Numeric Relational Reasoning Learning Progressions: K-2 Protocols

MARCH 2, 2019
Research Council on Mathematics Learning

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This project is funded by the National Science Foundation, grant #1721100. Any opinions, findings, and conclusions or recommendations expressed in these materials are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.
Agenda

- **Measuring Early Mathematics Reasoning Skills (MMaRS)**
  - Project Background
  - Team Members
  - MMaRS Project Overview
- **Numerical Relational Reasoning (NRR)**
  - Learning Progression Development & Examples
  - Protocol Development & Examples
- **Preliminary Findings**
MMaRS Project Team
Research in Mathematics Education (RME)

• Leanne Ketterlin Geller, Ph.D.
• Lindsey Perry, Ph.D.
• Eloise Aniag Kuehnert, Ph.D.
• Qadeer Haider, Ph.D.
• Josh Geller, M.Ed.
• Marilea Jungman
• Toni Buttner
• Tina Barton
MMaRS Project Goals

• Develop and gather validity evidence for K-2 assessment tools measuring:

**Numeric Relational Reasoning**

\[ 4 + 1 = \square + 2 \quad \quad 22 + 8 - 8 = \square \]

**Spatial Reasoning**

<table>
<thead>
<tr>
<th>Question</th>
<th>Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Are the two objects the same?</td>
<td><img src="image1" alt="Diagram" /> <img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>Are the two objects the same?</td>
<td><img src="image3" alt="Diagram" /> <img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>How many cubes were used in this object?</td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td>How many squares were used to make this shape?</td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
</tbody>
</table>

(Perry, 2017, p. 92) [RCML Proceedings Paper]
Empirically Recovered Learning Progression
(van Rijn, Graf, & Deanne, 2014)

What level of evidence exists to confirm or disconfirm the ordering, content, and developmental appropriateness of the learning progressions?
Cognitive Interviews
Numeric Relational Reasoning

• *Relational reasoning*: “ability to recognize or derive meaningful relations between and among pieces of information that would otherwise be unrelated”
  
  (Dumas, Alexander, & Grossnickle, 2013, p. 392)

• *Numeric relational reasoning*: ability to mentally analyze relationships between numbers or expressions, often using knowledge of properties of operations, decomposition, and known facts
  

15 + 28 = ____ + 15

22 + 13 = 10 + ____

___ = 8 + 7
Numeric Relational Reasoning

Properties of Operations:
Operations have properties that can be used to solve equations.

Composition & Decomposition:
Any whole number can be composed in many different ways.

Relations:
Whole numbers have an order that can be used to make comparisons (less than or greater than).
Numeric Relational Reasoning

Relations

Whole numbers have an order that can be used to make comparisons (less than or greater than).

1. Comparison 7 skills
2. Ordinality 2 skills
3. Transitivity 6 skills
4. Representations 6 skills

Composition & Decomposition

Any whole number can be composed in many different ways.

5. Composition 5 skills
6. Decomposition 6 skills
7. Applying and Representing Composition and Decomposition 5 skills

Properties of Operations

Operations have properties that can be used to solve equations.

8. Equivalence of Quantity and Number 7 skills
9. Equal Sign as a Relational Symbol 4 skills
10. Maintaining Equality 4 skills
11. Solving for Unknown Values 7 skills
### NRR Core Concept Example

#### Grade Band

<table>
<thead>
<tr>
<th>Code</th>
<th>Kindergarten</th>
<th>Grade 1</th>
<th>Grade 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRR.B.5.a.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRR.B.5.b.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRR.B.5.c.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRR.B.5.d.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRR.B.5.e.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Core Concept

- **NRR.B.5.a.** Compose a **number** with single objects.

- **NRR.B.5.b.** Compose a **number** with **two** parts.

- **NRR.B.5.c.** Compose a **number** with **three or more** parts.

- **NRR.B.5.d.** Compose a **number** with two or more parts using **different number combinations**.

- **NRR.B.5.e.** Compose a **number** with two or more parts using **concepts of place value**.

#### Essentialized Skill Statements

- **World Changers Shaped Here**
- **SMU**
## Participants & Sampling

*Participant sample by school, grade, and support level as identified by classroom teacher.*

<table>
<thead>
<tr>
<th>Support Level</th>
<th>Kindergarten</th>
<th>First Grade</th>
<th>Second Grade</th>
<th>Third Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>B</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>School</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
## 5. Composition

### NRR.B.5.a. (single object)
- Give child a pile of the same color counters.
- Document child’s response (verbal and/or actions)
- Allow children to come up with a couple of numbers if they choose. However, once they have given a few responses, move to the next task. You can say something like this:

### Actions

<table>
<thead>
<tr>
<th><strong>NRR.B.5.a.</strong> [single object]</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Give child a pile of the same color counters.</td>
</tr>
<tr>
<td>- Document child’s response (verbal and/or actions)</td>
</tr>
<tr>
<td>- Allow children to come up with a couple of numbers if they choose. However, once they have given a few responses, move to the next task. You can say something like this:</td>
</tr>
</tbody>
</table>

### Questions

What different numbers could you make using these counters?

You can make a lot of different values! Let’s do the same thing with numbers.

### # Range

<table>
<thead>
<tr>
<th># Range</th>
<th>Number in pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>5</td>
</tr>
<tr>
<td>0-10</td>
<td>10</td>
</tr>
<tr>
<td>0-19</td>
<td>15</td>
</tr>
<tr>
<td>0-50</td>
<td>20</td>
</tr>
</tbody>
</table>

### Student Responses

Describe child’s verbal response and/or actions:

_0-5_  
_0-10_  
_0-19_  
_0-50_

---

### Can you show me what you are doing in your head?

Can you show me what you are doing in your head?

<table>
<thead>
<tr>
<th><strong># Range</strong></th>
<th><strong>Question</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>Can you show me 3 using these counters?</td>
</tr>
<tr>
<td>0-10</td>
<td>Can you show me 8 using these counters?</td>
</tr>
<tr>
<td>0-19</td>
<td>Can you show me 11 using these counters?</td>
</tr>
<tr>
<td>0-50</td>
<td>Can you show me 15 using these counters?</td>
</tr>
</tbody>
</table>

Describe child’s verbal response and/or actions:

_0-5_  
_0-10_  
_0-19_  
_0-50_

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### [Probing thinking/reasoning]

Regardless if child is correct, ask:

*Are there other numbers you can make using these counters? How do you know that there are ___ counters?*

Describe child’s verbal response and/or actions:

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### [Initial thinking & scaffolding]

Wait 5-10 seconds. If child is not responding, check if a tool would be helpful or show an example.

- If child is still unresponsive, then ask child to create a specific number.
- Highlight question(s) asked.
- Skip this section if child begins to respond without prompting.

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### [Unscripted questions here]
Selected References


