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Overview of the Project

This five-year project (2014-2019) aims to identify, from a cross-cultural perspective, essential algebraic knowledge for teaching (AKT) that fosters students’ algebraic thinking in elementary school. Focusing on two fundamental mathematical ideas that are early algebra topics – inverse relations and properties of operations (Common Core State Standards Initiative, 2010) – this study explores AKT based on integrated insights of U.S. and Chinese expert teachers’ classroom performance. This study is innovative because it is among the very first to seek AKT focusing on fundamental mathematical ideas from a cross-cultural perspective. The conceptual framework for identifying AKT is aligned with high-quality cognitive research recommendations on worked examples, representations, and deep questions (Pashler et al., 2007). It is expected that the identified AKT along with these aspects will contribute to students’ deep understanding of fundamental mathematical ideas and thus algebraic readiness.

Objectives of the Project

Objective #1: Identify AKT that facilitates algebraic thinking and develop preliminary findings into teaching materials (Ys1-3).
Objective #2: Disseminate preliminary findings and refine research-based teaching materials based on evaluative data (Ys3-4).
Objective #3: Integrate research with education through course development at Temple and teacher outreach in Philadelphia (Ys3-5).

Participants

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<tbody>
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<td>Inverse Relations</td>
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<td>Properties of Operations</td>
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<td>Both topics</td>
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Materials and Instruments

Each teacher taught 4 lessons on either inverse relations or the basic properties of operations based on teachers’ existing textbooks. In years 1-2, the US textbooks included Investigations, Go Math, and My Math. However, in year 4, the school district adopted two new textbook series: enVisionmath2.0 and Math Expressions. Investigations remained in some classrooms. The Chinese textbook series was Jiang Su Educational Press (JSEP), which was not changed.

Coding

Videos: A 0-2 scale was used to rate each lesson in terms of three dimensions aligned with the IES recommendations (Pashler et al., 2007), which were detailed into six sub-categories:

- Worked examples: Introducing worked examples with problem-solving exercises (example + practice)
- Representations: Making connections between concrete and abstract representations (concrete + abstract)
- Deep questions: Asking deep questions to elicit student self-explanations (question + explanation)

Student Tests: The pre- and post-tests were also rated for both math topics.

Results: Identifying AKT (Ys1-3)

A video analysis of 132 US and Chinese lessons on inverse relations and the properties of operations indicates instructional differences. Below shows the mean score differences (out of 2):

- Inverse Relations: US 84, China 78
- Properties of Operations: US 88, China 92

Based on qualitative and quantitative analyses, we concluded with three AKT components:

- Worked example: Unpacking one worked example sufficiently to illustrate the targeted concept big idea. It is not effective to teach many repetitive examples without making the big idea explicit.
- Representation: Situating the new teaching in a real-world context which should be gradually faded out into abstract. It is not effective to use concrete representations to find computational answers. Rather, these tools should be used to illustrate quantitative relationships (Ding, in press; Ding, Chen, & Hassler, 2013).
- Deep question: Asking deep follow-up questions that targeted the meaning of operations and quantitative relationships. One type of deep questions is comparison with follow-ups. Illustration of these results include 25 annotated video clips that contain merits of various aspects.

Partial Results on the Impact of AKT (Y1)

Using the Y1 teaching and learning data on inverse relations, we examined whether instruction aligned with IES recommendations predicts student learning of early algebra in elementary classrooms. Instructional quality was determined in an opportunity-propensity analysis (Ding, Byrnes, Barnett, and Hassler, 2018).

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Results of Disseminating/Refining AKT (Ys3-4)

To disseminate and refine our findings, we conducted an one-month US-China online video-forum through the platforms of Youtube (US) and YouKu (China). Teachers in both countries watched and commented 25 annotated video clips with pedagogical merits, followed by 20-hour summer workshops. This serves as a project intervention, which led to teachers’ re-teaching of the targeted lessons in Y4 (Ding Manfredonia, & Luo, 2018).

An analysis of teachers’ videos comments indicates:

- Teachers in both countries showed great interests and learning desire from watching international peers’ videos.
- US Teachers were particularly impressed by the depth of mathematical ideas children engaged in Chinese classrooms and teachers’ instructional approaches (e.g., concreteness fading, deep follow-up questions) to pursue such depth.
- Chinese Teachers were more interested in US relaxed classroom climate and teachers’ use of concrete representations to enable students to naturally explore ideas. They reflected why Chinese classrooms seemed to lack such atmosphere and whether the type of mathematical depth pursued in Chinese classrooms was necessary for students’ well-rounded growth.

These findings inform us that our identified AKT is feasible for practice. We are currently analyze Y4 videos to understand how teachers actually transform what they’ve learned to classrooms. We are also in the process of disseminating the findings to more teachers in the school districts through PD workshops.

For more information, please watch NSF2018 video showcase: http://stemtoral2018.videoshow.com/presentations/1143

References


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