Numeric Relational Reasoning Learning Progressions: K-2 Protocols

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Agenda

- **Ma**easuring Early **Ma**thematics **R**easoning **S**kills (MMaRS)
  - Project Background
  - Team Members
  - MMaRS Project Overview
- Numerical Relational Reasoning (NRR)
  - Learning Progression Development & Examples
  - Protocol Development & Examples
- Preliminary Findings

MMaRS Project Team
Research in Mathematics Education (RME)

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- Lindsey Perry, Ph.D.
- Eloise Aniag Kuehnert, Ph.D.
- Qadeer Haider, Ph.D.
- Josh Geller, M.Ed.
- Marilea Jungman
- Toni Buttner
- Tina Barton

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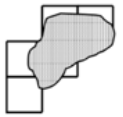
MMaRS Project Goals

- Develop and gather validity evidence for K-2 assessment tools measuring:

Numeric Relational Reasoning

$4 + 1 = \square + 2$ $22 + 8 - 8 = \square$

Spatial Reasoning

Are the two objects the same?	
Are the two objects the same?	
How many cubes were used in this object?	
How many squares were used to make this shape?	

(Perry, 2017, p. 92) [RCML Proceedings Paper]

Research Question

Empirically Recovered Learning Progression

(van Rijn, Graf, & Deanne, 2014)

What level of evidence exists to confirm or disconfirm the ordering, content, and developmental appropriateness of the learning progressions?

