C a group?							15	5			D
33 Why are E, F, D, and C one large group?							25	5			D
34 Can there be different slopes with the same points?							1				653
35 Why are M, E, and A a group?							154	5			D
36 What is the equation for card M?							145				
37 Where do we see 50 cents in the equation y=-1/2x+6?							15				
38 Why are F and B a group?							15	5		5	D
39 Why do all of the cards go together?								254	5	5	5
40 Why are L and F a group?								15		5	D
41 How can we use the transitive property with these cards?								145	4		D
42 Why are L, F, and B a group?								145		5	D
43 Why do J and H go together?								1		5	D
44 How do N, G, I, H, and J connect with the other cards?									15	5	D
45 Why do G and N go together?									15	5	D
46 Why do G and I match?									25	5	D
47 Why do G, N, and I all belong together?									15		D
48 Why does F belong with G, I, and N?									15		D
49 Why are G and F a match?									15		D
50 How can we show that all of the cards have a y-intercept of 6?										25D	
51 Does B have a y-intercept of (0, 6)?										15D	
52 How do you know what the y-intercept is if you can't see it?										15	
53 Why are E, B, and F a group?										1D	
54 What is the pattern that all of the cards have?											253
55 Do they all have the same slope?											25653
56 What is the slope of each of the cards?											15
57 What is slope?											143
58 Do two lines have the same slope even if their fractions are different?											153

Ms. Spruce's Non-Captivating Mathematical Story - Linear Inequalities

KSy2o4p4	1	2	3	4	5	6	7	8	9
1 What is the graph of y<-(2/3)x+4?	15	5D							
2 Is (-2 ,-5) a solution to y<-(2/3)x+4?	154		5D						
3 Do you shade below the line when you graph y<(-2/3)x+4?	25	D							
4 How do I use algebra to test if (-2, -5) is a solution to y<(-2/3)x+4?	24		45D						
5 How do you decide where to shade for the solution to a linear inequality?	15	154						5	
6 How do I know when a linear inequality will have a dashed or solid line?	15	15D							
7 Is the line dashed for y<(-2/3)x+4?	25	5D							
8 Do I have to make table for y<(-2/3)x+4?		2D							
9 What do we call the type of line that is not solid in an inequality?		15D							
10 Can you go about graphing y<(-2/3)x+4 in more than one way?		245D							
11 How do you know if (-2,-5) is a solution to y<(-2/3)x+4?			15D						
12 Is (-2,-5) in the shaded region of y<(-2/3)x+4?			15D						
13 Does having a graph make testing solutions of inequalities easier?			1D						
14 How do you algebraically test whether (-2,-5) is a solution of y<(-2/3)+4?			15D						
15 What is the right side of -5<(-2/3)(-2)+4 equal to?			16D						
16 What are the different types of forms that linear inequalities could be in?				15D					
17 How do you graph an equation in standard form, such as 2x+3y=12?				154					
18 What are we finding if we plug in zero for x and y in a linear inequality?				15D					
19 How do you graph 2x+3y>12?					1445	5	5		
20 What are the x- and y-intercepts of 2x+3y>12?					145	5D			
21 Does the sign (< vs ≤) have any impact on the inequality's graph?					2D				
22 If I plug in zero for x into 2x+3y>12, what would y need to be for it to equal 12?					15	5D			
23 How did S10 get (0,4) and (6,0) as the intercepts for 2x+3y>12?						15D			
24 How do you decide whether the line in 2x+3y>12 is solid or dashed?							15D		
25 Do you shade above or below the line in 2x+3y>12?							15		
26 What is a point that is a solution to 2x+3y>12?								15	
27 How did S14 know (4,6) is a solution to 2x+3y>12 by looking at the graph?								15D	
28 How do I algebraically prove that (4,6) is a solution to 2x+3y>12?								154	
29 What is a point that is not a solution to 2x+3y>12?								154	
30 Is (-2,-2) a solution to 2x+3y>12?								25	
31 How do I graph 4x+5y≤20?									15
32 What are the x- and y-intercepts of 4x+5y≤20?									15
33 When graphing 4x+5y≤20, is the line solid or dashed?									15
34 When graphing 4x+5y≤20, do you shade above or below the line?									15
35 What is a point that is a solution of 4x+5y≤20?									15