Theoretical Background

Research Questions

1. What language do secondary (middle and high school) mathematics teachers use to describe and explain routines commonly used in algebra? To what extent does this discourse contain explicit descriptions of procedures, concepts, and relationships?

2. What changes in algebraic discourse, in general, and with respect to specific concepts and procedures, occur in teachers who participate in an intensive, content-focused professional development program?

3. What opportunities can teacher-designed lessons, created in the context of such a professional development program, offer for the development of students’ algebraic reasoning and discourse?

Research Design

To investigate the language that secondary mathematics teachers use to describe and explain algebraic concepts, we have developed a Survey of Algebraic Language and Reasoning (SALR) to be completed by teachers both before and after participation in the PD program. In developing the SALR, we focused on key algebraic concepts in the Texas Essential Knowledge and Skills (TEKS) for Grade 7 Math, Grade 8 Math, and Algebra I and created 15 items that present realistic scenarios that arise in the teaching of these concepts, often including hypothetical student work based on known conceptions that algebra learners exhibit.

In analyzing teachers’ responses to the SALR, we attended to respondents’ algebraic language and discourse at three levels:

1. Respondents’ use of words and mediators (symbols and visual representations) to refer to mathematical objects. Specifically, we observed whether respondents’ questions emphasized algebraic objects and their relationships (e.g., “Because 2x + 3 is equal to 17, we know that 2x must be equal to 14”) or stories about personalized actions on symbols (e.g., “We move the 3 to the right side of the equation”).

2. To learn mathematical objects and respondents’ meta-rules for endorsing or rejecting these narratives. Specifically, we consider whether participants endorse narratives based on mathematical definitions and deductive reasoning or based on textual consistency with other endorsed narratives.

3. Respondents’ use of discourse routines (such as problem-solving procedures) and descriptions of these routines. Specifically, we consider the extent to which participants analyze routines as chains of deductive reasoning that generate endorsed narratives about mathematical objects.

To investigate the changes in discourse that occur in a PD program focused on algebraic reasoning and discourse, we plan to capture video recordings of teacher participants as they work on a variety of activities, including:

- Mathematical problem-solving activities that involve algebraic reasoning
- Reflections on the algebraic reasoning opportunities inherent in these mathematical tasks
- Analysis and critique of available curriculum materials (such as instructional lessons and worksheets) based on the opportunities they afford for students to engage in algebraic discourse
- Design and planning of lessons for the 2023–2024 school year that provide rich opportunities for development of students’ algebraic reasoning

We are interested in the algebraic language that teachers use to explain and justify their own mathematical thinking as well as the language they use to describe opportunities for algebraic reasoning for their students.

To investigate the opportunities for algebraic reasoning and discourse that occur in lessons collaboratively planned by teacher participants, we plan to observe classes in which teachers implement these lessons and take field notes and transcripts to capture the language that teachers use as they orchestrate these lessons (specifically, how this language invites students into mathematical inquiry or performance-based routines). We will also analyze the language that students use as they explore problem, explain their reasoning, and justify findings so that we can compare the algebraic reasoning opportunities as hypothesized by teacher participants during the planning process with these opportunities as enacted by teachers and students in the classroom.

Research Background

Reframing language for teaching secondary algebra (Weinert, 2007) provides a perspective on how teachers' linguistic actions can influence students' understandings of algebraic concepts. This perspective highlights the importance of attending to students' cultural and knowledge assets in order to leverage their engagement in algebraic content.

By comparing the SALR results with those from previous studies, we can draw insights into the extent to which teachers' discourse about equation-solving tends to contain elements of both approaches. Figures 1 through 3 show examples of teacher responses to a question about an equation-solving process that illustrate how these discourses can interchange.

Implications

Rather than seeing algebraic discourse about equation-solving as a superior alternative to extractive discourse, we see the two as distinct approaches to equation-solving, each with unique benefits. We hypothesize that algebraic discourse allows a person to investigate a particular "variable" that occur in the process of equation-solving, such as degenerate cases and extraneous solutions. On the other hand, extractive discourse seems better suited to talking about each step in organizing the steps of an equation-solving process (e.g., "Isolate the variable "). Our goal is to illustrate benefits of both approaches in algebra teaching and learning.

We hope, through our PD program, to set the stage for algebraic discourse about equation-solving (as well as conceptually grounded discourse about other algebraic topics) in the classroom so that students can learn to use routines with greater awareness of their underlying assumptions and meanings of individual steps.

References

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Stohl, A. (2005). Doing wrong with words: Understanding the degree of authenticity of the hypothetical student responses. We summarize teachers’ responses to these questions here:

- Question 1: On a scale of 0 to 5, with 0 meaning “totally unrealistic” and 5 meaning “I have seen exactly these student conceptions in my own classes,” how authentic did these items feel? (Mean: 4.5)
- Question 2: On a scale of 0 to 5, with 0 meaning “totally unrealistic” and 5 meaning “extremely relevant,” how relevant did the topics of the questions feel to your teaching of algebra content to students? (Mean: 4.4)
- Question 3: On a scale of 0 to 5, with 0 meaning “totally unrealistic” and 5 meaning “I have seen exactly these student conceptions in my own classes,” how authentic did these items feel? (Mean: 4.5)

We interpret these results as evidence of the content validity of the survey as a research tool for investigating the reasoning and discourse that teachers have at their disposal and they explain concepts and questions in algebra.

In our preliminary analysis of the completed surveys, we have identified distinct approaches to explaining processes for solving equations, inequalities, and systems, which we call extractive and inferential discourses. Extractive discourse tends to treat equation-solving as a process of manipulating symbols in order to isolate a set of values for a variable (or variables), whereas inferential discourse tends to treat an equation-solving process as a sequence of steps, starting with an assumption that a solution exists, that deductively motivates and produces the solution. We have found that teachers’ discourse about equation-solving tends to contain elements of both approaches. Figures 1 through 3 show examples of teacher responses to a question about an equation-solving process that illustrate how these discourses can interchange.