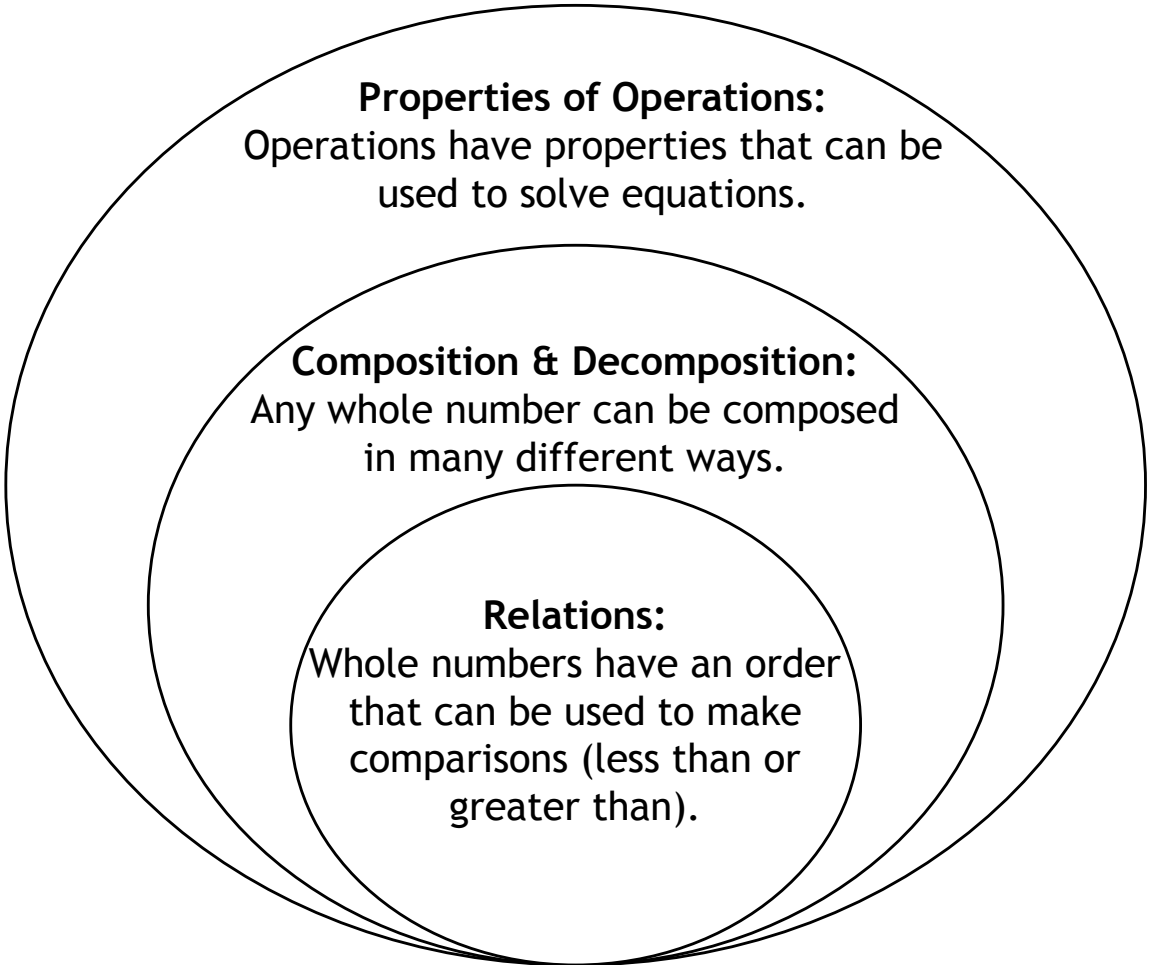
Cognitive Interviews

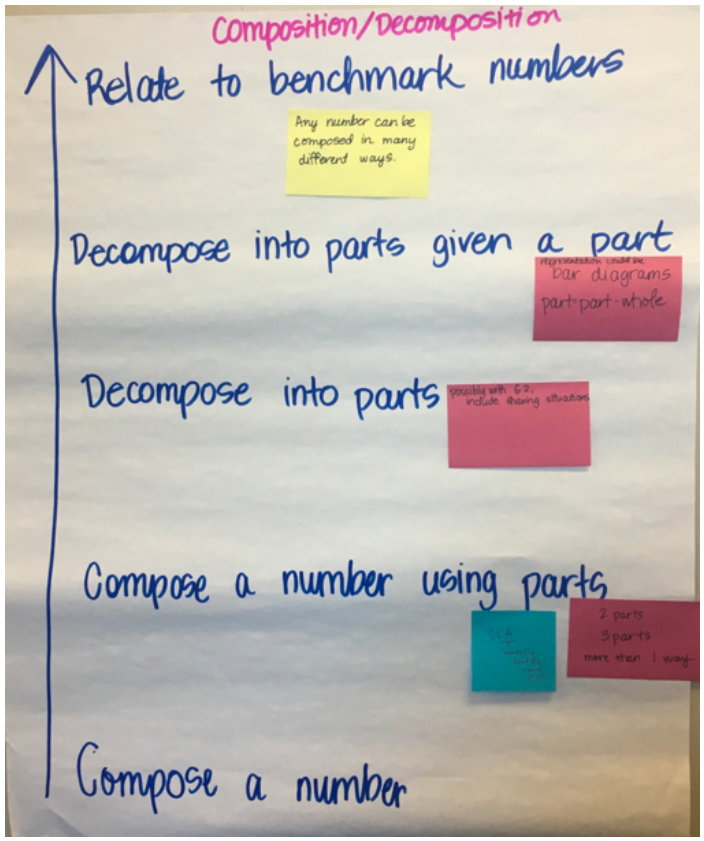
Numeric Relational Reasoning

- *Relational reasoning*: "ability to recognize or derive meaningful relations between and among pieces of information that would otherwise be unrelated"
  (Dumas, Alexander, & Grossnickle, 2013, p. 392)
- *Numeric relational reasoning*: ability to mentally analyze relationships between numbers or expressions, often using knowledge of properties of operations, decomposition, and known facts (Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Carpenter, Franke, & Levi, 2003; Farrington-Flint, Canobi, Wood, & Faulkner, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007)

