Assessment of K-2 Numeric Relational Reasoning Skills: Strengths and Limitations of Item Types and Formats

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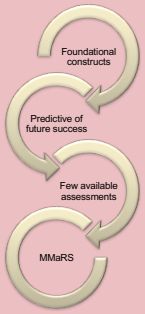
Measuring Early Mathematical Reasoning Skills (MMaRS)

Project purpose

- To develop and gather validity evidence for mathematics assessments for Grades K-2 that measure students' abilities in numeric relational reasoning and spatial reasoning that can be used to assist teachers in instructional decision making.

Why is MMaRS important?

- Numeric relational reasoning and spatial reasoning are foundational mathematical constructs that support the development of other mathematics skills, such as algebraic thinking (Carpenter et al., 2003), place value (Cheng & Mix, 2012), problem solving (Battista, 1990), and number line knowledge (Gunderson et al., 2012), among others.
- Improving students' numeric relational reasoning and spatial reasoning abilities by intervening based on results from the assessments may positively impact mathematics education and STEM outcomes over time.

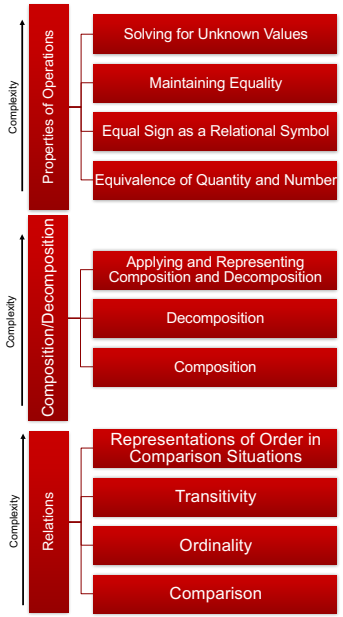


Numeric Relational Reasoning

General Definitions

- Numeric relational reasoning is often defined as the ability to recognize and analyze relationships between numbers or expressions (Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Jacobs, Franke, Carpenter, Levi, & Battey, 2007).
- Students using numeric relational reasoning can use known facts to derive new facts (e.g., using 5 + 5 to solve 6 + 5), solve complex equations by transforming expressions using composition and properties of operations (6 + 5 = ☐ + 4), and recognize when calculations aren't necessary (5 + 8 = ☐ + 5).
- Three primary components are used when reasoning relationally:
  1. **Relations**
  2. **Composition/ Decomposition**
  3. **Properties of Operations**

Learning Progressions



Purpose of Current Study

The purpose of this literature search was to

(1) identify what instruments currently exist that assess K-2 students' numeric relational reasoning competence,
(2) determine different item types and formats for assessing this construct, and
(3) determine the depth of reasoning needed by students for each item type.

Methods

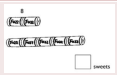
An assessment inventory was created to catalog current assessments and items that test K-2 students' numeric relational reasoning abilities. This work builds on the inventory created by Perry (2016). The inventory was created by:

- searching various education research and test review databases
- examining articles on numeric relational reasoning to find instruments that were used in the research

An assessment was included in the inventory if any items within the assessment measured Numeric Relational Reasoning components. Only formalized assessments were included. Instruments created just for limited research studies were not included.

Assessment Inventory: Numeric Relational Reasoning

An item inventory was created to catalog current assessments and items that test K-2 students' numeric relational reasoning.

Assessment	Author(s)	Example Items
Assessment of Mathematical Equivalence Knowledge	Rittle-Johnson, et al., 2011	67 + 84 = ___ + 83 5 + __ = 6 + 2 True or false: 89 + 44 = 87 + 46
Open Equations	Powell, 2007, 2015	___ + 4 = 5 + 2 6 – ___ = 7 – 3
Preschool Early Numeracy Skills (PENS)	Purpura, 2010; Purpura & Lonigan, 2013	• "Children were presented with a set of objects on the table and informed verbally of the quantity. The objects were hidden and more objects were either added or subtracted from the initial set. The new set was presented and the child was asked to identify how many objects were either added or subtracted from the initial set." (Purpura & Lonigan, 2013, p. 192) • "Most – 5, 3, 8" -- dots or pictures, (Purpura, 2010, p. 99)
Quantitative Reasoning Test	Nunes et al., 2015	• "The children see a row of bricks and count with the tester to establish the number of bricks in the row (in the scheme, represented by (a); the row is then covered so that counting is no longer possible but the endings of the row are visible; the tester adds a number of bricks to the row (b) and either subtracts the same number or the number plus or minus 1. The children are asked how many bricks are now on the table. In the second set of items, the children see a picture of a box and are told that there are n objects inside (for example, 8 books). The number of objects is also written on the box. The children are then shown the next picture, which is laid to the right of the first one, to form a sequence, and are told how many objects were added to the box; a third picture is then added and the tester says how many objects were taken out. The child is asked how many books are now in the box."
Research-Based Early Maths Assessment (REMA)	Clements et al., 2008	• "I can make 6 with 3 and 3…Show me a different way to make 6." (p. 465) • "Which is closer to 6, 9 or 4?" (p. 468)
Test of Mathematical Reasoning	Nunes et al., 2001; Nunes et al., 2012	"The roll on top has 8 sweets. How many sweets do you think there are in the big roll below?" (Nunes et al., 2012, p. 143) 
Test of Pre-algebraic Reasoning, Subtest: Equations	Fuchs, Seethaler, & Powell, 2009; Powell & Fuchs, 2014	1 + 5 = 4 + X 5 = Y + 4 8 – X = 3 + 3

Results

Assessments that Currently Exist

- We found few assessments that exist to test students' numeric relational reasoning abilities in K-2.
- Many of the assessments tested numeric relational reasoning skills within a larger numeracy assessment.
- Additional instruments were created for specific research projects but were not designed for teachers' use, such as Powell (2015), Sherman & Bisanz (2009), Howell & Kemp (2010), Bryant et al. (1999), and Farrington-Flint et al. (2007).

Common Item Types

- Items with concrete or visual representations
- Word problems
- Items with abstract notation only

Reasoning within Items

- Many items could be solved using numeric relational reasoning. However, not all of the items allowed students' reasoning to be visible. Therefore, conclusions specific to students' numeric relational reasoning may be inaccurate.
- Future assessments should focus on making students' reasoning visible to strengthen the validity of the claims about students' numeric relational reasoning abilities.

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References available upon request.

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