15 + 28 = ___ + 15 22 + 13 = 10 + ____ ___ = 8 + 7

Numeric Relational Reasoning





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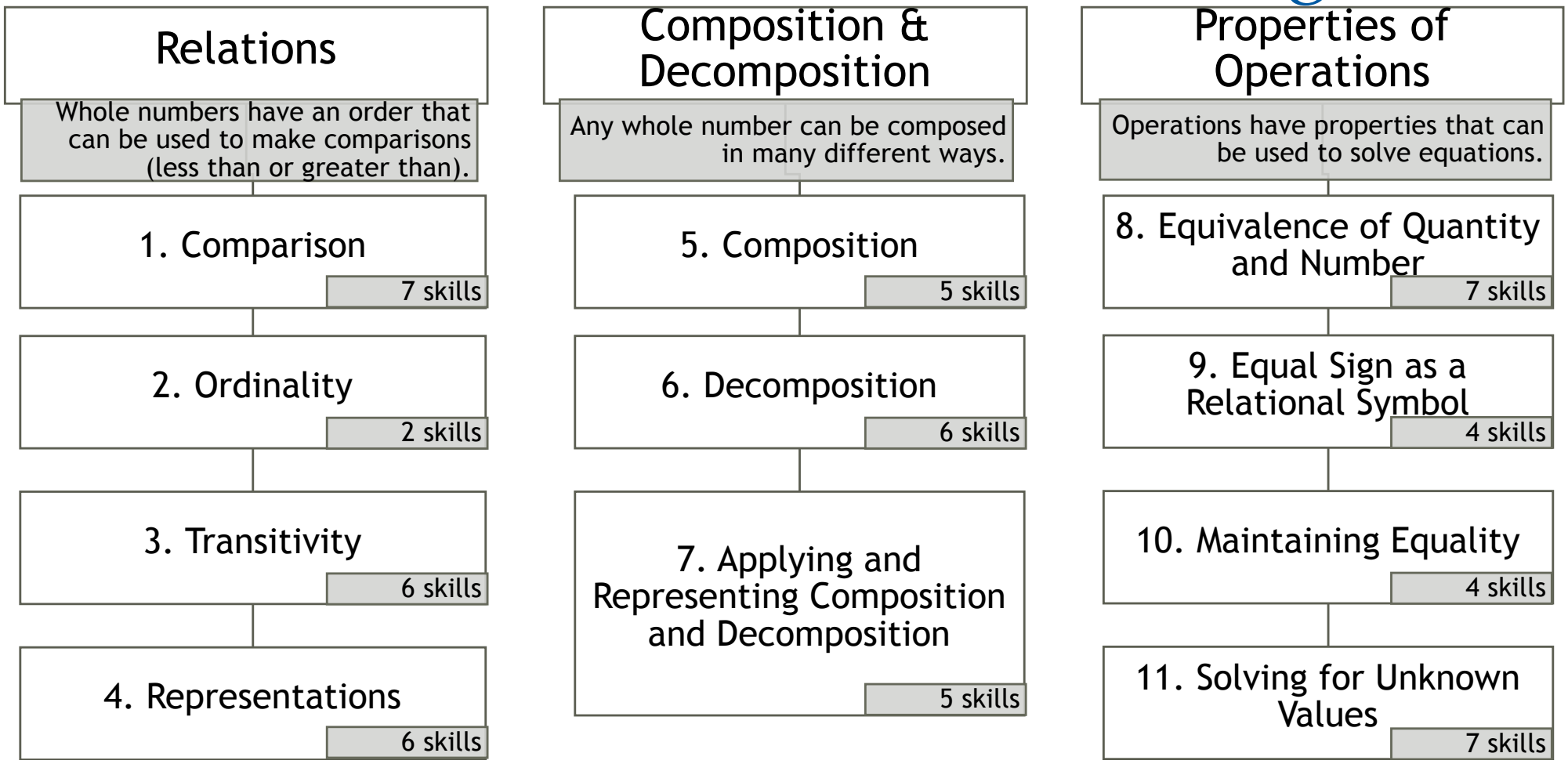
Numeric Relational Reasoning



NRR Core Concept Example

Grade Band

Core Concept

5. Composition									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a **number** with **single** objects.								
NRR.B.5.b.		Compose a **number** with **two** parts.							
NRR.B.5.c.			Compose a **number** with **three or more** parts.						
NRR.B.5.d.			Compose a **number** with two or more parts using **different number combinations**.						
NRR.B.5.e.			Compose a **number** with two or more parts using **concepts of place value**.						

Essentialized Skill Statements

Participants & Sampling

Participant sample by school, grade, and support level as identified by classroom teacher.

	Kindergarten			First Grade			Second Grade			Third Grade		
Support Level												
	F	B	T	F	B	T	F	B	T	F	B	T
School												
A	1	2	1	1	2	2	3	2	2	1	0	0
B	3	2	1	0	1	0	1	2	0	0	0	0
C	0	0	0	3	1	0	0	0	0	1	0	0
Total	4	4	2	4	4	2	4	4	2	2	0	0

5. Composition

Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
NRR.B.5.a.	Compose a **number** with **single** objects.								

5. Composition			
Skill	**Actions**	**Questions**	**Student Responses**
NRR.B.5.a. [single object]	• Give child a pile of the same color counters. • Document child's response (verbal and/or actions) --- • [Allow children to come up with a couple of numbers if they choose. However, once they have given a few responses, move to the next task. You can say something like this:	**# Range** / **Number in pile** 0-5 ▲ — *5* 0-10 ♦ — *10* 0-19 ♥ — *15* 0-50 ♠ — *20* *What different numbers could you make using these counters?* --- *You can make a lot of different values! Let's do the same thing with numbers.*	**# Range** / **Child's Numbers** / **Initially Correct** / **Self Corrects** 0-5 ▲ 0-10 ♦ 0-19 ♥ 0-50 ♠ **Describe child's verbal response and/or actions:**
[initial thinking & scaffolding]	• Wait 7-10 seconds. If child is not responding, check if a tool would be helpful or show an example. • If child is still unresponsive, then ask child to create a specific number. • Highlight question(s) asked. • Skip this section if child begins to respond without prompting.	*Can you show me what you are doing in your head?* **# Range** / **Question** 0-5 ▲ — *Can you show me 3 using these counters?* 0-10 ♦ — *Can you show me 8 using these counters?* 0-19 ♥ — *Can you show me 11 using these counters?* 0-50 ♠ — *Can you show me 15 using these counters?*	**# Range** / **Child's Number** / **Initially Correct** / **Self Corrects** 0-5 ▲ 0-10 ♦ 0-19 ♥ 0-50 ♠
[Probing thinking/ reasoning]	• Regardless if child is correct, ask:	**Are there other numbers you can make using these counters?* *How do you know that there are __ counters?*	**Describe child's verbal response and/or actions:**
	• Record unscripted questions here.		**Describe child's verbal response and/or actions:**

Selected References

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